1. Simplify each of the following

   a) 6^{-2} b) $32^{3/5}$ c) $64^{-2/3}$ d) $\log_9 \sqrt{27}$ e) $\log_4 \left(\frac{1}{\sqrt{2}}\right)$ f) $\log_2 (\sin 45°)$

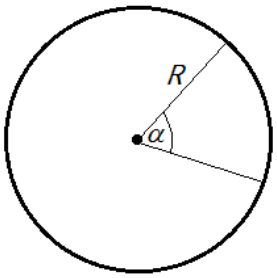
2. Solve the equation $9x^2 - 12x = 11$ and check one of your solution using exact values.

3. Solve each of the following equations. Present exact values of all solutions, in radians.

   a) $\tan^2 x + \tan x = 0$
   b) $\sin^2 x = \frac{1}{2}$
   c) $\sin x = 2 \sin x \cos x$
   d) $\sin x + 2 \cos^2 x = 1$
   e) $\cos x - 2 \sin^2 x = 1$

4. Suppose that α is a central angle (less than 360°) in a circle with radius R.

   a) Express the length of the arc subtended by the central angle α in terms of α and R. Assume that α is measured in degrees.
   b) Express the area of the sector determined by the central angle α in terms of α and R. Assume that α is measured in degrees.
   c) Express the length of the arc subtended by the central angle α in terms of α and R. Assume that α is measured in radians.
   d) Express the area of the sector determined by the central angle α in terms of α and R. Assume that α is measured in radians.



5. Solve each of the following inequalities.

   a) $x^2 \geq 4x$ b) $8x + x^2 < 33$ c) $x^2 < -2x + 2$ d) $4x^2 \leq 4x - 1$ e) $x^2 - 6x > -10$

6. Find the coordinates of all points where the graphs of $f(x) = x^2 - 2x - 26$ and $g(x) = 2x - 5$ intersect each other.

7. a) Find the center and radius of a circle with equation $2y - 6x + x^2 + y^2 = 10$
   b) Find an equation for the tangent line drawn to the circle $(x+2)^2 + (y-4)^2 = 10$ to the point $(-5, 3)$

8. a) Solve the formula $V = 2\pi r^3 + \frac{1}{2}\pi r^2 h$ for h.
   b) A right pyramid has a square base with sides 30 meters long. The pyramid is 24 meters tall. At what height is the perpendicular cross section a square with sides 10 meters?

9. Graph each of the following pairs of functions in the same coordinate system.

   a) $f(x) = 2^x$ and $g(x) = \log_2 x$
   b) $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$
   c) $f(x) = \log_2 x$ and $g(x) = \log_{1/2} x$
   d) $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \log_{1/2} x$

10. For each of the following functions given, give a complete analysis and sketch its graph.

    a) $f(x) = 9 - 3x^2 - 6x$ on $[-2, 3]$
    b) $f(x) = \sqrt{x+3}$
    c) $f(x) = \log_3 x$
    d) $f(x) = 0.7^x$
    e) $f(x) = 1 + \sqrt{16 - x^2}$

11. An object's height (measured in feet) is defined by $s(t) = 0.4t + 12$ where t is the time, measured in seconds.
    a) Find the location of the object at $t = 10$ seconds.
    b) Find the average velocity of the object between $t = 0$ and $t = 3$ seconds
    c) Find the average velocity of the object between $t = 5$ and $t = 10$ seconds

12. An object's height (measured in feet) is defined by $s(t) = t^3 - 12t$ where t is the time, measured in seconds.
    a) Find the location of the object at $t = 3$ seconds.
    b) Find the average velocity of the object between
    i) $t = 0$ and $t = 2$ seconds ii) $t = 1$ s and $t = 2$ s iii) $t = 1.5$ s and $t = 2$

13. A bus travels between cities A and B. The distance between these cities is 60 miles. It takes the bus 2 hours to get from A to B. On its way back, the traveling time was only 1.5 hours. Find the average speed of the bus for
    a) the trip from A to B b) the trip from B to A c) for the roundtrip.
    d*) A bus travels between cities A and B. From A to B, the bus has an average speed of v_1. On its way back, the average speed is v_2. Express the average speed of the bus in terms of v_1 and v_2.

14. Compute each of the following limits.

a) $\lim_{x\to-\infty}\left(-2x^5+8x^2\right)$

b) $\lim_{x\to\infty}\left(-2x^5+8x^2\right)$

c) $\lim_{x\to-\infty}\left(-2x^5+8x^6\right)$

d) $\lim_{x\to\infty}\left(-2x^5+8x^6\right)$

e) $\lim_{x\to-\infty}\dfrac{3x^2-1}{5x^2-3x+2}$

f) $\lim_{x\to-\infty}\dfrac{100x-1}{5x^2-3x+2}$

g) $\lim_{x\to-\infty}\log_2 x$

h) $\lim_{x\to\infty}\dfrac{2x^2+3x+1}{3x^2-5x+2}$

i) $\lim_{x\to\infty}\dfrac{-x^3+2x+1}{x-3}$

j) $\lim_{x\to-\infty}2^x$

k) $\lim_{x\to\infty}\left(\log_2\left(x^2-5x+17\right)\right)$

l) $\lim_{x\to\infty}\dfrac{12+\log_7 3x}{15+\log_7 x}$

m) $\lim_{x\to\infty}\dfrac{2^{x+5}}{4^{x-1}}$

n) $\lim_{x\to\infty}\dfrac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

o) $\lim_{x\to\infty}x\left(\dfrac{1}{3}-\dfrac{1}{3-\frac{1}{x}}\right)$

p) $\lim_{x\to\infty}\dfrac{\sqrt{4-\frac{1}{x}}-2}{\frac{1}{x}}$

q) $\lim_{x\to-\infty}\dfrac{\cos x-2}{x^3+1}$

r) $\lim_{x\to\infty}\tan x$

s) $\lim_{x\to\infty}\dfrac{\ln 2x}{\ln 3x}$

t) $\lim_{\theta\to\infty}\left(\sin^2\theta+\cos^2\theta\right)$

u) $\lim_{x\to\infty}\dfrac{\sqrt{2x^2-1}}{x}$

v) $\lim_{x\to-\infty}\tan^{-1}x$

15. Compute the exact value of each of the following.
    a) $\sin 300°$ b) $\cos\left(-\dfrac{7\pi}{4}\right)$ c) $\tan(900°)$ d) $\sec(-300°)$ e) $\cot\left(\dfrac{\pi}{6}\right)$

16. Find an equation for the curve that consists of points $P(x, y)$ with the following property: they are twice as far from point $A(2, -5)$ as from point $B(-1, 1)$. Describe the curve.

17. (Enrichment) Consider the function $f(x) = 2mx - 8m + mx^2 + 3$ where m is a fixed real number.
    a) Find all values of m for which the graph of f is NOT a parabola.
    b) Graph $y = f(x)$ in the same coordinate system when $m = 0, 1$, and -1.
    c) Prove that there exist two points P and Q that are on the graph of $f(x)$ for all values of m.

Answers

1. a) $\frac{1}{36}$ b) 8 c) $\frac{1}{16}$ d) $\frac{3}{4}$ e) $-\frac{1}{4}$ f) $-\frac{1}{2}$

2. $\frac{2\pm\sqrt{15}}{3}$ Check: if $x=\frac{2-\sqrt{15}}{3}$, then the left-hand side of the equation is

$$\text{LHS} = 9x^2-12x = 9\left(\frac{2-\sqrt{15}}{3}\right)^2-12\left(\frac{2-\sqrt{15}}{3}\right) = 9\cdot\frac{\left(2-\sqrt{15}\right)^2}{9}-12\cdot\frac{2-\sqrt{15}}{3}$$
$$= \left(2-\sqrt{15}\right)^2-4\left(2-\sqrt{15}\right) = 4+15-4\sqrt{15}-8+4\sqrt{15} = 19-8 = 11 = \text{RHS}$$

3. a) $-\frac{\pi}{4}+k\pi$ and $k\pi$ where $k\in\mathbb{Z}$ b) $\frac{\pi}{4}+\frac{k\pi}{2}$ where $k\in\mathbb{Z}$ c) $k\pi$ or $\pm\frac{\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$
   d) $x=\frac{\pi}{2}+2k\pi$ or $x=-\frac{\pi}{6}+2k\pi$ or $x=-\frac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$ e) $x=2k\pi$ where $k\in\mathbb{Z}$

4. a) $s=\frac{2\pi R\alpha}{360^\circ}$ b) $A=\frac{\pi R^2\alpha}{360^\circ}$ c) $s=R\alpha$ d) $A=\frac{1}{2}R^2\alpha$

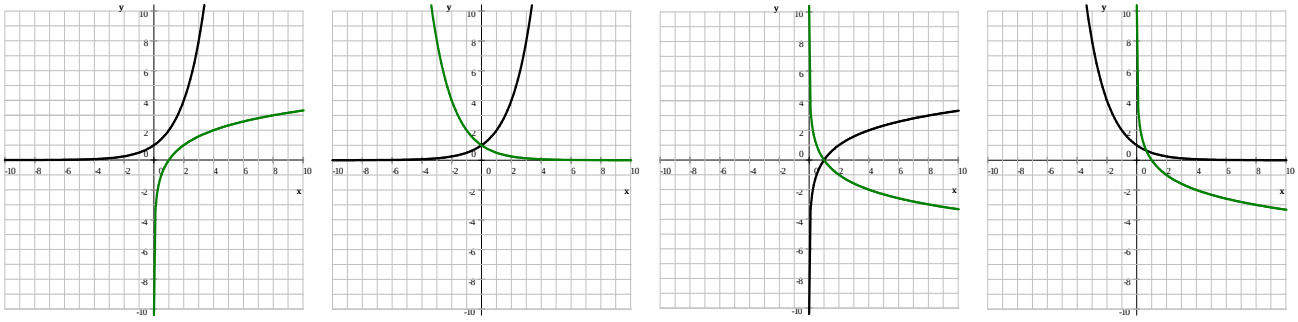
5. a) $(-\infty,0]\cup[4,\infty)$ b) $(-11,3)$ c) $\left(-\sqrt{3}-1,\sqrt{3}-1\right)$ d) $x=\frac{1}{2}$ e) $\mathbb{R}$

6. $(7,9)$ and $(-3,-11)$

7. a) center: $(3,-1)$ radius: $2\sqrt{5}$ b) $y=-3x-12$

8. a) $h=\frac{2\left(V-2\pi r^3\right)}{\pi r^2}$ or $h=\frac{2V}{\pi r^2}-4r$ b) At a height of 16 meters

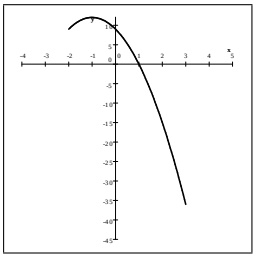
9. a) b) c) d)



10. a) $f(x)=9-3x^2-6x$ on $[-2,3]$

domain: $[-2,3]$
range: $[-36,12]$
$x-$intercept: $(1,0)$
$y-$ intercept: $(0,9)$
maximum: $(-1,12)$
minimum: $(3,-36)$

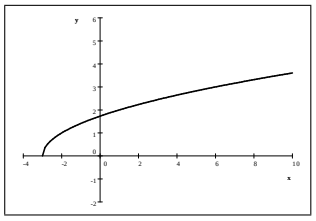
and decreasing on $(-1,3)$
one-to-one: no
end-behavior: none
i.e. $\lim\limits_{x\to\pm\infty} f(x)=$ undefined



b) $f(x) = \sqrt{x+3}$

domain: $[-3, \infty)$
range: $[-3, \infty)$
$x-$intercept: $(-3, 0)$
$y-$ intercept: $\left(0, \sqrt{3}\right)$
maximum: none
minimum: $(-3, 0)$

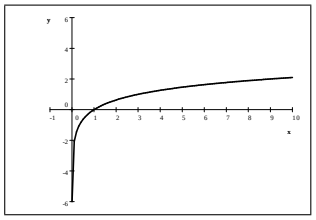
one-to-one: yes
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
and $\lim_{x \to \infty} f(x) = \infty$



c) $f(x) = \log_3 x$

domain: $(0, \infty)$
range: $\mathbb{R}$
$x-$intercept: $(1, 0)$
$y-$ intercept: none
maximum: none
minimum: none

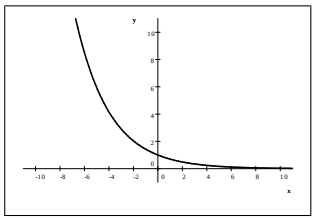
one-to-one: yes
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
and $\lim_{x \to \infty} f(x) = \infty$



d) $f(x) = 0.7^x$

domain: $\mathbb{R}$
range: $(0, \infty)$
$x-$intercept: none
$y-$ intercept: $(0, 1)$
maximum: none
minimum: none

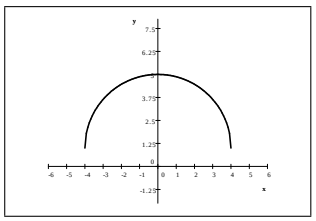
one-to-one: yes
end-behavior:
$\lim_{x \to -\infty} f(x) = \infty$
and $\lim_{x \to \infty} f(x) = 0$



e) $f(x) = 1 + \sqrt{16 - x^2}$

domain: $[-4, 4]$
range: $[1, 5]$
$x-$intercept: none
$y-$ intercept: $(0, 5)$
maximum: $(0, 5)$
minimum: $(-4, 1)$ and $(4, 1)$

one-to-one: no
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
and $\lim_{x \to \infty} f(x) =$ undefined



11. a) $16\,\text{ft}$ b) $0.4\frac{\text{ft}}{\text{s}}$ c) $0.4\frac{\text{ft}}{\text{s}}$

12. a) $-9\,\text{ft}$ b) i) $-8\frac{\text{ft}}{\text{s}}$ ii) $-5\frac{\text{ft}}{\text{s}}$ iii) $-2.75\frac{\text{ft}}{\text{s}}$

13. a) $30\frac{\text{mi}}{\text{h}}$ b) $40\frac{\text{mi}}{\text{h}}$ c) $\frac{240}{7}\frac{\text{mi}}{\text{h}} \approx 34.285\,7143\frac{\text{mi}}{\text{h}}$ d) $\frac{2v_1v_2}{v_1 + v_2}$

14. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) undefined h) $\frac{2}{3}$ i) $-\infty$ j) 0 k) ∞
l) 1 m) 0 n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0 r) undefined s) 1 t) 1 u) $\sqrt{2}$ v) $-\frac{\pi}{2}$

15. a) $-\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{2}$ c) 0 d) 2 e) $\sqrt{3}$ f) $\frac{\pi}{6}$ g) 0 h) $\frac{2\pi}{3}$ i) $-\frac{\pi}{6}$

16. $(x+2)^2 + (y-3)^2 = 20$ a circle centered at $(-2, 3)$, with radius $\sqrt{20}$

17. a) 0 b) see below c) $y = m(x-2)(x+4)+3$ No matter what m is, $(2,3)$ and $(-4,3)$ will be on the graph