Review Problems

1. Simplify each of the following

   a) 6^{-2} c) $64^{-2/3}$ e) $8^{\log_2 5}$ g) $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 - 1$

   b) $32^{3/5}$ d) $\log_9 \sqrt{27}$ f) $3^{\log_9 10}$ h) $\log_2 (4x^2) - 3\log_2 \left(\frac{6}{x}\right) + \log_4 (4x^6)$

2. Let $x = \log_3 2$. Express each of the following in terms of x.

   a) $\log_3 72$ b) $\log_2 3$ c) $\log_6 72$

3. Suppose that $x = \log_2 5$ and $y = \log_{10} 9$. Express $\log_2 3$ in terms of x and y.

4. Factor $3x^2 - 4x - 319$ by completing the square.

5. In case of each polynomial given, determine (by completing the square) whether it factors or not. (You do not have to actually factor.)

   a) $20x + 2x^2 + 44$ b) $20x - 5x^2 - 25$

6. What is the remainder when we divide $3x^4 - 5x^2 + 2x - 3$

   a) by $x - 1$? b) by $x + 2$? c) $x^2 - 3$?

7. Solve each of the following equations.

   a) $2x^3 = 6x$ d) $9^x - 3^{x+1} = 18$ g) $\log_2 (x+5) - \log_2 (x-7) = -1$

   b) $2x^2 - 3x - 1 = 0$ e) $2^{3x-1} = 3^{x+2}$ h) $e^{2x} + 2xe^{2x} = 0$

   c) $\log_3 (7-x) + \log_3 (1-x) = 3$ f) $\log_6 (-8-x) + \log_6 (8-x) = 2$

8. Find the domain for each of the following functions.

   a) $f(x) = \ln\left(x^2 - 10x + 29\right)$ c) $h(x) = \sqrt{\dfrac{2x-1}{x+3}}$ e) $k(x) = \dfrac{1}{\log_5 (x^2 - 10x + 21)}$

   b) $g(x) = \dfrac{1}{\log_2 (4-x)}$ d) $f(x) = \log_5\left(x^2 - 10x + 21\right)$

9. Solve the inequality $\dfrac{2x-1}{x+3} \geq 5$

10. Find an equation for the inverse of each of the following functions.

    a) $f(x) = 3^{5x-1}$ c) $f(x) = \ln(2x-1)$ e) $p(x) = \dfrac{1}{3x^7 - 5}$

    b) $g(x) = \dfrac{x+4}{3x-5}$ d) $h(x) = \sqrt[3]{\dfrac{1}{2}x + 1}$

11. Let $f(x) = 2x - 1$ and $g(x) = \dfrac{x+3}{2-x}$. Find the formulas for the functions $f \circ g$ and $g \circ f$.

12. Find the value of the real number a so that the function $f(x) = \dfrac{-2x+5}{a+3x}$ is its own inverse.

13. We placed \$3000 in a bank account, with an annual compound interest rate of 8% (compounded annually). How long does it take until there is \$60 000 in the account?

14. Suppose that x and y are real numbers with $x + 3y = 20$. Compute the exact value of
    a) the smallest value of $x^2 + y^2$.
    b) the greatest value of xy.
    c) the smallest value of $(x - y)^2$.
    d) the greatest value of $y^2 - x^2$.

15. A citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. Find the total number of trees the grower should plant to maximize yield.

16. The cost of manufacturing q units of a product is given by $C(q) = 6q^2 + 10q$. Suppose we can sell all q units for a total of $142q + 1674$ dollars. Find the maximum profit we can achieve.

17. Graph each of the following equations in the same coordinate system.

$$f(x) = x + 3 \qquad g(x) = x - 3 \qquad h(x) = (x + 3)(x - 3) \qquad k(x) = \frac{1}{(x + 3)(x - 3)}$$

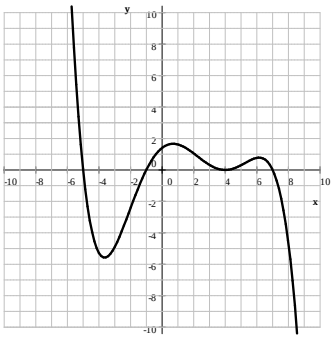
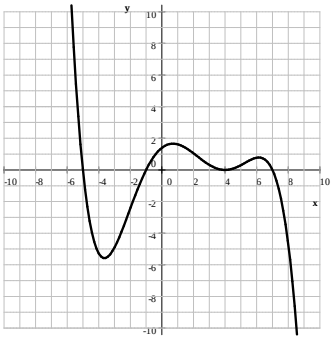
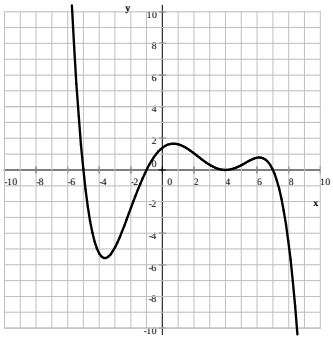
18. Graph each of the following pairs of functions in the same coordinate system.
    a) $f(x) = 2^x$ and $g(x) = \log_2 x$
    b) $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$
    c) $f(x) = \log_2 x$ and $g(x) = \log_{1/2} x$
    d) $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \log_{1/2} x$

19. An object's height (measured in feet) is defined by $s(t) = t^3 - 12t$ where t is the time, measured in seconds.
    a) Find the location of the object at $t = 3$ seconds.
    b) Find the average velocity of the object between
        i) $t = 0$ and $t = 2$ seconds
        ii) $t = 1$ second and $t = 2$ seconds
        iii) $t = 1.5$ seconds and $t = 2$ seconds

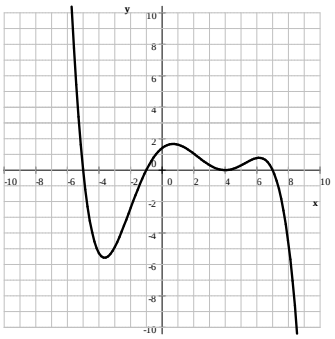
20. Find the coordinates of all points where the graphs of $f(x) = x^2 - 2x - 26$ and $g(x) = 2x - 5$ intersect each other.

21. Find an equation of the tangent line drawn to the circle $(x - 2)^2 + (y + 3)^2 = 50$ at the point $P(9, -4)$.

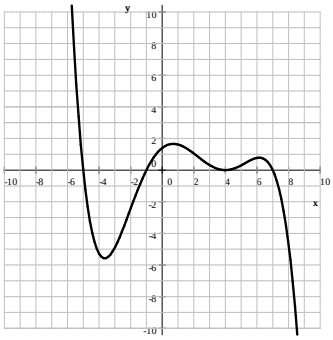
22. The picture below shows the graph of a function, $f(x)$. Graph each of the following.
    a) $g(x) = f(x - 2)$
    b) $g(x) = f(-x)$
    c) $g(x) = -2f(x)$

d) $g(x) = \dfrac{f(x) + |f(x)|}{2}$



e) $g(x) = \dfrac{1}{f(x)}$



23. Find each of the following limits.

a) $\lim\limits_{x\to-\infty}\left(-2x^5+8x^2\right)$

b) $\lim\limits_{x\to\infty}\left(-2x^5+8x^2\right)$

c) $\lim\limits_{x\to-\infty}\left(-2x^5+8x^6\right)$

d) $\lim\limits_{x\to\infty}\left(-2x^5+8x^6\right)$

e) $\lim\limits_{x\to-\infty}\dfrac{3x^2-1}{5x^2-3x+2}$

f) $\lim\limits_{x\to-\infty}\dfrac{100x-1}{5x^2-3x+2}$

g) $\lim\limits_{x\to-\infty}\log_2 x$

h) $\lim\limits_{x\to\infty}\dfrac{2x^2+3x+1}{3x^2-5x+2}$

i) $\lim\limits_{x\to\infty}\dfrac{-x^3+2x+1}{x-3}$

j) $\lim\limits_{x\to-\infty}2^x$

k) $\lim\limits_{x\to\infty}\left(\log_2\left(x^2-5x+17\right)\right)$

l) $\lim\limits_{x\to\infty}\dfrac{12+\log_7 3x}{15+\log_7 x}$

m) $\lim\limits_{x\to\infty}\dfrac{2^{x+5}}{4^{x-1}}$

n) $\lim\limits_{x\to\infty}\dfrac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

o) $\lim\limits_{x\to\infty} x\left(\dfrac{1}{3}-\dfrac{1}{3-\frac{1}{x}}\right)$

p) $\lim\limits_{x\to\infty}\dfrac{\sqrt{4-\frac{1}{x}}-2}{\frac{1}{x}}$

q) $\lim\limits_{x\to-\infty}\dfrac{\cos x-2}{x^3+1}$

Review Problems - Answers

1. a) $\dfrac{1}{36}$ b) 8 c) $\dfrac{1}{16}$ d) $\dfrac{3}{4}$ e) 125 f) $\sqrt{10}$ g) $\log_3 2 = \dfrac{\ln 2}{\ln 3}$ h) $\log_2\left(\dfrac{x^8}{27}\right)$

2. a) $3x+2$ b) $\dfrac{1}{x}$ c) $\dfrac{3x+2}{x+1}$

3. $\dfrac{1}{2}y(x+1)$ Solution: $y = \log_{10} 9 = \dfrac{\log_2 9}{\log_2 10} = \dfrac{\log_2 3^2}{\log_2(2\cdot 5)} = \dfrac{2\log_2 3}{\log_2 2+\log_2 5} = \dfrac{2\log_2 3}{1+x}$

$y = \dfrac{2\log_2 3}{1+x} \implies \log_2 3 = \dfrac{1}{2}y(x+1)$

4. $3\left(x+\dfrac{29}{3}\right)(x-11) = (3x+29)(x-11)$

5. a) factors b) does not factor

6. a) -3 b) 21 c) $2x+9$

7. a) $0, -\sqrt{3}, \sqrt{3}$ b) $\dfrac{3 \pm \sqrt{17}}{4}$ c) -2 d) $\log_3 6 = \dfrac{\ln 6}{\ln 3}$ e) $\log_{8/3} 18 = \dfrac{\ln 18}{\ln 8 - \ln 3}$
   f) -10 g) no solution h) $-\dfrac{1}{2}$

8. a) $\mathbb{R}$ b) $x < 4$ but $x \neq 3$ c) $x < -3$ or $x \geq \dfrac{1}{2}$ d) $x < 3$ or $x > 7$
   e) $x < 3$ but $x \neq 5 - \sqrt{5}$ or $x > 7$ but $x \neq 5 + \sqrt{5}$

9. $-\dfrac{16}{3} \leq x < -3$

10. a) $f^{-1}(x) = \dfrac{1}{5}(\log_3 x + 1)$ b) $g^{-1}(x) = \dfrac{5x+4}{3x-1}$ c) $f^{-1}(x) = \dfrac{1}{2}(e^x + 1)$
    d) $h^{-1}(x) = 2(x^3 - 1)$ e) $p^{-1}(x) = \sqrt[7]{\dfrac{5x+1}{3x}}$

11. $(f \circ g)(x) = \dfrac{3x+4}{-x+2}$ $(g \circ f)(x) = \dfrac{-2x-2}{2x-3}$

12. 2

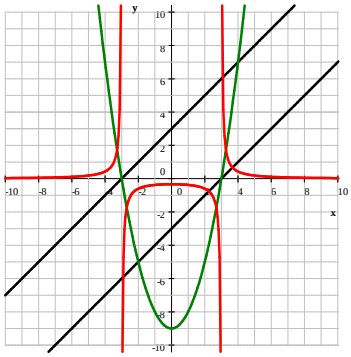
13. In the 39th year $(\log_{1.08} 20 = \dfrac{\ln 20}{\ln 1.08} \approx 38.925\,31$ years $)$

14. a) 40 when $x = 2, y = 6$ b) $\dfrac{100}{3}$ when $x = 10, y = \dfrac{10}{3}$
    c) 0 when $x = 5, y = 5$ d) 50 when $x = -\dfrac{5}{2}, y = \dfrac{15}{2}$

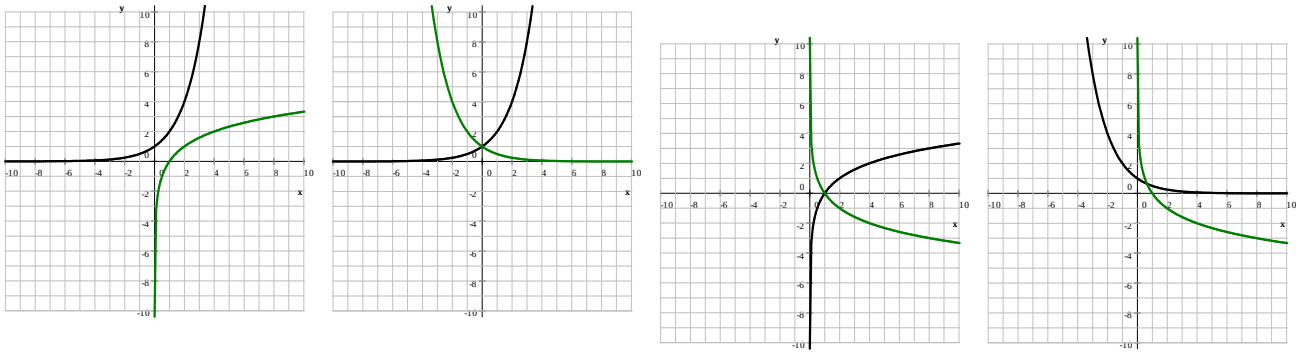
15. If we plant 80 trees, then we will obtain a maximal yield of 25 600 oranges.

16. \$2400

17. $f(x) = x + 3$ - black graph $h(x) = (x+3)(x-3)$ - green graph
    $g(x) = x - 3$ - black graph $k(x) = \dfrac{1}{(x+3)(x-3)}$ - red graph



18. a) b) c) d)

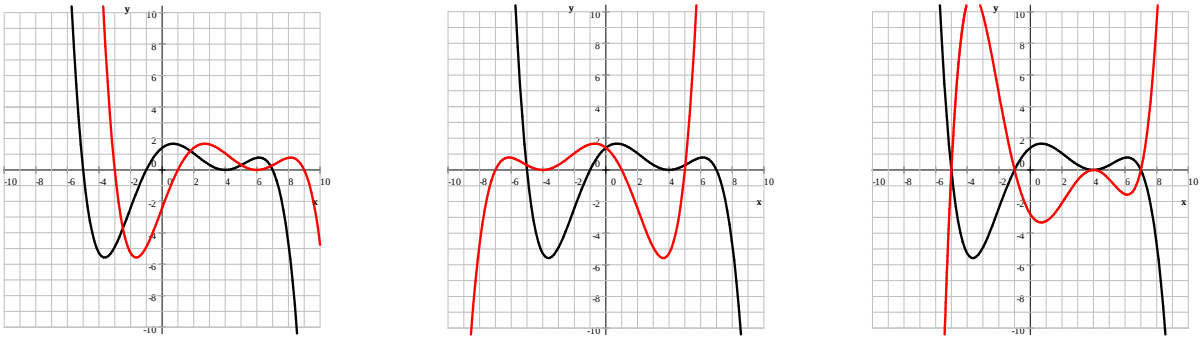


19. a) $-9\,\text{ft}$ b) i) $-8\frac{\text{ft}}{\text{s}}$ ii) $-5\frac{\text{ft}}{\text{s}}$ iii) $-2.75\frac{\text{ft}}{\text{s}}$

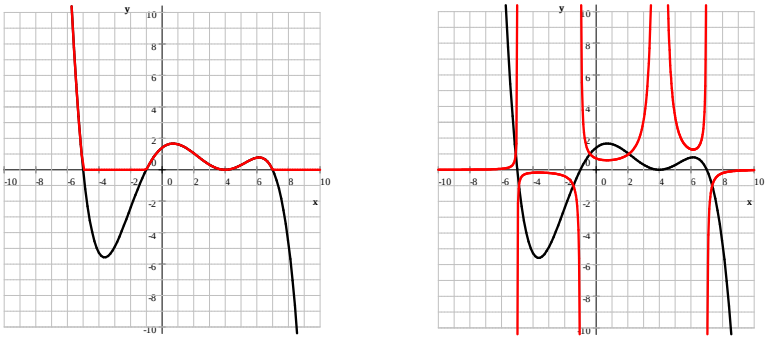
20. $(7, 9)$ and $(-3, -11)$

21. $y = 7x - 67$

22. a) $g(x) = f(x-2)$ b) $g(x) = f(-x)$ c) $g(x) = -2f(x)$



d) $g(x) = \dfrac{f(x) + |f(x)|}{2}$ e) $g(x) = \dfrac{1}{f(x)}$



23. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) undefined h) $\frac{2}{3}$ i) $-\infty$ j) 0
k) ∞ l) 1 m) 0 n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0