1. Simplify: a) $\dfrac{x^3-9x}{x^2-7x+12}$ b) $\dfrac{x^2-2x-8}{x^3+x^2-2x}$ c) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ d) $\dfrac{9-x^{-2}}{3+x^{-1}}$

2. Rationalize the denominator: a) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$ b) $\dfrac{4}{1-\sqrt{5}}$

3. Assume that all variables represent positive numbers. Write each of the following expressions in the form $c\,a^p b^q$ where c, p, q are numbers:

   a) $\dfrac{\left(2a^2\right)^3}{b}$ b) $\sqrt{9ab^3}$ c) $\dfrac{a\left(\frac{2}{b}\right)}{\frac{3}{a}}$ d) $\dfrac{ab-a}{b^2-b}$ e) $\dfrac{a^{-1}}{\left(b^{-1}\right)\sqrt{a}}$ f) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2\left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

4. Solve for x. Do not use a calculator.

   a) $5^{x+1}=25$ b) $\dfrac{1}{3}=3^{2x+2}$ c) $\log_2 x=3$ d) $\log_3 x^2=2\log_3 4-4\log_3 5$

5. Simplify a) $\log_2 5+\log_2\left(x^2-1\right)-\log_2(x-1)$ b) $3^{2\log_3 5}$ c) $2\log_4 9-\log_2 3$

6. Simplify a) $\log_{10}\left(10^{1/2}\right)$ b) $\log_{10}\left(\dfrac{1}{10^x}\right)$ c) $2\log_{10}\sqrt{x}+3\log_{10}x^{1/3}$

7. Solve the following equations for the indicated variables.

   a) $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$, for a b) $S=2(ab+bc+ac)$, for a

   c) $A=2\pi r^2+2\pi rh$, for positive r d) $A=P+nPr$, for P

   e) $2x-2yd=y+xd$, for d f) $\dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0$ for x

8. Find the vertex for each of the following parabolas.

   a) $y=x^2+4x+3$ b) $3x^2+3x+2y=0$ c) $9y^2-6y-9-x=0$

9. Find all real solutions to each of the following equations.

   a) $x^6-16x^4=0$ b) $4x^3-8x^2-25x+50=0$ c) $8x^3+27=0$ d) $x^4=1$

10. Solve the equations: a) $4x^2+12x+3=0$ b) $2x+1=\dfrac{5}{x+2}$ c) $\dfrac{x+1}{x}-\dfrac{x}{x+1}=0$

11. Find the remainders on division of

    a) $x^5-4x^4+x^3-7x+1$ by $x+2$ b) $x^5-x^4+x^3+2x^2-x+4$ by x^3+1

12. The equation $12x^3-23x^2-3x+2=0$ has a solution $x=2$. Find all other solutions.

13. Solve the inequalities a) $x^2+2x-3\le 0$ b) $\dfrac{2x-1}{3x-2}\le 1$ c) $x^2+x+1>0$

14. Solve for x : a) $|5x-2|=8$ b) $|2x+1|=|x+3|$

15. Determine the equations of the following lines:

    a) the line through $(-1,3)$ and $(2,-4)$

    b) the line through $(-1,2)$ and perpendicular to the line $2x-3y+5=0$

    c) the line through $(2,3)$ and the midpoint of the line segment from $(-1,4)$ to $(3,2)$.

16. a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$

    b) Shade the region in the xy-plane that is described by the inequalities $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$.

17. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find

    a) the center and radius b) the equation of the tangent line at $(-2, 5)$

18. Find the equations of the following circles:

    a) the circle with center at $(1, 2)$ that passes through the point $(-2, -1)$

    b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the x- and y-axes, respectively.

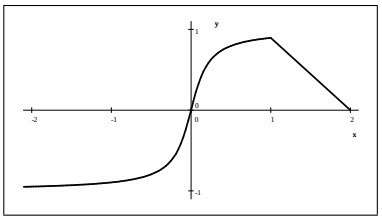
    c) A circle is tangent to the $y-$axis at $y = 3$ and has one $x-$intercept at $x = 1$.

19. a) Find the domain and range of the functions: i) $f(x) = 7$ ii) $g(x) = \dfrac{5x - 3}{2x + 1}$

    b) Find the domain of the function $f(x) = \dfrac{3x + 1}{\sqrt{x^2 + x - 2}}$

20. Simplify $\dfrac{f(x + h) - f(x)}{h}$ where a) $f(x) = 2x + 3$ b) $f(x) = x^2$ c) $f(x) = \dfrac{1}{x + 1}$

21. The graph of a function $y = f(x)$ is given as follows:



    Plot the graph of each of the following functions: a) $f(x + 1)$ b) $f(-x)$ c) $|f(x)|$ d) $f(|x|)$

22. Sketch the graph of the functions: a) $g(x) = |3x + 2|$ b) $h(x) = |x(x - 1)|$

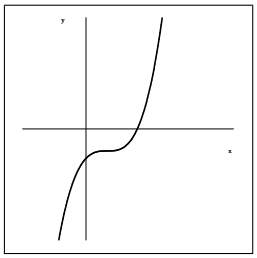
23. a) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$

    b) The graph of a quadratic function (a parabola) has $x-$intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.

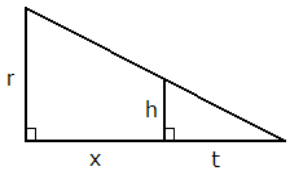
24. Find the inverse of the following functions.

    a) $f(x) = 2x + 3$ b) $f(x) = \dfrac{x + 2}{5x - 1}$ c) $f(x) = x^2 + 2x - 1, \quad x > 0$

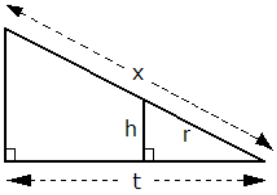
25. A function $f(x)$ has the following graph. Sketch the graph of the inverse function $f^{-1}(x)$.



26. Express x in terms of the other variables in the picture.



a)

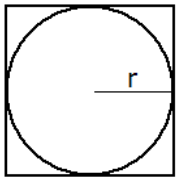


b)

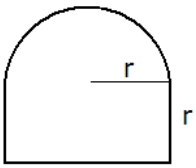
27. A curve is traced by a point $P(x, y)$ which moves such that its distance fom the point $A(-1, 2)$ is twice its distance from the point $B(2, -1)$. Determine the equation of the curve.

28. Let $f(x) = \dfrac{|x|}{x}$. Show that $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$. Find the domain and range of $f(x)$.

29. a) Find the ratio of the area inside the square but outside the circle to the area of the square in picture (a) below.



a)



b)

b) Find a formula for the perimeter of a window of the shape on picture (b) above.

c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10 m high and has a radius of 3 m at the top. If the water is 5 m deep (in the middle) what is the surface area of the top of the water?

d) Two cars start moving from the same point. One travels south at $100\dfrac{\text{km}}{\text{h}}$ (kilometer per hour), the other west at $50\dfrac{\text{km}}{\text{h}}$. How far apart are they two hours later?

e) A kite is 100 m above the ground. If there is 200 m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)

30. Without using a calculator, evaluate each of the following:

a) $\cos 210°$ b) $\sin \dfrac{5\pi}{3}$ c) $\tan^{-1}(-1)$ d) $\sin^{-1}(-1)$

e) $\cos \dfrac{9\pi}{4}$ f) $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ g) $\tan\left(\dfrac{7\pi}{6}\right)$ h) $\cos^{-1}(-1)$

31. Solve for x : a) $3\sin^2 x = \cos^2 x; \quad 0 \le x \le 2\pi$ b) $\cos^2 x - \sin^2 x = \sin x; \quad -\pi \le x \le \pi$
c) $\tan x + \sec x = 2\cos x; \quad -\infty < x < \infty$

32. Assume the following. For all real numbers x and y,

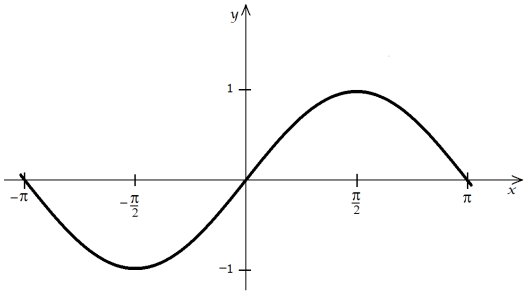
$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y \quad \text{and} \\ \sin(x+y) &= \sin x \cos y + \cos x \sin y \end{aligned}$$

Use the statements above to prove each of the following identities.

a) $\sin 2x = 2\sin x \cos x$ b) $\cos 2x = \cos^2 x - \sin^2 x$ c) $\cos 2x = 2\cos^2 x - 1$

d) $\cos 2x = 1 - 2\sin^2 x$ e) $\left|\cos \frac{x}{2}\right| = \sqrt{\frac{1+\cos x}{2}}$ f) $\left|\sin \frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{2}}$

33. Given the graph of $y = \sin x$, sketch the graphs of



a) $y = \sin\left(x - \frac{\pi}{4}\right)$ b) $y = \sin\left(\frac{x}{2}\right)$ c) $y = 2\sin x$ d) $y = \cos x$ e) $y = \frac{1}{\sin x}$