1. Prove that if a function is differentiable at a number $x = a$, then it is also continuous there.

2. Find an equation for the tangent line drawn to the graph of $f(x) = x^2 - \dfrac{2}{x}$ at $x = -2$.

3. Compute each of the following limits.

a) $\displaystyle\lim_{x\to 0^-} \frac{\sin 3x}{x}$ b) $\displaystyle\lim_{x\to 6} \frac{x^2-6x}{x^2-36}$ c) $\displaystyle\lim_{x\to 5} \frac{x^2-25}{2-\sqrt{x-1}}$

4. Compute each of the following limits.

a) $\displaystyle\lim_{x\to -\infty} \left(-2x^5 + 8x^2\right)$

b) $\displaystyle\lim_{x\to \infty} \left(-2x^5 + 8x^2\right)$

c) $\displaystyle\lim_{x\to -\infty} \left(-2x^5 + 8x^6\right)$

d) $\displaystyle\lim_{x\to \infty} \left(-2x^5 + 8x^6\right)$

e) $\displaystyle\lim_{x\to -\infty} \frac{3x^2-1}{5x^2-3x+2}$

f) $\displaystyle\lim_{x\to -\infty} \frac{100x-1}{5x^2-3x+2}$

g) $\displaystyle\lim_{x\to -\infty} \log_2 x$

h) $\displaystyle\lim_{x\to \infty} \frac{2x^2+3x+1}{3x^2-5x+2}$

i) $\displaystyle\lim_{x\to \infty} \frac{-x^3+2x+1}{x-3}$

j) $\displaystyle\lim_{x\to -\infty} 2^x$

k) $\displaystyle\lim_{x\to \infty} \left(\log_2\left(x^2-5x+17\right)\right)$

l) $\displaystyle\lim_{x\to \infty} \frac{12+\log_7 3x}{15+\log_7 x}$

m) $\displaystyle\lim_{x\to \infty} \frac{2^{2x+5}}{3^{x-1}}$

n) $\displaystyle\lim_{x\to \infty} \frac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

o) $\displaystyle\lim_{x\to \infty} x\left(\frac{1}{3} - \frac{1}{3-\frac{1}{x}}\right)$

p) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{4-\frac{1}{x}}-2}{\frac{1}{x}}$

q) $\displaystyle\lim_{x\to -\infty} \frac{\cos x - 2}{x^3+1}$

5. Differentiate each of the following, using the definition of the derivative.

a) $f(x) = \sqrt{2x-1}$ b) $f(x) = \dfrac{1}{x^2-1}$ c) $f(x) = \sqrt{1-x^2}$

6. Differentiate each of the following functions.

a) $f(x) = (\sin x - \cos x)\sqrt{x^5}$ b) $f(y) = \dfrac{1}{y^2} + \dfrac{1}{y} + \dfrac{1}{\sqrt{y}} + \sqrt{y}$ c) $f(x) = \dfrac{\cos x - \sqrt{x}}{x^2}$

7. Find the point on the line $y = \frac{1}{2}x - 2$ that is closest to the point $(3, 1)$.

8. We know the following things about a function f. $f'(x) = 20x^3 - 3$ and $f(-1) = 16$. Find f.

9. Find the value of c for which the following is true: the tangent line drawn to the graph of $y = \dfrac{1}{x}$ has $x-$intercept $(6, 0)$.

10. Find the equation for the function f if we know that f is a polynomial of degree 3, and has a relative minimum at $x = -2$ and a relative maximum at $x = 3$.

11. Prove that the function given is one-to-one.

a) $f(x) = \dfrac{2x-3}{5x+1}$ b) $f(x) = 3x^5 - 50x^3 + 390x - 1200$

12. Find the value of a so that the line $y = 2x$ is a tangent line to the parabola $y = ax^2 + 5$.

13. Let $P(x, y)$ be a point on the graph of $y = 4 - x^2$ with $0 \le x \le 2$. Let $PQRS$ be a rectangle with one side on the $x-$axis and two vertices on the graph of $y = 4 - x^2$. Find the exact value of the greatest possible area of such a rectangle.

14. We would like to construct an open box with a square base. We are to use of $600\,\text{ft}^2$ of material. What is the greatest volume for such a box possible?

15. Find a third degree polynomial $P(x)$ such that $P(0) = -5$, $P'(0) = 3$, $P''(0) = -6$ and $P'''(0) = 60$.

16. We want to construct a cylindrical soda can with volume $100\,\text{cm}^3$. If the material for the side costs 2 cents per cm^2, and the material for the top and bottom costs 5 cents per cm^2, what dimensions would guarantee a minimal cost of producing such a can? What would be the minimal cost?

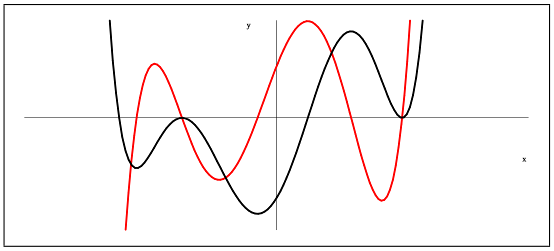
17. Find the $x-$coordinate of all relative maximums and minimums of each of the functions given below.

    a) $f(x) = x^3(5x-2)$ b) $f(x) = 2x + \dfrac{18}{x}$ c) $f(x) = x^3 + 3x^2 - 24x + 24$

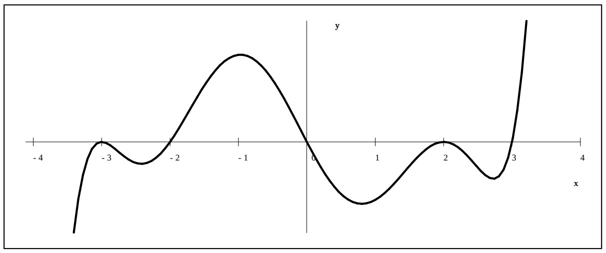
18. Find all relative and absolute maximums and minimums for the $f(x) = 6x^5 - 15x^4 + 5x^6 + 60$ on $[-3, 3]$. Sketch the graph of both f and f' on this domain.

19. Prove that the function $f(x) = \sin x - x$ does not have any relative minimums or maximums.

20. The graph below shows a function f and its first derivative, f'. Which is which?



21. The graph below shows f', the first derivative of a function f.
    a) Find all values of x for which the function f has a local maximum at x.
    b) Find all values of x for which the function f has a local minimum at x.



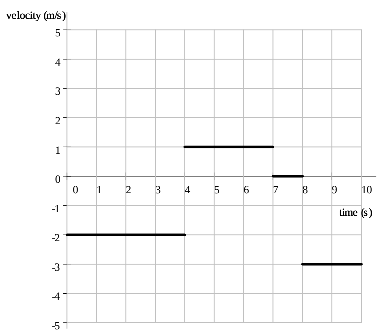
22. We shoot a small object upward, from the top of a tower. The acceleration function of the object is $a(t) = -10$. (Location is measured in meters, velocity in $\frac{\text{m}}{\text{s}}$, acceleration in $\frac{\text{m}}{\text{s}^2}$.)

    a) Given that $v(0) = 160$, find $v(t)$, the velocity function of the object.

    b) Given that $h(0) = 525$, find $h(t)$, the location function of the object.

    c) Find the maximum height that the object reaches.

23. The picture shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward.).
    a) Suppose that the object starts at a height of 5 m. Graph its location function.
    b) Suppose that the object starts at a height of 9 m. Graph its location function.



24. What is the greatest volume of a right circular cone that can be written into a sphere with radius R?

Answers

1.) See handout 2.) $y = -\frac{7}{2}x - 2$ 3.) a) 3 b) $\frac{1}{2}$ c) -40

4.) a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) ∞ h) $\frac{2}{3}$ i) $-\infty$ j) 0

k) ∞ l) 1 m) ∞ n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0 5.) see handout

6.) a) $f'(x) = \frac{5}{2}(\sin x - \cos x)\, x\sqrt{x} + (\cos x + \sin x)\, x^2\sqrt{x}$ b) $f'(y) = -\frac{2}{y^3} - \frac{1}{y^2} - \frac{1}{2y\sqrt{y}} + \frac{1}{2\sqrt{y}}$

c) $f'(x) = -\frac{2}{x^3}(\cos x - \sqrt{x}) + \frac{1}{x^2}\left(-\sin x - \frac{1}{2\sqrt{x}}\right)$ 7.) $\left(\frac{18}{5}, -\frac{1}{5}\right)$

8.) $f(x) = 5x^4 - 3x + 8$ 9.) 3 10.) Let $a > 0$ and $c \in \mathbb{R}$ any number $f(x) = -a\left(-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x\right) + c$

11.) a) Write $y = \frac{2x-3}{5x+1}$ and solve for x: $x = \frac{y+3}{-5y+2}$ so for each y, there is a **unique** x for which the function takes that $y-$value.

b) $f'(x) = 15\left(\left(x^2 - 5\right)^2 + 1\right)$ is always positive, hence f is always strictly increasing. Thus one-to-one.

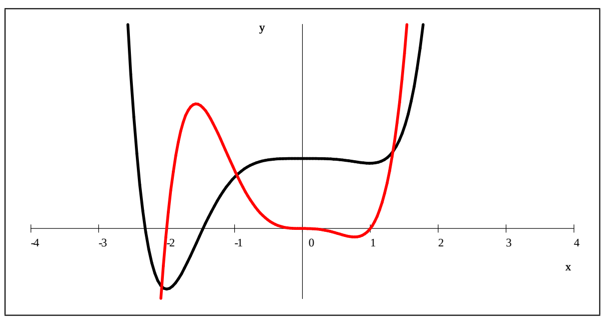
12.) $\frac{1}{5}$ 13.) $\frac{32\sqrt{3}}{9}$ 14.) $1000\sqrt{2}\,\text{ft}^3$ 15.) $P(x) = 10x^3 - 3x^2 + 3x - 5$

16.) $r = \sqrt[3]{\frac{20}{\pi}}\,\text{cm}$ $h = 5\sqrt[3]{\frac{20}{\pi}}\,\text{cm}$ cost: 323.74 cents

17.) a) no relative max, relative min at $x = \frac{3}{10}$ b) relative max at $x = -3$ relative min at $x = 3$

c) relative max at $x = -4$, relative min at $x = 2$

18.) a) $f'(x) = 30x^5 + 30x^4 - 60x^3$
$= 30(x+2)x^3(x-1)$
rel min: $(-2, -52)$ and $(1, 56)$
abs min: $(-2, -52)$
rel max: $(0, 60)$
abs max: $(-3, 332)$

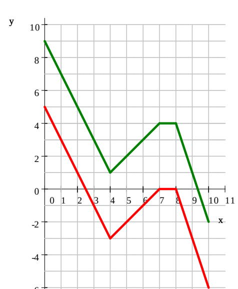


19.) $f'(x) = \cos x - 1$ is always negative or zero. Thus f' never changes sign, so f has no relative max or min

20.) the black graph is f, the red graph is f' 21.) a) $x = 0$ b) $x = -2$ and $x = 3$

22.) a) $v(t) = -10t + 160$ b) $h(t) = -5t^2 + 160t + 525$ c) $h_{\max} = 1805\,\text{m}$

23.) a) red graph b) green graph



24.) $V_{\max} = \frac{32}{81}\pi R^3$