Review Problems

1. Simplify each of the following

   a) 6^{-2} c) $64^{-2/3}$ e) $8^{\log_2 5}$ g) $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 - 1$

   b) $32^{3/5}$ d) $\log_9 \sqrt{27}$ f) $3^{\log_9 10}$ h) $\log_2\left(4x^2\right) - 3\log_2\left(\frac{6}{x}\right) + \log_4\left(4x^6\right)$

2. Let $x = \log_3 2$. Express each of the following in terms of x.

   a) $\log_3 72$ b) $\log_2 3$ c) $\log_6 72$

3. Factor $3x^2 - 4x - 319$ by completing the square.

4. In case of each polynomial given, determine (by completing the square) whether it factors or not. (You do not have to actually factor.)

   a) $20x + 2x^2 + 44$ b) $20x - 5x^2 - 25$

5. Solve the equation $9x^2 - 12x = 11$ and check one of your solution using exact values.

6. Solve each of the following equations.

   a) $2x^3 = 6x$

   b) $2x^2 - 3x - 1 = 0$

   c) $\log_2(x+5) - \log_2(x-7) = -1$

   d) $\log_3(7-x) + \log_3(1-x) = 3$

   e) $\log_6(-8-x) + \log_6(8-x) = 2$

   f) $\frac{2x-1}{3} - \frac{x-1}{2} = x - 4$

7. Solve each of the following inequalities.

   a) $x^2 \geq 4x$ b) $8x + x^2 < 33$ c) $x^2 < -2x + 2$ d) $4x^2 \leq 4x - 1$ e) $x^2 - 6x > -10$

8. Find the domain for each of the following functions.

   a) $f(x) = \ln\left(x^2 - 10x + 29\right)$

   b) $g(x) = \frac{1}{\log_2(4-x)}$

   c) $f(x) = \log_5\left(x^2 - 10x + 21\right)$

   d) $k(x) = \frac{1}{\log_5\left(x^2 - 10x + 21\right)}$

9. a) Find the center and radius of a circle with equation $2y - 6x + x^2 + y^2 = 10$

   b) Find an equation for the tangent line drawn to the circle $(x+2)^2 + (y-4)^2 = 10$ to the point $(-5, 3)$

10. Graph each of the following pairs of functions in the same coordinate system.

    a) $f(x) = 2^x$ and $g(x) = \log_2 x$

    b) $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$

    c) $f(x) = \log_2 x$ and $g(x) = \log_{1/2} x$

    d) $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \log_{1/2} x$

11. For each of the following functions given, give a complete analysis and sketch its graph.

    a) $f(x) = 9 - 3x^2 - 6x$ on $[-2, 3]$ b) $f(x) = \sqrt{x+3}$ c) $f(x) = \log_3 x$ d) $f(x) = 0.7^x$

12. An object's height (measured in feet) is defined by $s(t) = 0.4t + 12$ where t is the time, measured in seconds.

    a) Find the location of the object at $t = 10$ seconds.

    b) Find the average velocity of the object between $t = 0$ and $t = 3$ seconds

    c) Find the average velocity of the object between $t = 5$ and $t = 10$ seconds

13. An object's height (measured in feet) is defined by $s(t) = t^3 - 12t$ where t is the time, measured in seconds.
    a) Find the location of the object at $t = 3$ seconds.
    b) Find the average velocity of the object between
        i) $t = 0$ and $t = 2$ seconds
        ii) $t = 1$ second and $t = 2$ seconds
        iii) $t = 1.5$ seconds and $t = 2$ seconds

14. Find the coordinates of all points where the graphs of $f(x) = x^2 - 2x - 26$ and $g(x) = 2x - 5$ intersect each other.

15. Find each of the following limits.

a) $\lim_{x\to-\infty} \left(-2x^5 + 8x^2\right)$

b) $\lim_{x\to\infty} \left(-2x^5 + 8x^2\right)$

c) $\lim_{x\to-\infty} \left(-2x^5 + 8x^6\right)$

d) $\lim_{x\to\infty} \left(-2x^5 + 8x^6\right)$

e) $\lim_{x\to-\infty} \dfrac{3x^2 - 1}{5x^2 - 3x + 2}$

f) $\lim_{x\to-\infty} \dfrac{100x - 1}{5x^2 - 3x + 2}$

g) $\lim_{x\to-\infty} \log_2 x$

h) $\lim_{x\to\infty} \dfrac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$

i) $\lim_{x\to\infty} \dfrac{-x^3 + 2x + 1}{x - 3}$

j) $\lim_{x\to-\infty} 2^x$

k) $\lim_{x\to\infty} \left(\log_2\left(x^2 - 5x + 17\right)\right)$

l) $\lim_{x\to\infty} \dfrac{12 + \log_7 3x}{15 + \log_7 x}$

m) $\lim_{x\to\infty} \dfrac{2^{x+5}}{4^{x-1}}$

n) $\lim_{x\to\infty} \dfrac{3^{x+1} \cdot \left(\dfrac{1}{3}\right)^{-x+2}}{9^{x-1}}$

o) $\lim_{x\to\infty} x\left(\dfrac{1}{3} - \dfrac{1}{3 - \dfrac{1}{x}}\right)$

p) $\lim_{x\to\infty} \dfrac{\sqrt{4 - \dfrac{1}{x}} - 2}{\dfrac{1}{x}}$

q) $\lim_{x\to-\infty} \dfrac{\cos x - 2}{x^3 + 1}$

Review Problems - Answers

1. a) $\frac{1}{36}$ b) 8 c) $\frac{1}{16}$ d) $\frac{3}{4}$ e) 125 f) $\sqrt{10}$ g) $\log_3 2 = \frac{\ln 2}{\ln 3}$ h) $\log_2\left(\frac{x^8}{27}\right)$

2. a) $3x+2$ b) $\frac{1}{x}$ c) $\frac{3x+2}{x+1}$

3. $3\left(x+\frac{29}{3}\right)(x-11) = (3x+29)(x-11)$

4. a) factors b) does not factor

5. $\frac{2\pm\sqrt{15}}{3}$

   Check: if $x = \frac{2-\sqrt{15}}{3}$, then the left-hand side of the equation is

$$\begin{aligned} 9x^2-12x &= 9\left(\frac{2-\sqrt{15}}{3}\right)^2 - 12\left(\frac{2-\sqrt{15}}{3}\right) = 9\frac{\left(2-\sqrt{15}\right)^2}{9} - 12\frac{2-\sqrt{15}}{3} \\ &= \left(2-\sqrt{15}\right)^2 - 4\left(2-\sqrt{15}\right) = 4+15-4\sqrt{15}-8+4\sqrt{15} = 19-8 = 11 \end{aligned}$$

   ant the right-hand side is 11 and so our solution is correct.

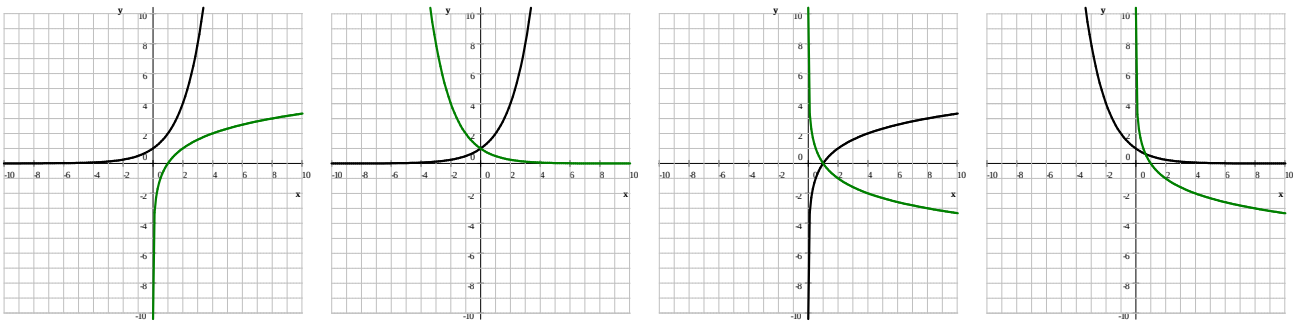
6. a) $0, -\sqrt{3}, \sqrt{3}$ b) $\frac{3\pm\sqrt{17}}{4}$ c) no solution d) -2 e) -10 f) 5

7. a) $(-\infty, 0] \cup [4, \infty)$ b) $(-11, 3)$ c) $\left(-\sqrt{3}-1, \sqrt{3}-1\right)$ d) $x = \frac{1}{2}$ e) $\mathbb{R}$

8. a) $\mathbb{R}$ b) $x < 4$ but $x \neq 3$ c) $x < 3$ or $x > 7$
   d) $x < 3$ but $x \neq 5-\sqrt{5}$ or $x > 7$ but $x \neq 5+\sqrt{5}$

9. a) center: $(3, -1)$ radius: $2\sqrt{5}$ b) $y = -3x - 12$

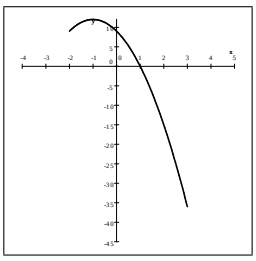
10. a) b) c) d)



11. a) $f(x) = 9 - 3x^2 - 6x$ on $[-2, 3]$

domain: $[-2, 3]$
range: $[-36, 12]$
$x-$intercept: $(1, 0)$
$y-$ intercept: $(0, 9)$
maximum: $(-1, 12)$
minimum: $(3, -36)$

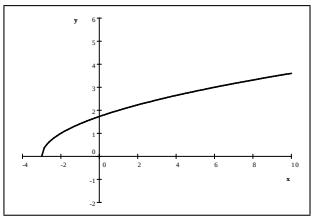
increasing on $(-2, -1)$
and decreasing on $(-1, 3)$
one-to-one: no
end-behavior: none
i.e. $\lim_{x \to \pm\infty} f(x) =$ undefined



b) $f(x) = \sqrt{x+3}$

domain: $[-3, \infty)$
range: $[-3, \infty)$
$x-$intercept: $(-3, 0)$
$y-$ intercept: $\left(0, \sqrt{3}\right)$
maximum: none
minimum: $(-3, 0)$

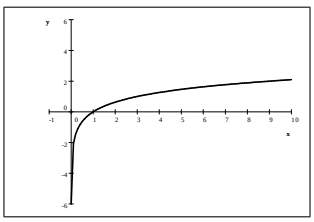
increasing on $[-3, \infty)$
one-to-one: yes
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
and $\lim_{x \to \infty} f(x) = \infty$



c) $f(x) = \log_3 x$

domain: $(0, \infty)$
range: $\mathbb{R}$
$x-$intercept: $(1, 0)$
$y-$ intercept: none
maximum: none
minimum: none

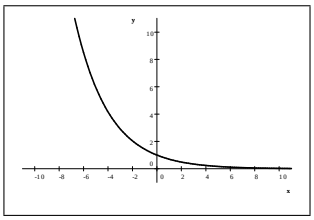
increasing on $(0, \infty)$
one-to-one: yes
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
and $\lim_{x \to \infty} f(x) = \infty$



d) $f(x) = 0.7^x$

domain: $\mathbb{R}$
range: $(0, \infty)$
$x-$intercept: none
$y-$ intercept: $(0, 1)$
maximum: none
minimum: none

decreasing on $\mathbb{R}$
one-to-one: yes
end-behavior:
$\lim_{x \to -\infty} f(x) = \infty$
and $\lim_{x \to \infty} f(x) = 0$



12. a) 16 ft b) $0.4 \frac{\text{ft}}{\text{s}}$ c) $0.4 \frac{\text{ft}}{\text{s}}$

13. a) -9 ft b) i) $-8 \frac{\text{ft}}{\text{s}}$ ii) $-5 \frac{\text{ft}}{\text{s}}$ iii) $-2.75 \frac{\text{ft}}{\text{s}}$

14. $(7, 9)$ and $(-3, -11)$

15. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) undefined h) $\frac{2}{3}$ i) $-\infty$ j) 0
k) ∞ l) 1 m) 0 n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0

