The following problems are interesting, thought-provoking exercises for YOU to think about them. Please do not take these problems to tutors at the Math Center or the Tutoring Center.

All problems are worth 1 point.

1. Simplify each of the following. Show all steps.

   a) $\left(\sqrt{5+\sqrt{21}}-\sqrt{5-\sqrt{21}}\right)^2$ b) $\dfrac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}$

2. Suppose that a is a number such that $\dfrac{1}{a^3}+a^3=5$. Find the exact value of $\dfrac{1}{a^6}+a^6$. (Hint: you do not need to find the value of a.)

3. Consider the equation $y=4mx-21m+mx^2+4$. Prove that for all values of m, the graphs will pass through the same two points.

4. A long train is moving slowly, with a constant speed. We walk next to the train, with a constant speed, higher than that of the train. When we walk from the end of the train to the font of it, it takes 200 steps. Then we turn around and walk from the front of the train to the end, and count 120 steps. How many steps long is the train?

5. a) Find the perimeter of a $15-$sided reguar polygon written into a circle with radius $10\,\mathrm{m}$.

   b) Find the perimeter of an $n-$sided regular polygon written into a circle with radius R. Use radians to measure angles.

   c) Find the limit of the perimeter of an $n-$sided regular polygon written into a circle with radius R as n approaches infinity. Use radians to measure angles.

6. a) Find the area of a $15-$sided regular polygon written into a circle with radius $10\,\mathrm{m}$.

   b) Find the area of an $n-$sided regular polygon written into a circle with radius R. Use radians to measure angles.

   c) Find the limit of the area of an $n-$sided regular polygon written into a circle with radius R as n approaches infinity. Use radians to measure angles.

7. Suppose that f and g are differentiable on $\mathbb{R}$. Let c be the point where the vertical distance between the graphs of $f(x)$ and $g(x)$ is the greatest. Prove that the tangent lines drawn to f and g at $x=c$ are parallel to each other.

8. Prove that for all values of c, the function $f(x)=x^3-15x+c$ has at most one zero in the interval $[-2,2]$.

9. We have differentiated the inverse trigonometric functions $f(x)=\arcsin x$ and $g(x)=\arccos x$ and found that they are opposites of each other. Find an argument supporting that fact that does not use any calculus. (Hint: what does it mean for functions f and g that their derivatives are opposites of each other?)

10. We have differentiated the function $f(x)=\arcsin x$ and found that $f'(x)=\dfrac{1}{\sqrt{1-x^2}}$. Consider the statement below.

$$\frac{d}{dx}(\arcsin x)=\frac{1}{\sqrt{1-x^2}}$$

    State the domain of the functions on the left-hand side and on the right-hand side. What is the geometric interpretation of your finding?

11. Suppose an object's location function is $L(t)=\dfrac{12}{1+3e^{-t}}$. Where is the object when its velocity is the greatest?

12. Consider the function $f(x) = x^3$ on the interval $[0,2]$.

You may need the following formula: $1^3 + 2^3 + 3^3 + ... + n^3 = \dfrac{n^2(n+1)^2}{4}$

a) Compute a left Riemann sum using a uniform partition of $n = 10$.
b) Compute a right Riemann sum using a uniform partition of $n = 10$.
c) Compute a left Riemann sum using a uniform partition of $n = 100$.
d) Compute a right Riemann sum using a uniform partition of $n = 100$.
e) Compute a left Riemann sum using a uniform partition of n. Express your answer in terms of n.
f) Compute the limit of the left Riemann sum you got in the previous part as n approaches infinity.
g) Compute a right Riemann sum using a uniform partition of n. Express your answer in terms of n.
h) Compute the limit of the right Riemann sum you got in the previous part as n approaches infinity.