

1. a) Prove that $\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$. b) Prove that if $f(x) = \cos x$, then $f'(x) = -\sin x$.

2. Find an equation for the tangent line drawn to the graph of $f(x) = x^2 - \frac{2}{x}$ at $x = -2$.

3. Compute each of the following limits.

a) $\lim_{x \rightarrow 0^-} \frac{\sin 3x}{x}$

d) $\lim_{x \rightarrow \pi/4} \frac{\tan x - 1}{x - \frac{\pi}{4}}$

f) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^{3x}$

b) $\lim_{x \rightarrow 6} \frac{x^2 - 6x}{x^2 - 36}$

g) $\lim_{x \rightarrow 0} \frac{1 - \cos^2(3x)}{x^2}$

c) $\lim_{x \rightarrow 5} \frac{x^2 - 25}{2 - \sqrt{x-1}}$

e) $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x$

4. Compute each of the following limits.

a) $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^2)$

h) $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$

n) $\lim_{x \rightarrow \infty} \frac{3^{x+1} \cdot \left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

b) $\lim_{x \rightarrow \infty} (-2x^5 + 8x^2)$

i) $\lim_{x \rightarrow \infty} \frac{-x^3 + 2x + 1}{x - 3}$

o) $\lim_{x \rightarrow \infty} x \left(\frac{1}{3} - \frac{1}{3 - \frac{1}{x}}\right)$

c) $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^6)$

j) $\lim_{x \rightarrow -\infty} 2^x$

d) $\lim_{x \rightarrow \infty} (-2x^5 + 8x^6)$

k) $\lim_{x \rightarrow \infty} (\log_2(x^2 - 5x + 17))$

e) $\lim_{x \rightarrow -\infty} \frac{3x^2 - 1}{5x^2 - 3x + 2}$

l) $\lim_{x \rightarrow \infty} \frac{12 + \log_7 3x}{15 + \log_7 x}$

p) $\lim_{x \rightarrow \infty} \frac{\sqrt{4 - \frac{1}{x}} - 2}{\frac{1}{x}}$

f) $\lim_{x \rightarrow -\infty} \frac{100x - 1}{5x^2 - 3x + 2}$

m) $\lim_{x \rightarrow \infty} \frac{2^{2x+5}}{3^{x-1}}$

q) $\lim_{x \rightarrow -\infty} \frac{\cos x - 2}{x^3 + 1}$

5. Differentiate each of the following, using the definition of the derivative.

a) $f(x) = \sqrt{2x-1}$

b) $f(x) = \frac{1}{x^2-1}$

c) $f(x) = \sqrt{1-x^2}$

6. Differentiate each of the following functions.

a) $f(x) = \frac{x^4 - 3x^2 + x - 2}{x^3}$

b) $f(y) = \frac{1}{y^2} + \frac{1}{y} + \frac{1}{\sqrt{y}} + \sqrt{y}$

c) $f(x) = 3x^5 - 2\sqrt[7]{x^3} + 8\sqrt[3]{x^7} - \pi^5$

7. Find the point on the line $y = \frac{1}{2}x - 2$ that is closest to the point $(3, 1)$.

8. We know the following things about a function f . $f'(x) = 20x^3 - 3$ and $f(-1) = 16$. Find a possible equation f .

9. Find the value of c for which the following is true: the tangent line drawn to the graph of $y = \frac{1}{x}$ has x -intercept $(6, 0)$.

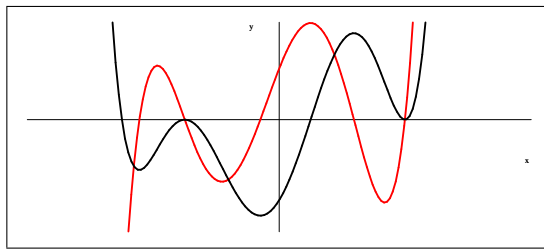
10. Suppose that $f(x) = 12x^3 + bx^2 + cx + 7$. Find the exact values of b and c if we know that f has a relative maximum at $x = -2$ and a relative minimum at $x = 3$.

11. Prove that the function given is one-to-one: $f(x) = 3x^5 - 50x^3 + 390x - 1200$.

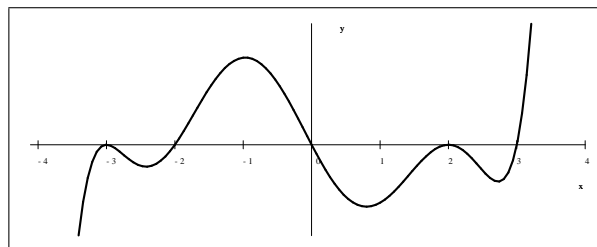
12. Find the value of a so that the line $y = 2x$ is a tangent line to the parabola $y = ax^2 + 5$.

13. Let $P(x, y)$ be a point on the graph of $y = 4 - x^2$ with $0 \leq x \leq 2$. Let $PQRS$ be a rectangle with one side on the x -axis and two vertices on the graph of $y = 4 - x^2$. Find the exact value of the greatest possible area of such a rectangle.
14. We would like to construct an open box with a square base. We are to use of 600 ft^2 of material. What is the greatest volume for such a box possible?
15. Find a third degree polynomial $P(x)$ such that $P(0) = -5$, $P'(0) = 3$, $P''(0) = -6$ and $P'''(0) = 60$.
16. We want to construct a cylindrical container. We have 300 cm^2 of metal. What dimensions would maximize the volume of the container?
17. Find the x -coordinate of all relative maximums and minimums of each of the functions given below.
- a) $f(x) = x^3(5x - 2)$ b) $f(x) = 2x + \frac{18}{x}$ c) $f(x) = x^3 + 3x^2 - 24x + 24$
18. Find all relative and absolute maximums and minimums for the $f(x) = 6x^5 - 15x^4 + 5x^6 + 60$ on $[-3, 3]$. Sketch the graph of both f and f' on this domain.
19. Prove that the function $f(x) = \sin x - x$ does not have any relative minimums or maximums.

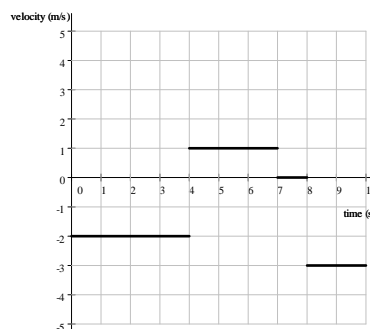
20. The graph below shows a function f and its first derivative, f' . Which is which?



21. The graph below shows f' , the first derivative of a function f .
- a) Find all values of x for which the function f has a local maximum at x .
- b) Find all values of x for which the function f has a local minimum at x .



22. The picture shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward.)
- a) Suppose that the object starts at a height of 5 m. Where is the object at $t = 10$?
- b) Suppose that the object starts at a height of 9 m. Where is the object at $t = 10$?



23. We shoot a small object upward, from the top of a tower. The acceleration function of the object is $a(t) = -10$. (Location is measured in meters, velocity in $\frac{\text{m}}{\text{s}}$, acceleration in $\frac{\text{m}}{\text{s}^2}$.)
- Given that $v(0) = 160$, find $v(t)$, the velocity function of the object.
 - Given that $h(0) = 525$, find $h(t)$, the location function of the object.
 - Find the maximum height that the object reaches.
24. What is the greatest volume of a right circular cone that can be written into a sphere with radius R ?

Answers

1.) See handout 2.) $y = -\frac{7}{2}x - 2$ 3.) a) 3 b) $\frac{1}{2}$ c) -40 d) 2 e) e^3 f) $\frac{1}{e^6}$ g) 9

4.) a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) undefined h) $\frac{2}{3}$ i) $-\infty$ j) 0

k) ∞ l) 1 m) ∞ n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0 5.) see handout

6.) a) $f'(x) = 1 + \frac{3}{x^2} - \frac{2}{x^3} + \frac{6}{x^4}$ b) $f'(y) = -\frac{2}{y^3} - \frac{1}{y^2} - \frac{1}{2y\sqrt{y}} + \frac{1}{2\sqrt{y}}$

c) $f'(x) = 15x^4 - \frac{6}{7x}\sqrt[3]{x^3} + \frac{56}{3x}\sqrt[3]{x^7}$ 7.) $\left(\frac{18}{5}, -\frac{1}{5}\right)$ 8.) $f(x) = 5x^4 - 3x + 8$ 9.) 3

10.) $b = -18, c = -216$

11.) $f'(x) = 15\left((x^2 - 5)^2 + 1\right)$ is always positive, hence f is always strictly increasing. Thus one-to-one.

12.) $\frac{1}{5}$ 13.) $\frac{32\sqrt{3}}{9}$ 14.) $1000\sqrt{2}\text{ft}^3$ 15.) $P(x) = 10x^3 - 3x^2 + 3x - 5$

16.) $r = \frac{5\sqrt{2}}{\sqrt{\pi}}$ and $h = \frac{10\sqrt{2}}{\sqrt{\pi}}$

- 17.) a) no relative max, relative min at $x = \frac{3}{10}$ b) relative max at $x = -3$ relative min at $x = 3$
 c) relative max at $x = -4$, relative min at $x = 2$

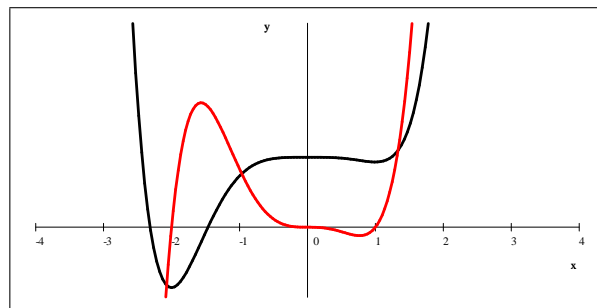
18.) a) $f'(x) = 30x^5 + 30x^4 - 60x^3$
 $= 30(x+2)x^3(x-1)$

rel min: $(-2, -52)$ and $(1, 56)$

abs min: $(-2, -52)$

rel max: $(0, 60)$

abs max: $(-3, 332)$



19.) $f'(x) = \cos x - 1$ is always negative or zero. Thus f' never changes sign, so f has no relative max or min

20.) the black graph is f , the red graph is f' 21.) a) $x = 0$ b) $x = -2$ and $x = 3$

22.) a) -6 b) -2 23.) a) $v(t) = -10t + 160$ b) $h(t) = -5t^2 + 160t + 525$ c) $h_{\max} = 1805\text{ m}$

24.) $V_{\max} = \frac{32}{81}\pi R^3$