Exam 2 will cover the following topics. All topics covered by Quizzes 1-5 and Exam 1. These topics include quadratic inequalities, functions and their graphs, exponents and logarithms, limits, differentiation by using the definition, the sum rule, the constant multiplier rule, the generalized power rule, the product rule, derivative of trigonometric functions, applications of the derivative: increasing and decreasing functions, relative and absolute extrema, optimization problems, tangent lines, and antiderivatives.

Students must be able to state the following theorems: Intermediate Value theorem, Rolle's Theorem, Mean Value Theorem.

Students must be able to prove the following theorems:

- Differentiating functions using the definition (limit of the differential quotient) of generalized polynomials, $f(x) = \sin x$ and $g(x) = \cos x$.
- If a function is differentiable at a number x, then it is continuous there.
- The sum and constant multiple rule for derivatives.
- The product rule for derivatives.

Review Problems

1. Prove that if a function is differentiable at a number $x = a$, then it is also continuous there.
2. Find an equation for the tangent line drawn to the graph of $f(x) = x^2 - \dfrac{2}{x}$ at $x = -2$.
3. Compute each of the following limits.

   a) $\displaystyle\lim_{x\to 0^-} \frac{\sin 3x}{x}$ b) $\displaystyle\lim_{x\to 6} \frac{x^2-6x}{x^2-36}$ c) $\displaystyle\lim_{x\to 5} \frac{x^2-25}{2-\sqrt{x-1}}$

4. Compute each of the following limits.

   a) $\displaystyle\lim_{x\to -\infty} \left(-2x^5+8x^2\right)$

   b) $\displaystyle\lim_{x\to \infty} \left(-2x^5+8x^2\right)$

   c) $\displaystyle\lim_{x\to -\infty} \left(-2x^5+8x^6\right)$

   d) $\displaystyle\lim_{x\to \infty} \left(-2x^5+8x^6\right)$

   e) $\displaystyle\lim_{x\to -\infty} \frac{3x^2-1}{5x^2-3x+2}$

   f) $\displaystyle\lim_{x\to -\infty} \frac{100x-1}{5x^2-3x+2}$

   g) $\displaystyle\lim_{x\to -\infty} \log_2 x$

   h) $\displaystyle\lim_{x\to \infty} \frac{2x^2+3x+1}{3x^2-5x+2}$

   i) $\displaystyle\lim_{x\to \infty} \frac{-x^3+2x+1}{x-3}$

   j) $\displaystyle\lim_{x\to -\infty} 2^x$

   k) $\displaystyle\lim_{x\to \infty} \left(\log_2\left(x^2-5x+17\right)\right)$

   l) $\displaystyle\lim_{x\to \infty} \frac{12+\log_7 3x}{15+\log_7 x}$

   m) $\displaystyle\lim_{x\to \infty} \frac{2^{2x+5}}{3^{x-1}}$

   n) $\displaystyle\lim_{x\to \infty} \frac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

   o) $\displaystyle\lim_{x\to \infty} x\left(\frac{1}{3}-\frac{1}{3-\frac{1}{x}}\right)$

   p) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{4-\frac{1}{x}}-2}{\frac{1}{x}}$

   q) $\displaystyle\lim_{x\to -\infty} \frac{\cos x-2}{x^3+1}$

5. Differentiate each of the following, using the definition of the derivative.

   a) $f(x) = \sqrt{2x-1}$ b) $f(x) = \dfrac{1}{x^2-1}$ c) $f(x) = \sqrt{1-x^2}$

6. Differentiate each of the following functions.

a) $f(x) = (\sin x - \cos x)\sqrt{x^5}$ b) $f(y) = \dfrac{1}{y^2} + \dfrac{1}{y} + \dfrac{1}{\sqrt{y}} + \sqrt{y}$ c) $f(x) = \dfrac{\cos x - \sqrt{x}}{x^2}$

7. Evaluate each of the following indefinite integrals.

a) $\displaystyle\int 10x - 3\; dx$ c) $\displaystyle\int \sqrt[3]{x^5} - \sqrt[5]{x^3}\; dx$ e) $\displaystyle\int 3abm - 2a + 1\; da$

b) $\displaystyle\int 3\sin x - \cos x\; dx$ d) $\displaystyle\int mx + b\; dx$ f) $\displaystyle\int 3abm - 2a + 1\; dm$

8. We know the following things about a function f. $f'(x) = 20x^3 - 3$ and $f(-1) = 16$. Find f.

9. Find the value of c for which the following is true: the tangent line drawn to the graph of $y = \dfrac{1}{x}$ has $x-$intercept $(6, 0)$.

10. Find the equation for the function f if we know that f is a polynomial of degree 3, and has a relative minimum at $x = -2$ and a relative maximum at $x = 3$.

11. Prove that the function given is one-to-one.

a) $f(x) = \dfrac{2x-3}{5x+1}$ b) $f(x) = 3x^5 - 50x^3 + 390x - 1200$

12. Find the value of a so that the line $y = 2x$ is a tangent line to the parabola $y = ax^2 + 5$.

13. Let $P(x, y)$ be a point on the graph of $y = 4 - x^2$ with $0 \le x \le 2$. Let $PQRS$ be a rectangle with one side on the $x-$axis and two vertices on the graph of $y = 4 - x^2$. Find the exact value of the greatest possible area of such a rectangle.

14. We would like to construct an open box with a square base. The box should have a volume of $100\,\mathrm{ft}^3$. The material for the sides costs 3 cents per square feet, and the material for the bottom costs 5 cents per square feet. What is the lowest cost for which such a box can be produced?

15. We want to construct a cylindrical soda can with volume $100\,\mathrm{cm}^3$. If the material for the side costs 2 cents per cm^2, and the material for the top and bottom costs 5 cents per cm^2, what dimensions would guarantee a minimal cost of producing such a can? What would be the minimal cost?

16. Find a third degree polynomial $P(x)$ such that $P(0) = -5$, $P'(0) = 3$, $P''(0) = -6$ and $P'''(0) = 60$.

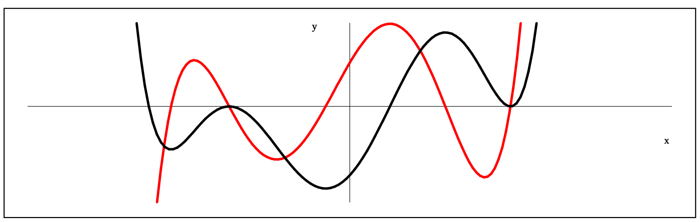
17. Find the $x-$coordinate of all relative maximums and minimums of each of the functions given below.

a) $f(x) = x^3(5x - 2)$ b) $f(x) = 2x + \dfrac{18}{x}$ c) $f(x) = x^3 + 3x^2 - 24x + 24$

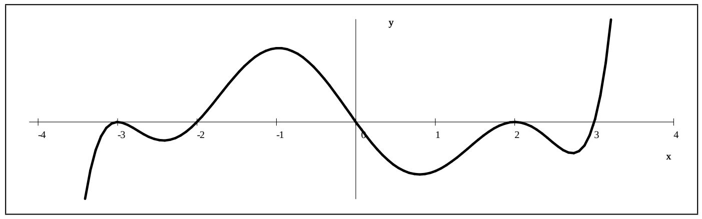
18. Find all relative and absolute maximums and minimums for the $f(x) = 6x^5 - 15x^4 + 5x^6 + 60$ on $[-3, 3]$. Sketch the graph of both f and f' on this domain.

19. Prove that the function $f(x) = \sin x - x$ does not have any relative minimums or maximums.

20. The graph below shows a function f and its first derivative, f'. Which is which?



21. The graph below shows f', the first derivative of a function f.



a) Find all values of x for which the function f has a local maximum at x.

b) Find all values of x for which the function f has a local minimum at x.

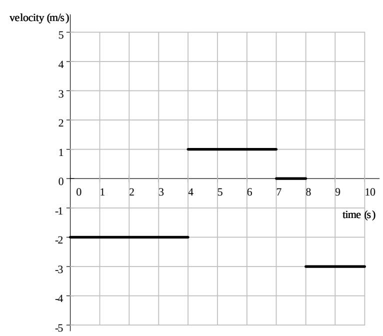
22. We shoot a small object upward, from the top of a tower. The acceleration function of the object is $a(t) = -10$. (Location is measured in meters, velocity in $\frac{\text{m}}{\text{s}}$, acceleration in $\frac{\text{m}}{\text{s}^2}$.)

a) Given that $v(0) = 160$, find $v(t)$, the velocity function of the object.

b) Given that $h(0) = 525$, find $h(t)$, the location function of the object.

c) Find the maximum height that the object reaches.

23. The picture below shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward.).



a) Suppose that the object starts at a height of 5 m. Graph its location function.

b) Suppose that the object starts at a height of 9 m. Graph its location function.

24. Find the value of c that satisfies the Mean Value Theorem for

a) $f(x) = x^2$ on $[-2, 5]$ b) $f(x) = \sqrt{x}$ on $[0, 9]$ c) $f(x) = x^3 - x$ on $[1, 3]$

Review Problems - Answers

1.) See handout 2.) $y = -\frac{7}{2}x - 2$ 3.) a) 3 b) $\frac{1}{2}$ c) -40

4.) a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) ∞ h) $\frac{2}{3}$ i) $-\infty$ j) 0

k) ∞ l) 1 m) ∞ n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0

5.) see handout

6.) a) $f'(x) = \frac{5}{2}(\sin x - \cos x)\, x\sqrt{x} + (\cos x + \sin x)\, x^2\sqrt{x}$ b) $f'(y) = -\frac{2}{y^3} - \frac{1}{y^2} - \frac{1}{2y\sqrt{y}} + \frac{1}{2\sqrt{y}}$

c) $f'(x) = -\frac{2}{x^3}(\cos x - \sqrt{x}) + \frac{1}{x^2}\left(-\sin x - \frac{1}{2\sqrt{x}}\right)$

7.) a) $5x^2 - 3x + C$ b) $-3\cos x - \sin x + C$ c) $\frac{3}{8}x^{8/3} - \frac{5}{8}x^{8/5} + C$

d) $\frac{1}{2}mx^2 + bx + C$ e) $\frac{3}{2}a^2bm - a^2 + a + C$ f) $\frac{3}{2}abm^2 - 2am + m + C$

8.) $f(x) = 5x^4 - 3x + 8$ 9.) 3 10.) Let $a > 0$ and $c \in \mathbb{R}$ any number $f(x) = -a\left(-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x\right) + c$

11.) a) Write $y = \frac{2x-3}{5x+1}$ and solve for x: $x = \frac{y+3}{-5y+2}$ so for each y, there is a **unique** x for which the function takes that $y-$value.

b) $f'(x) = 15\left(\left(x^2-5\right)^2 + 1\right)$ is always positive, hence f is always strictly increasing. Thus one-to-one.

12.) $\frac{1}{5}$ 13.) $\frac{32\sqrt{3}}{9}$ 14.) \$3.65 with base $\sqrt[3]{120}$ ft by $\sqrt[3]{120}$ ft

15.) $r = \sqrt[3]{\frac{20}{\pi}}$ cm $h = 5\sqrt[3]{\frac{20}{\pi}}$ cm cost: 323.74 cents 16.) $P(x) = 10x^3 - 3x^2 + 3x - 5$

17.) a) no relative maximum, relative minimum at $x = \frac{3}{10}$

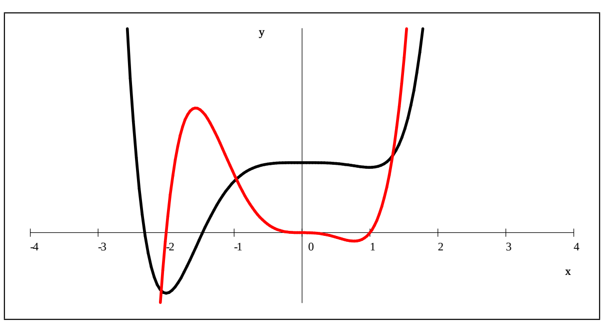
b) relative maximum at $x = -3$ relative minimum at $x = 3$

c) relative maximum at $x = -4$, relative minimum at $x = 2$

18.) a) $f'(x) = 30x^5 + 30x^4 - 60x^3 = 30(x+2)x^3(x-1)$

relative minimums: $(-2,-52)$ and $(1,56)$ absolute minimum: $(-2,-52)$

relative maximum: $(0,60)$ absolute maximum: $(-3,332)$



19.) $f'(x) = \cos x - 1$ is always negative or zero. Thus f' never changes sign, so f has no relative minimums or maximums.

20.) the black graph is f, the red graph is f' 21.) a) $x = 0$ b) $x = -2$ and $x = 3$

22.) a) $v(t) = -10t + 160$ b) $h(t) = -5t^2 + 160t + 525$ c) $h_{\text{max}} = 1805\,\text{m}$

23.) a) red graph b) green graph 24.) a) $\dfrac{3}{2}$ b) $\dfrac{9}{4}$ c) $\sqrt{3}$