

1. Prove each of the following identities.

a) $(\sin x + \cos x)^2 = 1 + \sin 2x$ b) $1 + \tan^2 x = \sec^2 x$

2. Differentiate each of the following.

a) $f(x) = (3x - 5)^{10}$ d) $f(x) = \frac{1}{5x + 1}$ g) $f(x) = \sqrt{r^2 - x^2}$

b) $f(x) = \frac{1}{2x - 10}$ e) $f(x) = \ln(x^2 - 6x + 9)$

c) $f(x) = \tan x$ f) $f(x) = 5(\ln x)^4 - 2(\ln x)^3 + \ln x - 5$

3. Compute the indefinite integrals.

a) $\int \sin 2x \, dx$ b) $\int (2x - 1)^3 \, dx$ c) $\int \frac{1}{3x - 1} \, dx$

4. Evaluate the indefinite integral $\int \tan^2 x \, dx$. Hint: look at problems 1b) and 2c).

5. Find an equation for the tangent line drawn to the graph of $f(x) = \frac{3x - 1}{x + 2}$ at $x = -1$.

6. Suppose that f is a differentiable odd function. Prove that then f' is an even function. (Recall that f is an even function if for all x in its domain, $f(-x) = f(x)$ and f is an odd function if for all x in its domain, $f(-x) = -f(x)$).

7. Find $g(x)$, where $g(x)$ is a polynomial of degree 4, and satisfies

$$f(0) = g(0), \quad f'(0) = g'(0), \quad f''(0) = g''(0), \quad f'''(0) = g'''(0), \quad \text{and} \quad f^{(4)}(0) = g^{(4)}(0)$$

where $f(x)$ is given as

a) $f(x) = \frac{1}{x + 1}$ b) $f(x) = \ln(x + 1)$

8. Find the x -coordinate of all relative extrema for

a) $f(x) = (x - 5)^8(x + 1)^3$ b) $f(x) = \frac{(x + 1)(x - 2)}{x^2}$

9. A company determines that if n is the number of items produced, they can all be sold at a price of $p(n) = \sqrt{9000 - 0.3n}$. How many items should be produced for a maximal revenue?