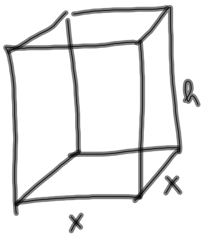


$12 = x^2 h$

$\frac{12}{x^2} = h$

Minimize surface area

$A = x^2 + 4xh$

$A(x) = x^2 + 4x \cdot \left(\frac{12}{x^2}\right)$

$A(x) = x^2 + \frac{48}{x}$

Mar 4-6:04 PM

$A(x) = x^2 + \frac{48}{x} = x^2 + 48x^{-1}$

$A'(x) = 2x + 48(-1)x^{-2}$

$A'(x) = 2x - \frac{48}{x^2}$

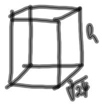
$x = ?$ $A'(x) = 0$

$2x - \frac{48}{x^2} = 0$

$2x = \frac{48}{x^2}$

$2x^3 = 48$

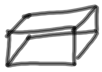
$x^3 = 24$



$x = \sqrt[3]{24}$

$h = \frac{12}{x^2} = \frac{12}{(\sqrt[3]{24})^2} \cdot \frac{\sqrt[3]{24}}{\sqrt[3]{24}} = \frac{12\sqrt[3]{24}}{24} = \frac{\sqrt[3]{24}}{2}$

$h = \frac{x}{2}$



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Review #14

$C(x) = 5x^2 + \frac{1200}{x}$

$C'(x) = 10x - \frac{1200}{x^2}$

$= \frac{10x^3 - 1200}{x^2} = \frac{10(x^3 - 120)}{x^3}$

$= \frac{10(x^3 - (\sqrt[3]{120})^3)}{x^3}$

$= \frac{10(x - \sqrt[3]{120})\overbrace{(x^2 + x\sqrt[3]{120} + (\sqrt[3]{120})^2)}^{\text{always } \oplus}}{x^3}$

Mar 4-6:41 PM

difference of cubes theorem:

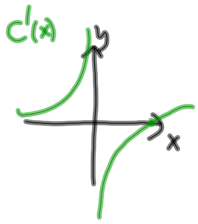
$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$(A + \frac{1}{2}B)^2 + \frac{3}{4}B^2$

never factors
always positive

Mar 4-6:42 PM

	$x<0$	0	$0<x<\sqrt[3]{120}$	$\sqrt[3]{120}$	$x>\sqrt[3]{120}$
x^3	$-$	0	$+$		$+$
$x-\sqrt[3]{120}$	$-$		$-$	0	$+$
$C'(x)$	$\oplus$	?	$\ominus$	min	$\oplus$



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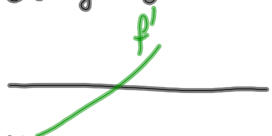
We are looking for a minimum

f' should change sign from $\ominus$ to $\oplus$



$\Rightarrow f'$ locally increases

$\Rightarrow f''$ must be $\oplus$

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Second derivative test:

① If $f'(a)=0$ and $f''(a)$ is ⊕ then f has a relative min. at $x=a$

a

② If $f'(a)=0$ and $f''(a)$ is ⊖ then f has a rel. max at $x=a$

Mar 4-6:56 PM

③ If $f'(a)=0$ and $f''(a)=0$ then the second derivative test gives us nothing!

Mar 4-7:02 PM

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}$$

$$\left[\lim_{x\to\infty}\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}}\right]^{\cdot 3}=e^{3}$$

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Differentiate $\ln x$

$$\lim_{h\to 0}\frac{\ln(x+h)-\ln x}{h}$$

$\ln a-\ln b=\ln\left(\frac{a}{b}\right)$

$$\lim_{h\to 0}\frac{1}{h}\cdot\ln\left(\frac{x+h}{x}\right)$$

$b\cdot\ln a=\ln a^{b}$

$$\lim_{h\to 0}\ln\left(\frac{x+h}{x}\right)^{1/h}$$

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$$\lim_{h\to 0}\ln\left(\frac{x+h}{x}\right)^{1/h}$$

Recall

$$\lim_{\substack{n\to\infty\\ -\infty}}\left(1+\frac{1}{n}\right)^{n}=e$$

$$\lim_{h\to 0}\ln\left(1+\frac{h}{x}\right)^{1/h}$$

$$\lim_{h\to 0}\ln\left(1+\frac{1}{x/h}\right)^{\frac{1}{h}\cdot\frac{x}{x}}$$

$$\lim_{h\to 0}\ln\left(1+\frac{1}{x/h}\right)^{\frac{x}{h}\cdot\frac{1}{x}}$$

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$$\lim_{h\to 0}\ln\left(1+\frac{1}{x/h}\right)^{\frac{x}{h}\cdot\frac{1}{x}}$$

$(a^{n})^{m}=a^{nm}$

$$\lim_{h\to 0}\ln\left[\left(1+\frac{1}{x/h}\right)^{x/h}\right]^{\frac{1}{x}}$$

$$\ln\left[\lim_{h\to 0}\left(1+\frac{1}{x/h}\right)^{x/h}\right]^{\frac{1}{x}}$$

$$\ln e^{1/x}=\frac{1}{x}$$

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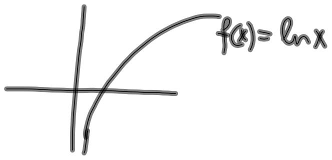
So, if $f(x) = \ln x$, then

$$f'(x) = \frac{1}{x}$$



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$$\left[\ln(x^{10})\right]'$$

$\ln a^b = b \cdot \ln a$

$$\left[10 \cdot \ln x\right]' = 10 \cdot \frac{1}{x} = \frac{10}{x}$$

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Other logarithmic functions?

$$\left(\log_3 x\right)' = \left(\frac{\ln x}{\ln 3}\right)' = \left(\frac{1}{\ln 3} \cdot \ln x\right)'$$

$$= \frac{1}{\ln 3} \cdot (\ln x)' = \frac{1}{\ln 3} \cdot \frac{1}{x}$$

$$= \frac{1}{x \ln 3}$$

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$$\left(\log_a x\right)' = \left(\frac{\ln x}{\ln a}\right)'$$

$$\frac{1}{\ln a} \cdot (\ln x)' = \frac{1}{x \cdot \ln a}$$

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$$(x^2)' = 2x \qquad \int 2x \, dx = x^2 + C$$

$$(\ln x)' = \frac{1}{x} \qquad \int \frac{1}{x} \, dx = \ln|x| + C$$

Important:

$$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

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Product Rule:

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

quotient rule

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