Topics:

1. How to find absolute max/min
2. Optimization

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Absolute max and min:

1. If $f(x)$ is continuous on a closed interval $[a,b]$, then it is easy.
2. If not, then it is often difficult and individual problems require different methods.

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Case 1. If f is continuous on a closed interval $[a,b]$, then f has an absolute minimum an absolute maximum, and f takes all values inbetween in $[a,b]$

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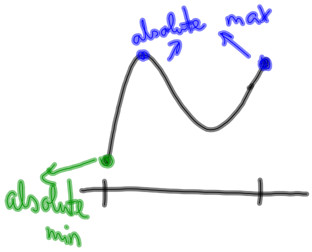
How do we find absolute max?



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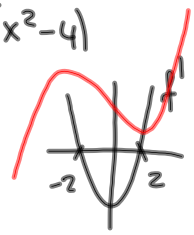
$f(x) = x^3 - 12x$ on $[-3, 10]$

$f'(x) = 3x^2 - 12 = 3(x^2-4)$
$= 3(x+2)(x-2)$

rel. max at $x=-2$
rel. min at $x=2$



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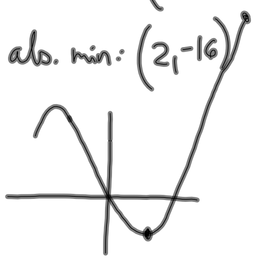
For absolute extrema, evaluate

$f(-3) = 9$
$f(-2) = 16$
$f(2) = -16$
$f(10) = 880$

abs. max: $(10, 880)$
abs. min: $(2, -16)$



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Step 1. Find all relative max/min
Step 2. Evaluate f at rel. max/min AND at endpoints.

The highest one is an abs. max
The lowest one is an abs. min.

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Steps to Optimization

1) Call something x (and write it down) There might be other variable labels such as y or z.
2) Use the constrains to eliminate all but 1 variables.
3) What are we asked to maximize/minimize? Express THAT as a function of x. (Discuss domain.)
4) Differentiate
5) Solve for the zero of the derivative
6) Is what we found a maximum or a minimum?
7*) Is what we found an absolute max/min?
8) Read the question again and make sure to provide the quantity that is asked.

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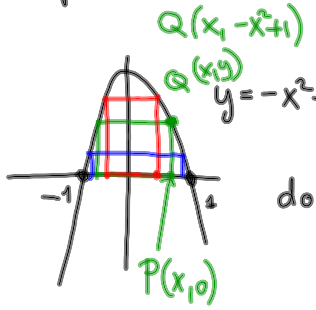
Optimization



domain: $0 \leq x \leq 1$

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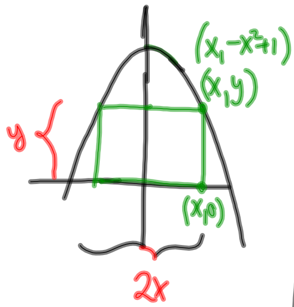
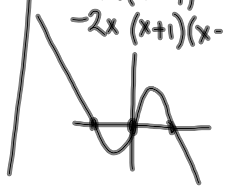


$A = (2x)\,y$

$A(x) = 2x(-x^2+1)$

upside down cubic
$-2x(x^2-1)$
$-2x(x+1)(x-1)$



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$A(x) = 2x(-x^2+1)$

$A(x) = -2x^3+2x$

$A'(x) = -6x^2+2$

$= -6\left(x^2-\frac{1}{3}\right) = -6\left(x^2-\left(\sqrt{\frac{1}{3}}\right)^2\right)$

$= -6\left(x+\frac{1}{\sqrt{3}}\right)\left(x-\frac{1}{\sqrt{3}}\right)$

also not in our domain

rel. min $\nearrow$ $\frac{-1}{\sqrt{3}}$ $\qquad \frac{1}{\sqrt{3}}$

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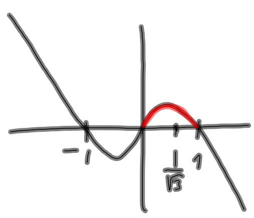

Make sure to answer the question that was asked?

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$A\left(\frac{1}{\sqrt{3}}\right)$

$$A(x) = 2x\left(-x^2+1\right) = 2\cdot\frac{1}{\sqrt{3}}\left(-\frac{1}{3}+1\right)$$

$$= 2\cdot\frac{\sqrt{3}}{3}\cdot\frac{2}{3} = \frac{4\sqrt{3}}{9}$$

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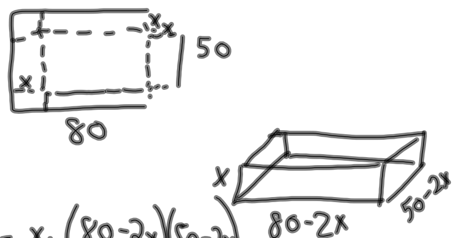

$$V(x) = x\cdot(80-2x)(50-2x)$$

domain: $0 < x < 25$

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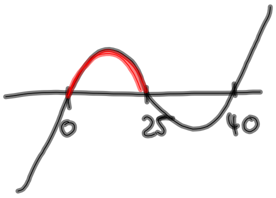
$$V(x) = x(80-2x)(50-2x)$$
$$= 4x(x-40)(x-25)$$

0 , 40 , 25



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$$V(x) = 4x(x-25)(x-40)$$
$$= 4x\left(x^2-65x+1000\right)$$
$$= 4x^3 - 260x^2 + 4000x$$

$$V'(x) = 12x^2 - 520x + 4000$$
$$0 = 4\left(3x^2 - 130x + 1000\right)$$

$$V_{1,2} = \frac{130 \pm \sqrt{130^2 - 4(3)1000}}{6} = \frac{130 \pm 70}{6}$$

$\frac{200}{6} = \frac{100}{3} = 33\frac{1}{3}$ → a min. and outside the domain

$\frac{60}{6} = 10$

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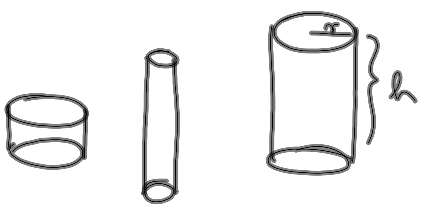
10) 100 cm^2

$$V = \pi r^2\cdot h$$



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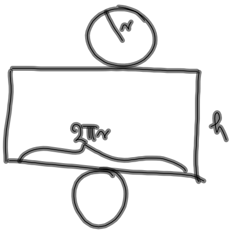

Bottom and top:
$2\pi r^2$

rectangle:
$2\pi r h$

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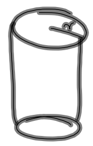
h



$V = \pi r^2 h$

$A_{su} = 2\pi r^2 + 2\pi r h = 100$

constrain

$100 = 2\pi r^2 + 2\pi r h$

$$\boxed{\begin{array}{l} \dfrac{100 - 2\pi r^2}{2\pi r} = h \\[1em] \dfrac{100}{2\pi r} - r = h \end{array}}$$

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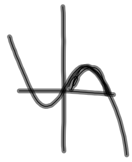
$V = \pi r^2 h$

$V(r) = \cancel{\pi} r^2 \dfrac{100 - 2\pi r^2}{2\cancel{\pi} r}$

$= r(50 - \pi r^2)$

$\boxed{V(r) = -\pi r^3 + 50r}$



$V'(r) = -3\pi r^2 + 50$

Solve for zero

$3\pi r^2 = 50$

$r^2 = \dfrac{50}{3\pi}$

$r = \pm\sqrt{\dfrac{50}{3\pi}}$

$r = \sqrt{\dfrac{50}{3\pi}}$

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