Quiz 12 will cover the following material (same as was for Exam 2)

All material covered in Classes 1-12

The following Sample Quiz is intended to demonstrate the difficulty level of the questions. It is not intended as a comprehensive review or list of the type of questions that can appear on the quiz.

Sample Quiz 12

1. Differentiate $f(x) = \dfrac{1}{2-x}$ by computing the limit of the difference quotient.

2. Compute each of the following limits.

   a) $\displaystyle\lim_{x \to 10^+} \frac{x+3}{\sqrt{x-10}}$ b) $\displaystyle\lim_{x \to \infty} \frac{\log_2(3x)}{\log_4(6x)}$ c) $\displaystyle\lim_{x \to \infty} \tan^{-1} x$

3. a) Prove the first version of the Intermediate Value Theorem.

   b) Prove the second version of the Intermediate Value Theorem.

   c) Prove the product rule of derivatives.

4. Differentiate each of the following.

   a) $f(x) = \sin x \left(2x^3 - x^2\right)$

   b) $g(x) = \dfrac{\cos x}{x}$

   c) $f(x) = \dfrac{x^6 - x^2}{x^4}$

   d) $f(x) = x^{10} \sin x$

5. Find all values of a and b for which the line $y = -3x - 18$ is a tangent line to $f(x) = 5x^3 + ax^2 + bx - 20$ at $x = 1$.

6. Suppose that an object's location function is given by $L(t) = t^3 - 9t^2 + 15t + 4$ on domain $[0, 5]$. Find the moment when the object is moving with the greatest speed. What is that greatest speed?

7. Find a degree 4 polynomial $P(x)$ with the following properties: $P(0) = -2$, $P'(0) = 5$, $P''(0) = -10$, $P^{(3)}(0) = 18$ and $P^{(4)}(0) = -48$

8. Find all relative maximums and minimums for $f(x) = 5x^4 - 10x^3 + 2x^5 - 2$.

Answers

1. Claim: $\dfrac{d}{dx}\left(\dfrac{1}{2-x}\right) = \dfrac{1}{(x-2)^2}$

   Proof:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\dfrac{1}{2-(x+h)} - \dfrac{1}{2-x}}{h} = \lim_{h\to 0} \frac{\dfrac{1}{2-x-h} - \dfrac{1}{2-x}}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{2-x}{(2-x-h)(2-x)} - \frac{2-x-h}{(2-x-h)(2-x)}\right) = \lim_{h\to 0} \frac{1}{h}\cdot\frac{2-x-(2-x-h)}{(2-x-h)(2-x)} \\
&= \lim_{h\to 0} \frac{1}{h}\cdot\frac{2-x-2-x+h}{(2-x-h)(2-x)} = \lim_{h\to 0} \frac{h}{h(2-x-h)(2-x)} \\
&= \lim_{h\to 0} \frac{1}{(2-x-h)(2-x)} = \frac{1}{(2-x)^2} = \boxed{\frac{1}{(x-2)^2}}
\end{aligned}$$

2. a) ∞ b) 2 c) $\dfrac{\pi}{2}$

3. see handout

4. a) $\cos x\left(2x^3 - x^2\right) + \sin x\left(6x^2 - 2x\right)$ b) $-\dfrac{1}{x}\sin x - \dfrac{1}{x^2}\cos x$ c) $2x + \dfrac{2}{x^3}$ d) $10x^9 \sin x + x^{10}\cos x$

5. $a = -12, b = 6$

6. $f'(0) = 15$

7. $P(x) = -2x^4 + 3x^3 - 5x^2 + 5x - 2$

8. relative minimum: $(1, -5)$ relative maximum: $(-3, 187)$

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