1. Compute each of the following integrals.

   a) $\displaystyle\int \frac{-2x^3+2x^2-x-2}{x^4+2x^3}dx$

   b) $\displaystyle\int (\sec x+\tan x)^2\,dx$

   c) $\displaystyle\int \sqrt{4-9x^2}dx$

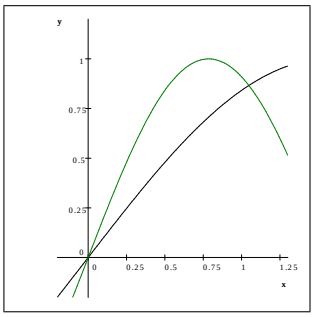
   d) $\displaystyle\int_0^{\pi/4} \sec^2 x\tan^3 x\;dx$

   e) $\displaystyle\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

   f) $\displaystyle\int_0^{\infty} x^4e^{-x^5}dx$

2. Let R be the region bounded by the curves $y=\sin x$ and $y=\sin 2x$ between $x=0$ and $x=\dfrac{\pi}{3}$. Compute the volume of the object we obtain by rotating R about

   a) the $x-$axis b) the $y-$axis



3. Compute the volume of the object we obtain when rotating the circle $(x-3)^2+y^2=4$ about the $y-$axis.

4. Compute the length of the curve of $y=\dfrac{x^4}{8}+\dfrac{1}{4x^2}$ between $x=1$ and $x=2$.

5. Prove that a bounded increasing sequence is convergent.

6. Determine whether each of the given series converges absolutely, converges conditionally, or diverges.

   a) $\displaystyle\sum_{n=0}^{\infty} n^2\;3^{-n^2}$

   b) $\displaystyle\sum_{n=0}^{\infty} \frac{3-n^2}{(n+3)^3}$

   c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{n^n}$

7. Find the Taylor polynomial of order 5 at $x=1$ of the function $f(x)=\sqrt{x}$.

8. Find a power series representation for the function $f(x)=\dfrac{1}{x+10}$ and determine the interval of convergence.

9. Compute the interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(x-5)^n}{3^n(n+1)}$

10. Find the sum of the infinite series

    a) $2^2+2^3+\dfrac{1}{2}\cdot 2^4+\dfrac{1}{6}\cdot 2^5+\dfrac{1}{24}\cdot 2^6+\ldots\ldots=\displaystyle\sum_{n=2}^{\infty}\frac{2^n}{(n-2)!}$

    b) $\dfrac{2}{3}-\dfrac{4}{9}+\dfrac{8}{27}-\dfrac{16}{81}+\ldots..$

    c) $1-3+\dfrac{9}{2}-\dfrac{9}{2}+\dfrac{27}{8}-\dfrac{81}{40}+\dfrac{81}{80}-\dfrac{243}{560}+\ldots..$

    d) $x+\dfrac{1}{3!}x^3+\dfrac{1}{5!}x^5+\dfrac{1}{7!}x^7+\dfrac{1}{9!}x^9+\ldots...$

Answers

1. a) $\ln x + \dfrac{1}{2x^2} - 3\ln|x+2| + C$ b) $2\tan x + 2\sec x - x + C$ c) $x\sqrt{1-\dfrac{9}{4}x^2} + \dfrac{2}{3}\sin^{-1}\left(\dfrac{3}{2}x\right) + C$

   d) $\dfrac{1}{4}$ e) 4 f) $\dfrac{1}{5}$

2. a) $\dfrac{3\sqrt{3}}{16}\pi$ b) $\dfrac{\pi^2}{2} - \dfrac{3\sqrt{3}}{4}\pi$

3. $24\pi^2$

4. $\dfrac{33}{16}$

5. see handout

6. a) converges absolutely (ratio test)

   b) diverges (sum of two series, one diverges by the comparison test)

   c) converges absolutely (ratio test)

7. $1 + \dfrac{1}{2}(x-1)^1 - \dfrac{1}{8}(x-1)^2 + \dfrac{1}{16}(x-1)^3 - \dfrac{5}{128}(x-1)^4 + \dfrac{7}{256}(x-1)^5$

8. $\dfrac{1}{10} - \dfrac{x}{100} + \dfrac{x^2}{1000} - \dfrac{x^3}{10\,000} + \dfrac{x^4}{100\,000} + ... = \displaystyle\sum_{n=0}^{\infty} \frac{1}{10}\left(-\frac{x}{10}\right)^n$ where $-10 < x < 10$

9. $(2, 8]$

10. a) $4e^2$ b) $\dfrac{2}{5}$ c) $\dfrac{1}{e^3}$ d) $\sinh x$

