1. Prove that if a function is differentiable at a number $x = a$, then it is also continuous there.

2. Find an equation for the tangent line drawn to the graph of $f(x) = x^2 - \dfrac{2}{x}$ at $x = -2$.

3. Compute each of the following limits.

a) $\displaystyle\lim_{x\to 0^-} \frac{\sin 3x}{x}$ b) $\displaystyle\lim_{x\to 6} \frac{x^2-6x}{x^2-36}$ c) $\displaystyle\lim_{x\to 5} \frac{x^2-25}{2-\sqrt{x-1}}$

4. Compute each of the following limits.

a) $\displaystyle\lim_{x\to -\infty} \left(-2x^5+8x^2\right)$

b) $\displaystyle\lim_{x\to \infty} \left(-2x^5+8x^2\right)$

c) $\displaystyle\lim_{x\to -\infty} \left(-2x^5+8x^6\right)$

d) $\displaystyle\lim_{x\to \infty} \left(-2x^5+8x^6\right)$

e) $\displaystyle\lim_{x\to -\infty} \frac{3x^2-1}{5x^2-3x+2}$

f) $\displaystyle\lim_{x\to -\infty} \frac{100x-1}{5x^2-3x+2}$

g) $\displaystyle\lim_{x\to -\infty} \log_2 x$

h) $\displaystyle\lim_{x\to \infty} \frac{2x^2+3x+1}{3x^2-5x+2}$

i) $\displaystyle\lim_{x\to \infty} \frac{-x^3+2x+1}{x-3}$

j) $\displaystyle\lim_{x\to -\infty} 2^x$

k) $\displaystyle\lim_{x\to \infty} \left(\log_2\left(x^2-5x+17\right)\right)$

l) $\displaystyle\lim_{x\to \infty} \frac{12+\log_7 3x}{15+\log_7 x}$

m) $\displaystyle\lim_{x\to \infty} \frac{2^{2x+5}}{3^{x-1}}$

n) $\displaystyle\lim_{x\to \infty} \frac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

o) $\displaystyle\lim_{x\to \infty} x\left(\frac{1}{3}-\frac{1}{3-\frac{1}{x}}\right)$

p) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{4-\frac{1}{x}}-2}{\frac{1}{x}}$

q) $\displaystyle\lim_{x\to -\infty} \frac{\cos x-2}{x^3+1}$

5. More limits.

a) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{x}}{\sqrt{x+1}-\sqrt{2x}}$

b) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{x}}{\sqrt{x+1}-\sqrt{2x}}$

c) $\displaystyle\lim_{x\to \infty} \frac{\sin x}{x}$

d) $\displaystyle\lim_{x\to 6^-} \frac{\frac{1}{x}-\frac{1}{6}}{x-6}$

e) $\displaystyle\lim_{x\to 6^+} \log_6(x-6)$

f) $\displaystyle\lim_{x\to 0} \frac{\left(\sqrt{x+25}-5\right)^2}{x^2}$

g) $\displaystyle\lim_{x\to -3^-} \frac{x^2-9}{x^2-2x-15}$

h) $\displaystyle\lim_{x\to 5^-} \frac{x^2-9}{x^2-2x-15}$

i) $\displaystyle\lim_{x\to \infty} \frac{3^{2x-1}}{2^{3x+1}}$

j) $\displaystyle\lim_{x\to \infty} \frac{3^{2x+1}}{2^{4x-1}}$

k) $\displaystyle\lim_{x\to \infty} \frac{2^{x-1}-2^{-x}}{2^{x+1}+2^{-x}}$

l) $\displaystyle\lim_{x\to \infty} x\left(\frac{1}{2}-\frac{1}{2+\frac{1}{x}}\right)$

m) $\displaystyle\lim_{x\to \infty} \left(5^{x+1}-5^x\right)$

n) $\displaystyle\lim_{x\to \infty} \frac{\cos^2 x}{x}$

o) $\displaystyle\lim_{x\to \infty} \left(\sqrt{2x}-\sqrt{x}\right)$

p) $\displaystyle\lim_{x\to \infty} \frac{3\cdot 2^{2x}-2^x+1}{-4^x+2^x-1}$

q) $\displaystyle\lim_{x\to \pi} \frac{\sin x}{x-\pi}$

6. Differentiate each of the following, using the definition of the derivative.

a) $f(x) = \sqrt{2x-1}$ b) $f(x) = \dfrac{1}{x^2-1}$ c) $f(x) = \sqrt{1-x^2}$

7. Differentiate each of the following functions.

a) $f(x) = (\sin x - \cos x)\sqrt{x^5}$ b) $f(y) = \dfrac{1}{y^2}+\dfrac{1}{y}+\dfrac{1}{\sqrt{y}}+\sqrt{y}$ c) $f(x) = \dfrac{\cos x-\sqrt{x}}{x^2}$

8. Evaluate each of the following indefinite integrals.

a) $\displaystyle\int 10x-3\;dx$ c) $\displaystyle\int \sqrt[3]{x^5}-\sqrt[5]{x^3}\;dx$ e) $\displaystyle\int 3abm-2a+1\;da$

b) $\displaystyle\int 3\sin x-\cos x\;dx$ d) $\displaystyle\int mx+b\;dx$ f) $\displaystyle\int 3abm-2a+1\;dm$

9. We know the following things about a function f. $f'(x) = 20x^3 - 3$ and $f(-1) = 16$. Find f.

10. Find the value of c for which the following is true: the tangent line drawn to the graph of $y = \dfrac{1}{x}$ has $x-$intercept $(6, 0)$.

11. Find the equation for the function f if we know that f is a polynomial of degree 3, and has a relative minimum at $x = -2$ and a relative maximum at $x = 3$.

12. Prove that the function given is one-to-one.

    a) $f(x) = \dfrac{2x-3}{5x+1}$ b) $f(x) = 3x^5 - 50x^3 + 390x - 1200$

13. Find the value of a so that the line $y = 2x$ is a tangent line to the parabola $y = ax^2 + 5$.

14. Sketch the graph and give a complete analysis for $f(x) = \sqrt{x+1} - 2$.

15. Let $P(x, y)$ be a point on the graph of $y = 4 - x^2$ with $0 \leq x \leq 2$. Let $PQRS$ be a rectangle with one side on the $x-$axis and two vertices on the graph of $y = 4 - x^2$. Find the exact value of the greatest possible area of such a rectangle.

16. We would like to construct an open box with a square base. The box should have a surface area of $600\,\text{ft}^2$. What is the greatest volume of such a box possible?

17. We want to construct a cylindrical soda can with volume $100\,\text{cm}^3$. If the material for the side costs 2 cents per cm^2, and the material for the top and bottom costs 5 cents per cm^2, what dimensions would guarantee a minimal cost of producing such a can? What would be the minimal cost?

18. Find a third degree polynomial $P(x)$ such that $P(0) = -5$, $P'(0) = 3$, $P''(0) = -6$ and $P'''(0) = 60$.

19. Find the $x-$coordinate of all relative maximums and minimums of each of the functions given below.

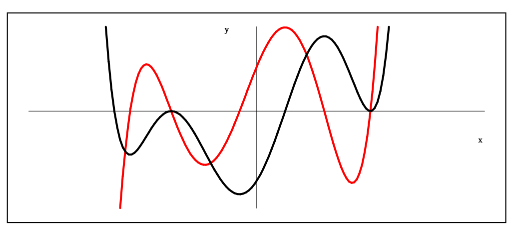
    a) $f(x) = x^3(5x - 2)$ b) $f(x) = 2x + \dfrac{18}{x}$ c) $f(x) = x^3 + 3x^2 - 24x + 24$

20. Find all relative and absolute maximums and minimums for the $f(x) = 6x^5 - 15x^4 + 5x^6 + 60$ on $[-3, 3]$. Sketch the graph of both f and f' on this domain.

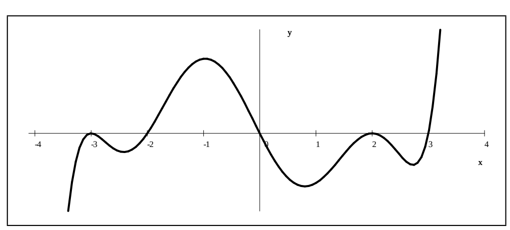
21. Prove that the function $f(x) = \sin x - x$ does not have any relative minimums or maximums.

22. Which point on the line $y = \dfrac{1}{2}x - 3$ is closest to the point $P(4, 3)$?

23. The graph shows a function f and its first derivative, f'. Which is which?



24. The graph below shows f', the first derivative of a function f.
    a) Find all values of x for which the function f has a local maximum at x.
    b) Find all values of x for which the function f has a local minimum at x.



25. We shoot a small object upward, from the top of a tower. The acceleration function of the object is $a(t) = -10$. (Location is measured in meters, velocity in $\frac{\text{m}}{\text{s}}$, acceleration in $\frac{\text{m}}{\text{s}^2}$.)

   a) Given that $v(0) = 160$, find $v(t)$, the velocity function of the object.

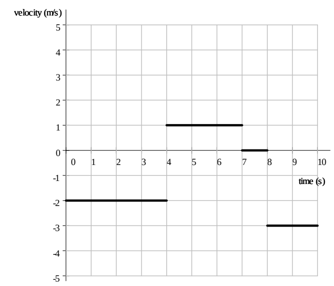
   b) Given that $h(0) = 525$, find $h(t)$, the location function of the object.

   c) Find the maximum height that the object reaches.

26. The picture shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward.).

   a) Suppose that the object starts at a height of 5 m. Graph its location function.

   b) Suppose that the object starts at a height of 9 m. Graph its location function.



27. Match each of the following graphs to the descriptions. Assume that the graphs you see are the location functions of an obejct moving along a vertical line.

   (a) The object is moving upward and its speed is increasing.

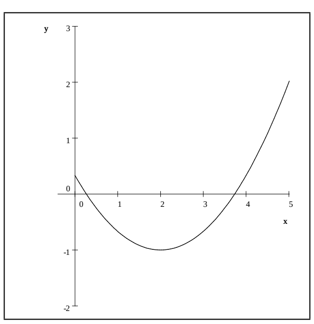
   (b) The object is moving upward and its speed is decreasing.

   (c) The object is moving downward and its speed is increasing.

   (d) The object is moving downward and its speed is decreasing.

   (e) The obejct has been moving upward but later is moving downward.

   (f) The obejct has been moving downward but later is moving upward.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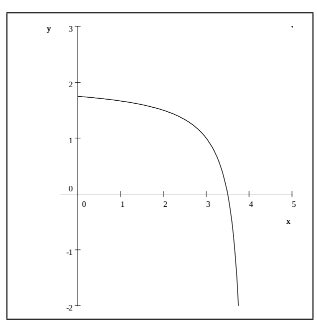
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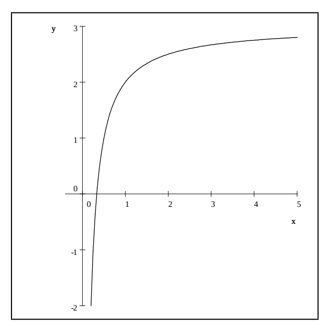
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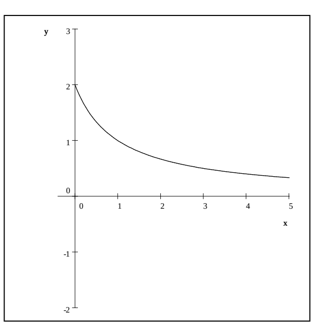
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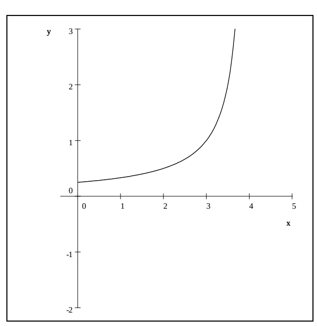
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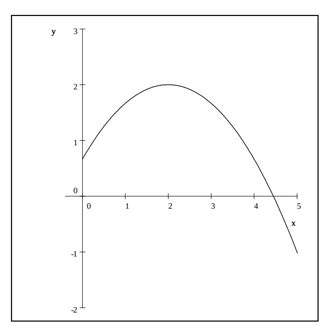

6.

28. Match each of the following graphs to the descriptions. Assume that the graphs you see are the velocity functions of an obejct moving along a vertical line.

(a) The object is moving upward and its speed is increasing.

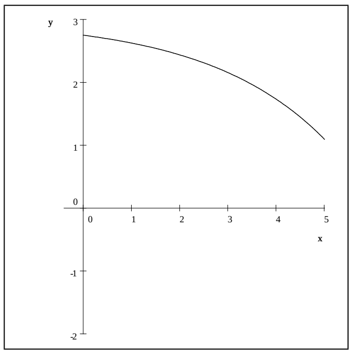
(b) The object is moving upward and its speed is decreasing.

(c) The object is moving downward and its speed is increasing.

(d) The object is moving downward and its speed is decreasing.

(e) The obejct has been moving upward but later is moving downward.

(f) The obejct has been moving downward but later is moving upward.



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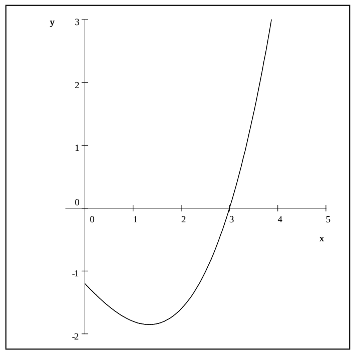

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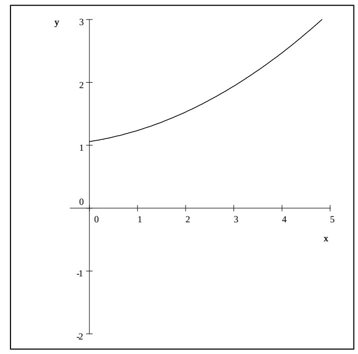

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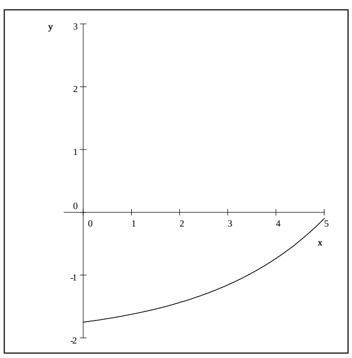

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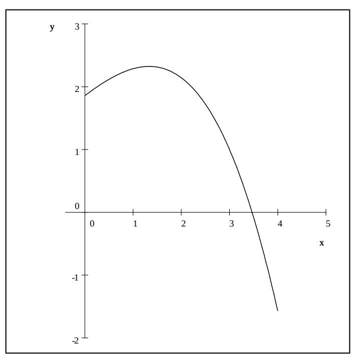

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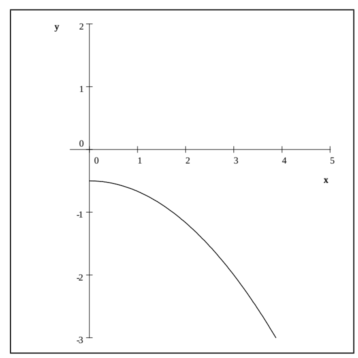


6.

Review Problems - Answers

1.) See handout 2.) $y=-\frac{7}{2}x-2$ 3.) a) 3 b) $\frac{1}{2}$ c) -40

4.) a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) ∞ h) $\frac{2}{3}$ i) $-\infty$ j) 0

k) ∞ l) 1 m) ∞ n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0

5.) a) $\frac{1}{1-\sqrt{2}}=-\sqrt{2}-1$ b) undefined c) 0 d) $-\frac{1}{36}$ e) $-\infty$ f) $\frac{1}{100}$ g) $\frac{3}{4}$ h) $-\infty$

i) ∞ j) 0 k) $\frac{1}{4}$ l) $\frac{1}{4}$ m) ∞ n) 0 o) ∞ p) -3 q) -1

Solution for q) : Let $y=x-\pi$. Then $y\to 0$ as $x\to\pi$ and also $x=y+\pi$

$$\lim_{x\to\pi}\frac{\sin x}{x-\pi}=\lim_{y\to 0}\frac{\sin(y+\pi)}{y}=\lim_{y\to 0}\frac{\sin y\cos\pi+\cos y\sin\pi}{y}=\lim_{y\to 0}\frac{\sin y(-1)+\cos y\cdot 0}{y}=\lim_{y\to 0}\frac{-\sin y}{y}=-\lim_{y\to 0}\frac{\sin y}{y}=-1$$

6.) see handout

7.) a) $f'(x)=\frac{5}{2}(\sin x-\cos x)x\sqrt{x}+(\cos x+\sin x)x^2\sqrt{x}$ b) $f'(y)=-\frac{2}{y^3}-\frac{1}{y^2}-\frac{1}{2y\sqrt{y}}+\frac{1}{2\sqrt{y}}$

c) $f'(x)=-\frac{2}{x^3}(\cos x-\sqrt{x})+\frac{1}{x^2}\left(-\sin x-\frac{1}{2\sqrt{x}}\right)$

8.) a) $5x^2-3x+C$ b) $-3\cos x-\sin x+C$ c) $\frac{3}{8}x^{8/3}-\frac{5}{8}x^{8/5}+C$

d) $\frac{1}{2}mx^2+bx+C$ e) $\frac{3}{2}a^2bm-a^2+a+C$ f) $\frac{3}{2}abm^2-2am+m+C$

9.) $f(x)=5x^4-3x+8$ 10.) 3 11.) Let $a>0$ and $c\in\mathbb{R}$ any number $f(x)=-a\left(-\frac{1}{3}x^3+\frac{1}{2}x^2+6x\right)+c$

12.) a) Write $y=\frac{2x-3}{5x+1}$ and solve for x: $x=\frac{y+3}{-5y+2}$ so for each y, there is a **unique** x for which the function takes that $y-$value.

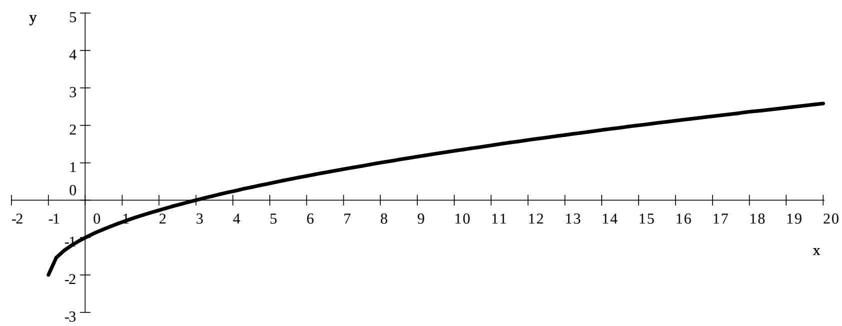
b) $f'(x)=15\left(\left(x^2-5\right)^2+1\right)$ is always positive, hence f is always strictly increasing. Thus one-to-one.

13.) $\frac{1}{5}$ 14.) $f(x)=\sqrt{x+1}-2$

domain: $[-1,\infty)$
range: $[-2,\infty)$
no horizontal asymptote
no vertical asymptote
$y-$intercept: $(0,-1)$

$x-$intercept: $(3,0)$
one-to-one
strictly increasing on on $[-1,\infty)$
no relative maximum
no absolute maximum

no relative minimum
absolute minimum: $(-1,-2)$
end behavior:
$\lim_{x\to-\infty}f(x)$ = undefined and $\lim_{x\to\infty}f(x)=\infty$

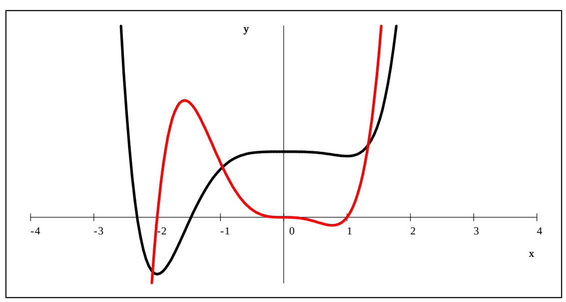


15.) $\dfrac{32\sqrt{3}}{9}$ 16.) $1000\sqrt{2}\,\text{ft}^3$ with base $\sqrt[3]{120}\,\text{ft}$ by $\sqrt[3]{120}\,\text{ft}$

17.) $r = \sqrt[3]{\dfrac{20}{\pi}}\,\text{cm}$ $h = 5\sqrt[3]{\dfrac{20}{\pi}}\,\text{cm}$ cost: 323.74 cents 18.) $P(x) = 10x^3 - 3x^2 + 3x - 5$

19.) a) no relative maximum, relative minimum at $x = \dfrac{3}{10}$
b) relative maximum at $x = -3$ relative minimum at $x = 3$
c) relative maximum at $x = -4$, relative minimum at $x = 2$

20.) a) $f'(x) = 30x^5 + 30x^4 - 60x^3 = 30(x+2)x^3(x-1)$
relative minimums: $(-2, -52)$ and $(1, 56)$ absolute minimum: $(-2, -52)$
relative maximum: $(0, 60)$ absolute maximum: $(-3, 332)$

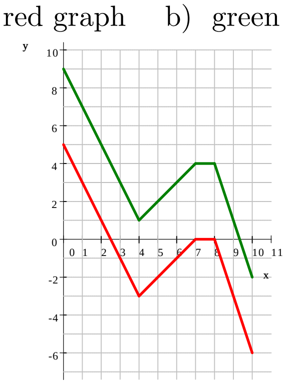


21.) $f'(x) = \cos x - 1$ is always negative or zero. Thus f' never changes sign, so f has no relative minimums or maximums.

22.) $\left(\dfrac{28}{5}, -\dfrac{1}{5}\right)$ 23.) the black graph is f, the red graph is f' 24.) a) $x = 0$ b) $x = -2$ and $x = 3$

25.) a) $v(t) = -10t + 160$ b) $h(t) = -5t^2 + 160t + 525$ c) $h_{\max} = 1805\,\text{m}$

26.) a) red graph b) green graph



27.) 1-(f) 2-(d) 3-(c) 4-(a) 5-(b) 6-(e)
28.) 1-(b) 2-(d) 3-(f) 4-(e) 5-(a) 6-(c)