1. Compute each of the following limits.

a) $\displaystyle\lim_{x\to-\infty}\left(-2x^5+8x^2\right)$

b) $\displaystyle\lim_{x\to\infty}\left(-2x^5+8x^2\right)$

c) $\displaystyle\lim_{x\to-\infty}\left(-2x^5+8x^6\right)$

d) $\displaystyle\lim_{x\to\infty}\left(-2x^5+8x^6\right)$

e) $\displaystyle\lim_{x\to-\infty}\frac{3x^2-1}{5x^2-3x+2}$

f) $\displaystyle\lim_{x\to-\infty}\frac{100x-1}{5x^2-3x+2}$

g) $\displaystyle\lim_{x\to-\infty}\log_2 x$

h) $\displaystyle\lim_{x\to\infty}\frac{2x^2+3x+1}{3x^2-5x+2}$

i) $\displaystyle\lim_{x\to\infty}\frac{-x^3+2x+1}{x-3}$

j) $\displaystyle\lim_{x\to-\infty}2^x$

k) $\displaystyle\lim_{x\to\infty}\left(\log_2\left(x^2-5x+17\right)\right)$

l) $\displaystyle\lim_{x\to\infty}\frac{12+\log_7 3x}{15+\log_7 x}$

m) $\displaystyle\lim_{x\to\infty}\frac{2^{2x+5}}{3^{x-1}}$

n) $\displaystyle\lim_{x\to\infty}\frac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

o) $\displaystyle\lim_{x\to\infty}x\left(\frac{1}{3}-\frac{1}{3-\frac{1}{x}}\right)$

p) $\displaystyle\lim_{x\to\infty}\frac{\sqrt{4-\frac{1}{x}}-2}{\frac{1}{x}}$

q) $\displaystyle\lim_{x\to-\infty}\frac{\cos x-2}{x^3+1}$

2. Compute each of the following limits.

a) $\displaystyle\lim_{x\to2}\frac{2-x}{\frac{1}{2}-\frac{1}{x}}$

b) $\displaystyle\lim_{x\to1^-}\frac{x^2-1}{2-\sqrt{3+x}}$

c) $\displaystyle\lim_{x\to3}\frac{|2x-4|}{x-8}$

d) $\displaystyle\lim_{x\to0^-}\frac{|x|}{x}$

e) $\displaystyle\lim_{x\to0}\frac{|x|}{x}$

f) $\displaystyle\lim_{x\to1^+}\frac{x^2-9}{2x^2-8x+6}$

g) $\displaystyle\lim_{x\to0}\frac{\sqrt{5+x}-\sqrt{5}}{x}$

h) $\displaystyle\lim_{x\to3^-}\frac{x^2-9}{2x^2-8x+6}$

i) $\displaystyle\lim_{x\to\pi}\frac{\cos x}{\sin x}$

j) $\displaystyle\lim_{x\to\pi^+}\frac{\cos x}{\sin x}$

k) $\displaystyle\lim_{x\to\pi/4}\tan x$

l) $\displaystyle\lim_{a\to0}\frac{\sqrt{5a+4}-2}{a}$

m) $\displaystyle\lim_{m\to1^-}\frac{m^2-1}{m^4-1}$

n) $\displaystyle\lim_{x\to0}\frac{1+x+\sin x}{\cos x}$

o) $\displaystyle\lim_{x\to\pi/3}\left(\tan^2 x-1\right)$

p) $\displaystyle\lim_{h\to0}\frac{\sqrt{h^2+100}-10}{h^2}$

q) $\displaystyle\lim_{x\to0^+}\ln x$

r) $\displaystyle\lim_{x\to0^-}\frac{\sin 3x}{x}$

s) $\displaystyle\lim_{x\to6}\frac{x^2-6x}{x^2-36}$

t) $\displaystyle\lim_{x\to5}\frac{x^2-25}{2-\sqrt{x-1}}$

u) $\displaystyle\lim_{x\to\pi/4}\frac{\tan x-1}{x-\frac{\pi}{4}}$

v) $\displaystyle\lim_{x\to0}\frac{1-\cos^2(3x)}{x^2}$

3. Compute each of the following limits.

a) $\displaystyle\lim_{x\to-\infty}\frac{x^2-9}{2x^2-8x+6}$

b) $\displaystyle\lim_{x\to-3^-}\frac{x^2-9}{2x^2-8x+6}$

c) $\displaystyle\lim_{x\to-3^+}\frac{x^2-9}{2x^2-8x+6}$

d) $\displaystyle\lim_{x\to-3}\frac{x^2-9}{2x^2-8x+6}$

e) $\displaystyle\lim_{x\to1^-}\frac{x^2-9}{2x^2-8x+6}$

f) $\displaystyle\lim_{x\to1^+}\frac{x^2-9}{2x^2-8x+6}$

g) $\displaystyle\lim_{x\to1}\frac{x^2-9}{2x^2-8x+6}$

h) $\displaystyle\lim_{x\to3^-}\frac{x^2-9}{2x^2-8x+6}$

i) $\displaystyle\lim_{x\to3^+}\frac{x^2-9}{2x^2-8x+6}$

j) $\displaystyle\lim_{x\to3}\frac{x^2-9}{2x^2-8x+6}$

k) $\displaystyle\lim_{x\to\infty}\frac{x^2-9}{2x^2-8x+6}$

4. Based on your results in the previous problem, sketch the graph of $f(x)=\dfrac{x^2-9}{2x^2-8x+6}$. (Plot a few additional points if needed.)

5. Based on the graph of a function f shown on the picture, determine each of the following limits.

a) $\lim\limits_{x\to 1^-} f(x)$

b) $\lim\limits_{x\to 1^+} f(x)$

c) $\lim\limits_{x\to 1} f(x)$

d) $\lim\limits_{x\to 2^-} f(x)$

e) $\lim\limits_{x\to 2^+} f(x)$

f) $\lim\limits_{x\to 2} f(x)$

g) $\lim\limits_{x\to 4^-} f(x)$

h) $\lim\limits_{x\to 4^+} f(x)$

i) $\lim\limits_{x\to 4} f(x)$

j) $\lim\limits_{x\to 6^-} f(x)$

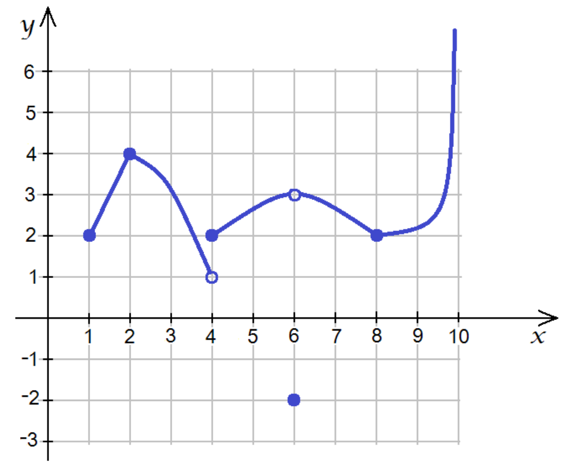
k) $\lim\limits_{x\to 6^+} f(x)$

l) $\lim\limits_{x\to 6} f(x)$

m) $\lim\limits_{x\to 10^-} f(x)$

n) $\lim\limits_{x\to 10^+} f(x)$

o) $\lim\limits_{x\to 10} f(x)$



6. a) Prove that $\lim\limits_{x\to 0^+} \dfrac{\sin x}{x} = 1$. b) Prove that if $f(x) = \cos x$, then $f'(x) = -\sin x$.

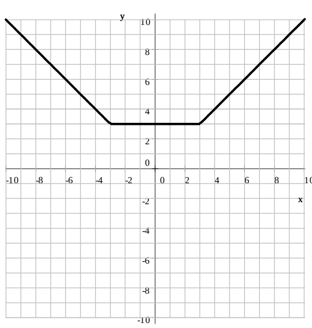
7. Consider the limit $\lim\limits_{x\to \pi/2} \dfrac{1-\tan^2 x}{1+\tan^2 x}$.

a) Explain why we can not evaluate the limit in its current form.

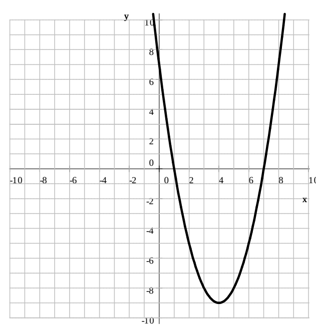
b) Prove that $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$ and use this to evaluate the limit.

8. In each case, the graph of a function is given. Sketch the graph of the inverse relation in the same coordinate system.

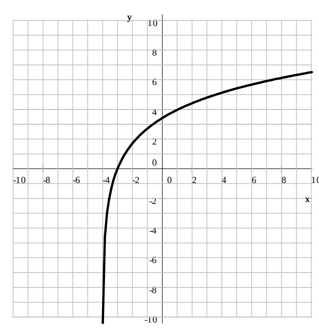
a)



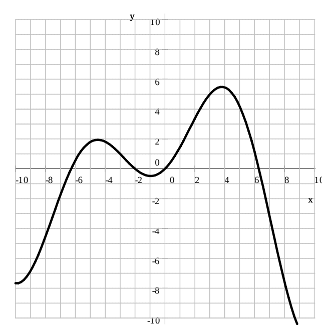
b)



c)



d)



9. Find the equation for the inverse for each of the following functions given.

a) $f(x) = 3x - 1$

b) $g(x) = \log_2(5x-1)$

c) $h(x) = 1 + 3e^{2x-7}$

d) $p(x) = (x-2)^3$

e) $m(x) = \dfrac{x-1}{2x+7}$

10. Differentiate each of the following, using the definition of the derivative.

a) $f(x) = \sqrt{2x-1}$ b) $f(x) = \dfrac{1}{x^2-1}$ c) $f(x) = \sqrt{1-x^2}$

11. Differentiate each of the following functions.

a) $f(x) = \dfrac{x^4 - 3x^2 + x - 2}{x^3}$ b) $f(y) = \dfrac{1}{y^2} + \dfrac{1}{y} + \dfrac{1}{\sqrt{y}} + \sqrt{y}$ c) $f(x) = 3x^5 - 2\sqrt[7]{x^3} + 8\sqrt[3]{x^7} - \pi^5$

12. Find an equation for the tangent line drawn to the graph of $f(x) = x^2 - \frac{2}{x}$ at $x = -2$.

13. We know the following things about a function f. $f'(x) = 20x^3 - 3$ and $f(-1) = 16$. Find a possible equation f.

14. Find the value of c for which the following is true: the tangent line drawn to the graph of $y = \frac{1}{x}$ has $x-$intercept $(6, 0)$.

15. Suppose that $f(x) = 12x^3 + bx^2 + cx + 7$. Find the exact values of b and c if we know that f has a relative maximum at $x = -2$ and a relative minimum at $x = 3$.

16. Prove that the function given is one-to-one: $f(x) = 3x^5 - 50x^3 + 390x - 1200$.

17. Find the value of a so that the line $y = 2x$ is a tangent line to the parabola $y = ax^2 + 5$.

18. Find a third degree polynomial $P(x)$ such that $P(0) = -5$, $P'(0) = 3$, $P''(0) = -6$ and $P'''(0) = 60$.

19. Find all values of p for which the given function is continuous on $\mathbb{R}$.

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < -3 \\ 2x^2 + px + 5 & \text{if } x \geq 3 \end{cases}$$

20. Find all values of p and q for which the given function is differentiable on $\mathbb{R}$.

$$f(x) = \begin{cases} -3x + 13 & \text{if } x < 2 \\ x^2 + px + q & \text{if } x \geq 2 \end{cases}$$

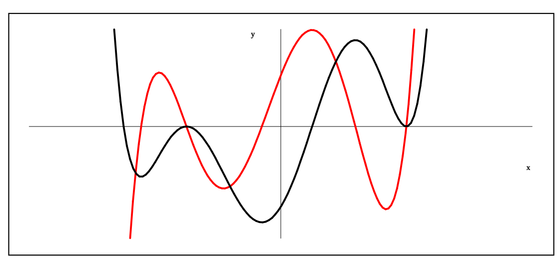
21. Find the $x-$coordinate of all relative maximums and minimums of each of the functions given below.

a) $f(x) = x^3(5x - 2)$ b) $f(x) = 2x + \frac{18}{x}$ c) $f(x) = x^3 + 3x^2 - 24x + 24$

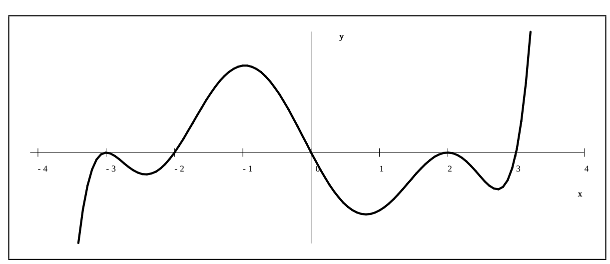
22. Find all relative and absolute maximums and minimums for the $f(x) = 6x^5 - 15x^4 + 5x^6 + 60$ on $[-3, 3]$. Sketch the graph of both f and f' on this domain.

23. Prove that the function $f(x) = \sin x - x$ does not have any relative minimums or maximums.

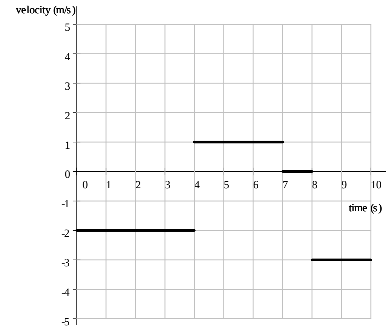
24. The graph below shows a function f and its first derivative, f'. Which is which?



25. The graph below shows f', the first derivative of a function f.
a) Find all values of x for which the function f has a local maximum at x.
b) Find all values of x for which the function f has a local minimum at x.



26. The picture shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\mathrm{m}}{\mathrm{s}}$. Positive direction is upward.).
a) Suppose that the object starts at a height of 5 m. Where is the object at $t = 10$?
b) Suppose that the object starts at a height of 9 m. Where is the object at $t = 10$?



27. We shoot a small object upward, from the top of a tower. The acceleration function of the object is $a(t) = -10$. (Location is measured in meters, velocity in $\frac{\mathrm{m}}{\mathrm{s}}$, acceleration in $\frac{\mathrm{m}}{\mathrm{s}^2}$.)

a) Given that $v(0) = 160$, find $v(t)$, the velocity function of the object.
b) Given that $h(0) = 525$, find $h(t)$, the location function of the object.
c) Find the maximum height that the object reaches.

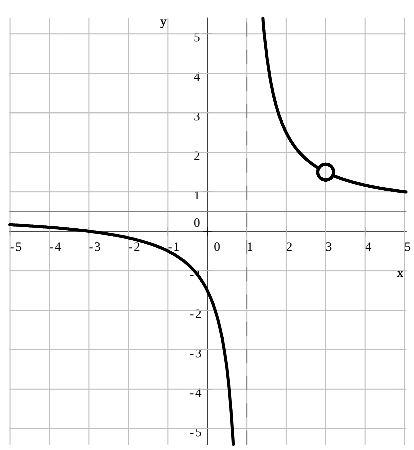
Answers

1.) a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) undefined h) $\frac{2}{3}$ i) $-\infty$ j) 0 k) ∞
l) 1 m) ∞ n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0

2.) a) -4 b) -8 c) $-\frac{2}{5}$ d) -1 e) undefined f) ∞ g) $\frac{\sqrt{5}}{10}$ h) $\frac{3}{2}$ i) undefined j) ∞
k) 1 l) $\frac{5}{4}$ m) $\frac{1}{2}$ n) 1 o) 2 p) $\frac{1}{20}$ q) $-\infty$ r) 3 s) $\frac{1}{2}$ t) -40 u) 2 v) 9

3.) a) $\frac{1}{2}$ b) 0 c) 0 d) 0 e) $-\infty$ f) ∞ g) undefined h) $\frac{3}{2}$ i) $\frac{3}{2}$ j) $\frac{3}{2}$ k) $\frac{1}{2}$

4.)



5.) a) undefined b) 2 c) undefined d) 4 e) 4
f) 4 g) 1 h) 2 i) undefined j) 3 k) 3
l) 3 m) ∞ n) undefined o) undefined

6.) See handout

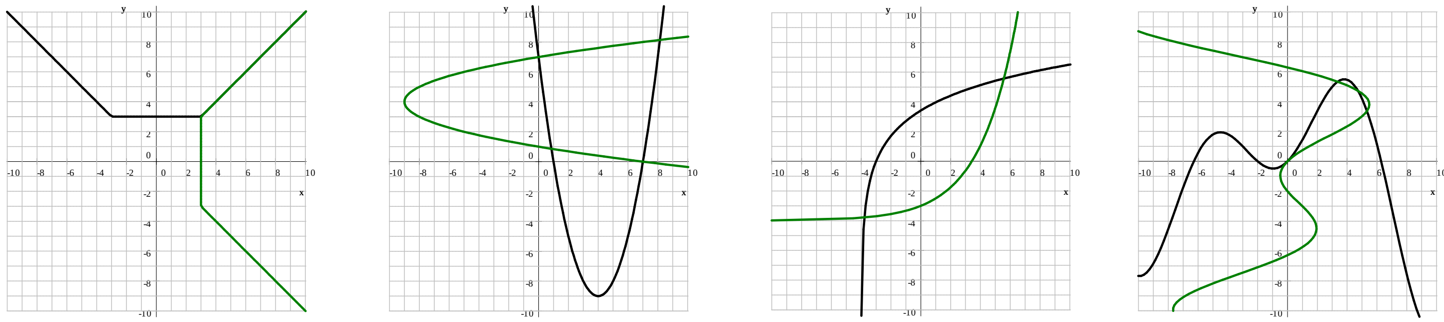
7.) a) $\tan\frac{\pi}{2}$ is undefined and the tangent function approaches infinity (and negative ininity) as x approaches $\frac{\pi}{2}$ In short, $\lim_{x\to\pi/2^-} \tan x = \infty$ and $\lim_{x\to\pi/2^+} \tan x = -\infty$. So the limit as stated is a $\frac{-\infty}{\infty}$ type of an indeterminate.

7.) b) $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \dfrac{1-\dfrac{\sin^2 x}{\cos^2 x}}{1+\dfrac{\sin^2 x}{\cos^2 x}}$ we multiply numerator and denominator by $\cos^2 x$.

$$\frac{1-\dfrac{\sin^2 x}{\cos^2 x}}{1+\dfrac{\sin^2 x}{\cos^2 x}} = \frac{1-\dfrac{\sin^2 x}{\cos^2 x}}{1+\dfrac{\sin^2 x}{\cos^2 x}} \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos 2x}{1} = \cos 2x$$

Thus the limit is $\displaystyle\lim_{x\to\pi/2} \frac{1-\tan^2 x}{1+\tan^2 x} = \lim_{x\to\pi/2} \cos 2x = \cos\pi = -1$

8.) a) b) c) d)



9.) a) $f^{-1}(x) = \dfrac{1}{3}x + \dfrac{1}{3}$ b) $g^{-1}(x) = \dfrac{1}{5}\left(2^x+1\right)$ c) $h^{-1}(x) = \dfrac{1}{2}\ln\left(\dfrac{x-1}{3}\right) + \dfrac{7}{2}$ d) $p^{-1}(x) = \sqrt[3]{x} + 2$

e) $m^{-1}(x) = \dfrac{7x+1}{-2x+1}$ 10.) see handout 11.) a) $f'(x) = 1 + \dfrac{3}{x^2} - \dfrac{2}{x^3} + \dfrac{6}{x^4}$

b) $f'(y) = -\dfrac{2}{y^3} - \dfrac{1}{y^2} - \dfrac{1}{2y\sqrt{y}} + \dfrac{1}{2\sqrt{y}}$ c) $f'(x) = 15x^4 - \dfrac{6}{7x}\sqrt[7]{x^3} + \dfrac{56}{3x}\sqrt[3]{x^7}$

12.) $y = -\dfrac{7}{2}x - 2$ 13.) $f(x) = 5x^4 - 3x + 8$ 14.) 3 15.) $b = -18, c = -216$

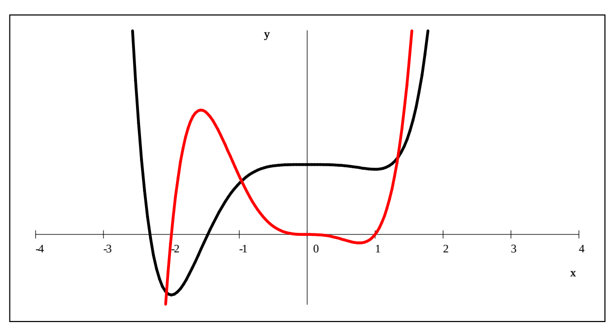
16.) $f'(x) = 15\left(\left(x^2-5\right)^2+1\right)$ is always positive, hence f is always strictly increasing. Thus one-to-one.

17.) $\dfrac{1}{5}$ 18.) $P(x) = 10x^3 - 3x^2 + 3x - 5$ 19.) $p = 10$ 20.) $p = -\dfrac{7}{2}$, $q = 10$

21.) a) no relative max, relative min at $x = \dfrac{3}{10}$ b) relative max at $x = -3$ relative min at $x = 3$

c) relative max at $x = -4$, relative min at $x = 2$

22.) a) $f'(x) = 30x^5 + 30x^4 - 60x^3$
$= 30(x+2)x^3(x-1)$
rel min: $(-2,-52)$ and $(1,56)$
abs min: $(-2,-52)$
rel max: $(0,60)$
abs max: $(-3,332)$



23.) $f'(x) = \cos x - 1$ is always negative or zero. Thus f' never changes sign, so f has no relative max or min

24.) the black graph is f, the red graph is f' 25.) a) $x = 0$ b) $x = -2$ and $x = 3$

26.) a) -6 b) -2 27.) a) $v(t) = -10t + 160$ b) $h(t) = -5t^2 + 160t + 525$ c) $h_{\max} = 1805\,\mathrm{m}$

