

Review Problems

1. What is the value of $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$?
A) 0 B) 1 C) $\frac{1}{3}$ D) 3 E) None of these
2. Find $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{2 + x - 4x^2}$.
A) -2 B) $\frac{1}{2}$ C) $-\frac{1}{4}$ D) 1 E) Does not exist
3. Given that $f(x) = \frac{x^2 - 4}{x + 2}$. Which of the following statements is true?
A) $f(x)$ is continuous everywhere.
B) $f(x)$ is removably discontinuous at $x = -2$.
C) $f(x)$ is nonremovably discontinuous at $x = -2$.
D) $f(x)$ is removably discontinuous at $x = -2$ and removably discontinuous at $x = 2$.
E) $f(x)$ is removably discontinuous at $x = -2$ and nonremovably discontinuous at $x = 2$.
4. Given that $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$, which of the following statements is true about $f(x)$?
A) $\lim_{x \rightarrow 1} f(x)$ exists. D) Both A and B are true
B) $f(1)$ is defined. E) All A, B, and C are true
C) f is continuous at $x = 1$.
5. Given that $f(x) = 2(\sin x)^3$. Find $f'(x)$.
A) $2(\cos x)^3$ B) $6(\sin x)^2$ C) $2(\sin x)^2 \cos x$ D) $6(\sin x)^2 \cos x$ E) none of these
6. Let $f(x) = 2 \ln(4e^{-3x})$. Find the value of $f'(0)$.
A) -6 B) 0 C) 1 D) 2 E) none of these
7. Let $f(x) = \arctan(\cos 2x)$. What is the value of $f'(\pi)$?
A) 0 B) 1 C) -1 D) -2 E) none of these

8. The area of the region in the first quadrant between the the graph of $y = x\sqrt{4 - x^2}$ and the x -axis is
- A) $\frac{2\sqrt{2}}{3}$ B) $\frac{8}{3}$ C) $2\sqrt{2}$ D) $2\sqrt{3}$ E) $\frac{16}{3}$
9. Given that $f(2) = 8$, $f'(2) = 3$, and $g(x) = \frac{1}{\sqrt[3]{(f(x))^2}}$. Find the exact value of $g'(2)$.
- A) -2 B) $\frac{1}{2}$ C) $\frac{2}{3\sqrt[3]{2}}$ D) $-\frac{1}{16}$ E) none of these
10. Let $f(x) = \ln \sqrt{x}$. Evaluate the second derivative of f at $x = 2$.
- A) $-\frac{1}{8}$ B) 0 C) -1 D) $\frac{3}{4}$ E) none of these
11. An antiderivative of $f(x) = \cos(3x + 6)$ is given by
- A) $\sin(3x + 6)$ C) $\frac{1}{3}\sin(3x + 6)$ E) none of these
- B) $-3\sin(3x + 6)$ D) $\sin\left(\frac{3}{2}x^2 + 6x\right)$
12. Find $\frac{dy}{dx}$ at the point $(1, 1)$ for the equation $x + 2xy = 3$.
- A) $-\frac{5}{2}$ B) $-\frac{3}{2}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) none of these
13. A particle with velocity at any time given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?
- A) $e^2 - 1$ B) $e - 1$ C) $2e$ D) e^2 E) none of these
14. If f is integrable on the interval $[a, b]$, then the **average value** of f on the interval $[a, b]$ is defined
- $$\frac{\int_a^b f(x) dx}{b - a}.$$
- as $\frac{\int_a^b f(x) dx}{b - a}$. Find the average value of $f(x) = \frac{1}{x + 1}$ on the interval $[0, 3]$.
- A) $\frac{1}{2}$ B) $\frac{\ln 2}{3}$ C) $\frac{\ln 2}{2}$ D) $\frac{2}{3}\ln 2$ E) none of these
15. Let f be the function defined by $f(x) = (x - 3)e^x$. Find all critical numbers of $f(x)$.
- A) $x = 1$ B) $x = 2$ C) $x = 3$ D) $x = -2$ E) none of these

16. Evaluate $\int \frac{x^2 + x + 1}{x^2 + 1} dx$
- A) $x + \ln(x^2 + 1) + C$ C) $1 + \frac{1}{2} \ln(x^2 + 1) + C$ E) none of these
B) $x + \frac{1}{2} \ln(x^2 + 1) + C$ D) $1 + \ln(x^2 + 1) + C$
17. Simplify the expression $\frac{d}{dx} \left[\int_1^x \frac{1}{\sqrt{2t^2 - 1}} dt \right]$
- A) $\frac{2x}{\sqrt{2x^2 - 1}}$ B) $\frac{1}{\sqrt{2x^2 - 1}}$ C) $2x\sqrt{2x^2 - 1}$ D) $\frac{1}{\sqrt{2x^2 - 1}} - 1$ E) none of these
18. Compute the slope of the tangent line drawn to the graph of $y = (\ln x)e^x$ at $x = 2$.
- A) e B) $\frac{1}{2}e^2$ C) $(2\ln 2 + 1)e$ D) $\left(\ln 2 + \frac{1}{2}\right)e^2$ E) none of these
19. Compute the area of the region bounded by the graph of $y = xe^{x^2}$, the x -axis, the lines $x = 0$ and $x = 2$.
- A) $\frac{1}{2}e^3$ B) $\frac{1}{2}(e^4 - e)$ C) $\frac{1}{2}(e^4 - 1)$ D) $\frac{1}{2}e^4$ E) none of these
20. Simplify $\int \tan^2 x dx$.
- A) $2 \tan x \sec x + C$ C) $\sin x - \tan x + C$ E) none of these
B) $\tan x - x + C$ D) $\frac{1}{3} \tan^3 x + C$
21. The equation of the tangent line to $f(x) = \sqrt{2x + 1}$ drawn to $x = 4$ is
- A) $y + 3 = -\frac{2}{3}(x + 4)$ C) $y + 3 = -\frac{1}{3}(x + 4)$ E) none of these
B) $y - 3 = \frac{1}{3}(x - 4)$ D) $y - 3 = \frac{2}{3}(x - 4)$
22. Find all values of x for which the function $f(x) = \frac{1}{x^2 - 1}$ is increasing.
- A) $x \in (-\infty, -1) \cup (-1, 0)$ C) $x \in (-\infty, -1) \cup (0, 1)$ E) $x \in (-\infty, \infty)$
B) $x \in (-\infty, -1) \cup (1, \infty)$ D) $x \in (-1, 1)$
23. Given that $f(x) = \arcsin(x^2)$, compute $f'(x)$.
- A) $\frac{1}{\sqrt{1 - x^2}}$ B) $\frac{1}{\sqrt{1 - x^4}}$ C) $\frac{2x}{\sqrt{1 - x^2}}$ D) $\frac{2x}{\sqrt{1 - x^4}}$ E) none of these

24. Oil is leaking from a container at the rate of $R(t) = 2000e^{-0.2t}$ gallons per hour, where t is time, measured in hours. How much oil has leaked out of the container after 10 hours? Round your answer to the nearest gallon.

- A) 54 gallons B) 271 gallons C) 865 gallons D) 8647 gallons E) 14 778 gallons

25. All edges of a cube are expanding at a rate of $3\frac{\text{cm}}{\text{min}}$. How fast is the surface area of the cube changing when each edge is 2 cm long?

- A) $12\frac{\text{cm}^2}{\text{min}}$ B) $24\frac{\text{cm}^2}{\text{min}}$ C) $36\frac{\text{cm}^2}{\text{min}}$ D) $72\frac{\text{cm}^2}{\text{min}}$ E) none of these

26. If $y = x + \sin(xy)$, then $\frac{dy}{dx} =$

- A) $1 + \cos(xy)$ C) $\frac{1 + y \cos(xy)}{1 - x \cos(xy)}$ E) none of these
B) $1 + y \cos(xy)$ D) $\frac{1}{1 - x \cos(xy)}$

27. The demand for life insurance, L , and the demand for health insurance, H , can be modeled as functions of time, t :

$$\begin{aligned}L(t) &= t^3 + 9t + 100 \quad \text{for } 0 \leq t \leq 4 \\H(t) &= 6t^2 + 102 \quad \text{for } 0 \leq t \leq 4\end{aligned}$$

During the time period for $0 \leq t \leq 4$, the greatest difference between the two demands occurs n times. Determine n .

- A) 1 B) 2 C) 3 D) 4 E) none of these

28. In a certain town, the rate of deaths at time t due to a particular disease is modeled by $\frac{10000}{(t+3)^3}$ where $t \geq 0$. What is the total number of deaths from this disease predicted by the model?

- A) 243 B) 370 C) 556 D) 1111 E) none of these

Answers

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|-------|-------|-------|
| 1. D | 11. C | 21. B |
| 2. C | 12. B | 22. A |
| 3. B | 13. A | 23. D |
| 4. E | 14. D | 24. D |
| 5. D | 15. B | 25. D |
| 6. A | 16. B | 26. C |
| 7. A | 17. B | 27. D |
| 8. B | 18. D | 28. C |
| 9. D | 19. C | |
| 10. A | 20. B | |