

1. Consider the function $f(x) = (x+1)^7(x-3)^8$
 - a) Sketch the graph of f .
 - b) Compute the derivative of f and factor it as much as possible.
 - c) Sketch the graph of the derivative.
 - d) Find the x -coordinate of all relative extrema for $f(x)$.

2. Find each of the following limits.

a) $\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{x^2 - 3x + 2}$	e) $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 - 9}$	i) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$	n) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos 3x}$
b) $\lim_{x \rightarrow 2^+} \frac{x^2 - 1}{x^2 - 3x + 2}$	f) $\lim_{x \rightarrow 3^+} \frac{x^2 + 3x}{x^2 - 9}$	j) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$	o) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^x$
c) $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$	g) $\lim_{x \rightarrow 0} \frac{\sqrt{16+x} - 4}{x}$	k) $\lim_{x \rightarrow \infty} \frac{3 + 2 \ln(x^3)}{2 - \ln 7x}$	p) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - 1}}$
d) $\lim_{x \rightarrow \infty} \frac{-x^3 + 2x + 1}{x - 3}$	h) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$	l) $\lim_{x \rightarrow \infty} \frac{3 \cdot 2^{2x-1}}{5 \cdot 3^{x+2}}$	q) $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 2x}$
		m) $\lim_{x \rightarrow \infty} \tan^{-1} x$	

3. Find the inverse for each of the following functions.

a) $f(x) = \sqrt[7]{3x-5}$ b) $f(x) = 2^{3x-1} + 1$ c) $f(x) = 2 \ln(x+8)$

4. Differentiate each of the following.

a) $f(x) = 7^x - x^7 + \sqrt{7x} - e^7$	f) $f(x) = \ln(\ln(x^5))$	k) $f(x) = \tan\left(2\pi x - \frac{\pi}{4}\right)$
b) $f(x) = \frac{e^x - e^{-x}}{2}$	g) $f(x) = \log_5\left(\sqrt[3]{5x^3 - 7x^2 + 8}\right)$	l) $f(x) = \sin^2 x + \cos^2 x$
c) $f(x) = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	h) $f(x) = \tan(3x)$	m) $g(x) = -\frac{1}{4x^2} - \frac{\ln x}{2x^2}$
d) $f(x) = \frac{8x^2 - 20x + 1}{4x - 10}$	i) $f(x) = \frac{1}{1+x^6}$	n) $f(\theta) = \frac{x^2}{x^2 + 1}$
e) $f(x) = \log_x(x^2 + 1)$	j) $f(x) = \frac{5^{2x-1}}{5^{x-4}}$	

5. Find the second derivative for the function $f(x) = \frac{5x-1}{5x+1}$.

6. Find all values of x for which the tangent line drawn to the graph of $f(x) = \frac{1}{2}x^2 - \ln x + 8$ is perpendicular to $x + 2y = -7$.

7. For each of the following functions, answer the following questions.

- i) For what interval(s) is f increasing and decreasing?
- ii) Find both coordinates of all relative extrema.
- iii) Find both coordinates of all absolute extrema.

- a) $f(x) = x^3 - 3x^2 - 9x + 5$ on $[-4, 5]$
- b) $f(x) = x(x-4)^4$ on $[-1, 5]$
- c) $f(x) = x^3e^{2x}$ on $[-2, 1]$

8. Compute each of the following indefinite integrals.

$$\begin{array}{llll}
 \text{a) } \int \frac{x-3}{x+2} dx & \text{e) } \int e^{2x} dx & \text{i) } \int 5^{2x-3} dx & \text{m) } \int (at+b) dt \\
 \text{b) } \int x^2+x+1+\frac{1}{x}+\frac{1}{x^2} dx & \text{f) } \int 5^x - x^5 dx & \text{j) } \int (3x-5)^{10} dx & \text{n) } \int (at+b) db \\
 \text{c) } \int x^5 - 2ax - a^2 dx & \text{g) } \int \sin 5x dx & \text{k) } \int \sqrt{7x+1} dx & \text{o) } \int \tan^2 x dx \\
 \text{d) } \int x^5 - 2ax - a^2 da & \text{h) } \int \frac{2x-1}{x+3} dx & \text{l) } \int \frac{3}{\sqrt[7]{x^4}} dx &
 \end{array}$$

9. Find the equation of $F(x)$ if $F'(x) = 10x^4 - 6x^2 + 4x - 5$ and $F(-1) = 8$.

10. An object is moving along a vertical line. Its acceleration, as a function of time, is $a(t) = 24t - 10$. After 5 seconds, the velocity of the object is $v(5) = -20$ and its height is $h(5) = 25$.

a) Find the velocity function $v(t)$. b) Find the location function $h(t)$.

11. Compute each of the following sums.

$$\text{a) } \sum_{n=1}^{25} (4n)^2 \quad \text{b) } \sum_{n=1}^{50} (3n^2 - 5n + 1) \quad \text{c) } \sum_{n=0}^{50} (3n^2 - 5n + 1) \quad \text{d) } \sum_{n=1}^{100} (n^2 + n + 1) \quad \text{e) } \sum_{n=50}^{100} (n^2 + n + 1)$$

12. We want to build a flowerbed in shape of a circular sector. If the perimeter is to be 20 feet, what radius and central angle would guarantee the maximal area for this flowerbed?

13. A box manufacturer plans to create an open box with a surface area of 30 ft^2 . What is the maximum size volume that can be formed by bending this material into a box? The box is to have a square base and rectangular sides.

14. Find all values of c that satisfy the statement of the Main Value Theorem for the given function.

$$\text{a) } f(x) = x^3 - 2x + 7 \text{ on domain } [-3, -1] \quad \text{b) } f(x) = x + \frac{3}{x} \text{ on } [1, 5] \quad \text{c) } f(x) = \sin x \text{ on } \left[0, \frac{\pi}{4}\right]$$

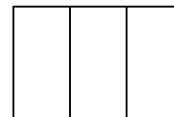
15. Let $P(x, y)$ be a point on the graph of $y = -x^2 + 1$ with $0 \leq x \leq 1$. Let $PQRS$ be a rectangle with one side on the x -axis and two vertices on the graph, as shown on the picture below. Find the exact value of the greatest possible area of such a rectangle.

16. Find two non-negative numbers x and y for which $2x + y = 30$, such that $x^5 y^3$ is maximized.

17. Find an equation of the tangent line drawn to the graph of $f(x) = \frac{1}{(3x-1)^2}$ at $x = 1$.

18. Which point on $y = 1 - x^2$ is closest to the origin?

19. We have P meters of fencing and want to create three adjacent rectangular enclosures as shown on the figure. What is the maximal area we can enclose this way?



20. Suppose that g is a function with $g(5) = 2$ and $g'(5) = -3$. Compute $f'(5)$ if $f(x)$ is defined as

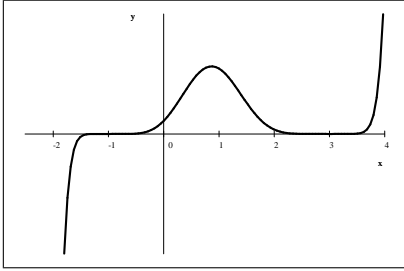
$$\text{a) } f(x) = [g(x)]^4 \quad \text{b) } f(x) = \ln(g(x)) \quad \text{c) } f(x) = \frac{1}{g(x)} \quad \text{d) } f(x) = \sqrt{g(x)}$$

21. Find the exact values of a and b so that the curve $y = x^2 + ax + b$ will be tangent to the line $y = 2x + 1$ at the point $(1, 3)$.

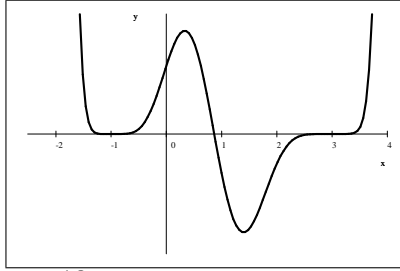
22. Find a polynomial function $P(x)$ such that P is of degree four (or less) and $f(0) = P(0)$, $f'(0) = P'(0)$, $f''(0) = P''(0)$, $f'''(0) = P'''(0)$, and $f^{(4)}(0) = P^{(4)}(0)$ if $f(x)$ is defined as
- a) $f(x) = \sin x$ b) $f(x) = e^{-x/2}$
23. Suppose that f is a function with derivative $f'(x) = (9 - x^2)(x + 2)^2(x - 3)(1 - x)x$
- a) Graph f' b) Find all x for which f has a relative maximum. c) Find all x for which f has a relative minimum.
24. A company finds that if x is the amount of money spent on research and y is the amount of money spent on advertising, then they can make a profit of $\sqrt[4]{xy^3}$. The company has \$50 000 to spend on these two activities. How much money should the company spend on research and on advertising to maximize its profit?
25. A company finds that if they produce q units of a product, then they can sell them all at a price of $p(q) = \sqrt{6300 - 0.02q}$. What production level would guarantee the maximum revenue?
26. Give a complete analysis of the function $f(x) = x\sqrt{1 - x^2}$
27. Prove the Mean Value Theorem.

Answers

1.) a) $f(x) = (x+1)^7(x-3)^8$



b-c) $f'(x) = (15x-13)(x+1)^6(x-3)^7$

d) relative min at $x = 3$ and relative max at $x = \frac{13}{15}$

2.) a) -2 b) ∞ c) $\frac{2}{3}$ d) $-\infty$ e) $\frac{1}{2}$ f) ∞ g) $\frac{1}{8}$ h) 1 i) $\frac{1}{6}$ j) e^2 h) -6 l) ∞

m) $\frac{\pi}{2}$ n) $\frac{2}{9}$ o) $\frac{1}{e^2}$ p) 1 q) $\frac{3}{2}$

3.) a) $f^{-1}(x) = \frac{x^7+5}{3}$ b) $f^{-1}(x) = \frac{1}{3}(\log_2(x-1)+1)$ c) $f^{-1}(x) = e^{x/2} - 8$

4.) a) $7^x \ln 7 - 7x^6 + \frac{7}{2\sqrt{7x}}$ b) $\frac{e^x + e^{-x}}{2}$ c) xe^{4x} d) $\frac{8x^2 - 40x + 49}{4x^2 - 20x + 25} = 2 - \frac{1}{(2x-5)^2}$

e) $\frac{\frac{2x \ln x}{x^2+1} - \ln(x^2+1) \left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{2x}{\ln x(x^2+1)} - \frac{\ln(x^2+1)}{x(\ln x)^2}$ f) $\frac{1}{x \ln x} - \frac{15x^2 - 14x}{3 \ln 5 (5x^3 - 7x^2 + 8)}$

h) $3 \tan^2 3x + 3 = \frac{3}{\cos^2(3x)}$ i) $\frac{-6x^5}{(x^6+1)^2}$ j) $\ln 5 \cdot 5^{x+3}$ k) $2\pi \sec^2\left(2\pi x - \frac{\pi}{4}\right)$

l) 0 m) $\frac{\ln x}{x^3}$ n) $\frac{2x}{(x^2+1)^2}$ 5.) $-\frac{100}{(5x+1)^3}$ 6.) $1 + \sqrt{2}$

7.) a) $f(x) = x^3 - 3x^2 - 9x + 5$ on the interval $[-4, 5]$

i) f increases on $(-4, -1)$, decreases on $(-1, 3)$, increases on $(3, 5)$ ii) rel. min: $(3, -22)$, rel. max: $(-1, 10)$
iii) abs. min: $(-4, -71)$ abs max: $(-1, 10)$ and $(5, 10)$

b) $f(x) = x(x-4)^4$ on the interval $[-1, 5]$

i) increases on $\left(-1, \frac{4}{5}\right)$, decreases on $\left(\frac{4}{5}, 4\right)$, and increases on $(4, 5)$ ii) rel. min: $(4, 0)$, rel. max: $\left(\frac{4}{5}, 83.886\right)$ iii) abs. min: $(-1, -625)$, abs. max: $\left(\frac{4}{5}, 83.886\right)$

c) $f(x) = x^3 e^{2x}$ on $[-2, 1]$

i) f decreases on $\left(-2, -\frac{3}{2}\right)$ and increases on $\left(-\frac{3}{2}, 1\right)$ ii) rel. min: $\left(-\frac{3}{2}, -\frac{27}{8}e^{-3}\right)$ there is no rel. max.iii) abs. min: $\left(-\frac{3}{2}, -\frac{27}{8}e^{-3}\right)$ abs max: $(1, e^2)$

8.) a) $x - 5 \ln|x+2| + C$ b) $\frac{x^3}{3} + \frac{x^2}{2} + x + \ln x - \frac{1}{x} + C$ c) $\frac{x^6}{6} - ax^2 - a^2x + C$ d) $x^5 a - xa^2 - \frac{a^3}{3} + C$

e) $\frac{1}{2}e^{2x} + C$ f) $\frac{5^x}{\ln 5} - \frac{x^6}{6} + C$ g) $-\frac{1}{5} \cos 5x + C$ h) $2x - 7 \ln|x+3| + C$ i) $\frac{5^{2x-3}}{2 \ln 5} + C$

j) $\frac{(3x-5)^{11}}{33} + C$ k) $\frac{2}{21}(7x+1)^{3/2} + C$ l) $7x^{3/7} + C$ m) $\frac{1}{2}at^2 + bt + C$ n) $\frac{1}{2}b^2 + atb + C$

o) $\tan x - x + C$ 9.) $F(x) = 2x^2 - 5x - 2x^3 + 2x^5 + 1$ 10.) a) $v(t) = 12t^2 - 10t - 270$

b) $h(t) = 4t^3 - 5t^2 - 270t + 1000$ 11. a) 88 400 b) 122 450 c) 122 451 d) 343 500 e) 301 801

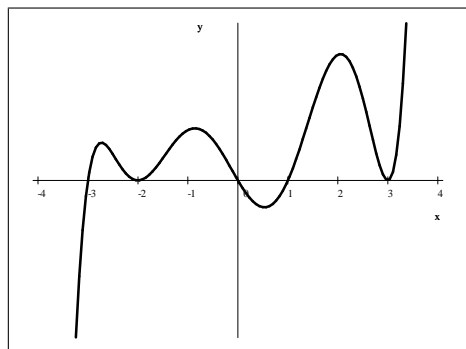
12.) $r = 5$ and $\alpha = 2\text{rad}$ 13.) $5\sqrt{10}\text{ft}^3 \approx 15.81139\text{ft}^3$

14.) a) $-\frac{\sqrt{39}}{3}$ b) $\sqrt{5}$ c) $\cos^{-1}\left(\frac{2\sqrt{2}}{\pi}\right) \approx 0.4503006\text{ rad}$ 15.) $\frac{4}{9}\sqrt{3}$ 16.) $x = \frac{75}{8}$ $y = \frac{45}{4}$

17.) $y = -\frac{3}{4}x + 1$ 18.) $\left(-\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$ and $\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$ 19.) $\frac{P^2}{32}$ 20.) a) -96 b) $-\frac{3}{2}$ c) $\frac{3}{4}$ d) $-\frac{3}{4}\sqrt{2}$

21.) $a = 0, b = 2$ 22.) a) $x - \frac{1}{6}x^3$ b) $1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{384}x^4$

23.) a) see below b) 0 c) -3, 1



24.) \$12 500 on research and \$37 500 on advertising. 25.) 210 000 units

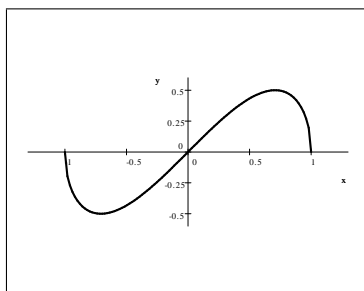
26.) $f(x) = x\sqrt{1-x^2}$ $f'(x) = \frac{-2x^2+1}{\sqrt{1-x^2}}$ $f''(x) = \frac{2x^3-3x}{(1-x^2)^{3/2}}$

domain: $[-1, 1]$ range: $\left[-\frac{1}{2}, \frac{1}{2}\right]$; continuous on $[-1, 1]$; y -intercept: $(0, 0)$;

x -intercepts: $(-1, 0)$, $(0, 0)$, and $(1, 0)$

decreasing on $\left(-1, -\frac{\sqrt{2}}{2}\right)$ and $\left(\frac{\sqrt{2}}{2}, 1\right)$, increasing on $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

relative and absolute minimum: $\left(-\frac{\sqrt{2}}{2}, -\frac{1}{2}\right)$, relative and absolute maximum: $\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$



27.) see handout