Quiz 11 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-10 and Exams 1, 2
2. Ratio test, root test, and Leibniz test
3. Conditional and absolute convergence

Sample Quiz 11

1. Prove that a convergent sequence is bounded.
2. In case of each of the following series given, determine whether it converges or diverges. Show computation and explain which test you are using. (Note that answers may vary.)

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$

b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{\sqrt{n}}{\sqrt{n+1}}\right)$

c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$

d) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^3+5}$

e) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^3-5}$

f) $\displaystyle\sum_{n=0}^{\infty} \frac{n^n}{n!}$

Answers

1. See handout.

2. a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$ converges absolutely by the ratio test, also alternating test works for $n \geq 2$

   b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{\sqrt{n}}{\sqrt{n+1}}\right)$ diverges, it is a telescoping sum; $\ln\left(\frac{\sqrt{n}}{\sqrt{n+1}}\right) = \ln\sqrt{n} - \ln\sqrt{n+1}$

   c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$ diverges by the comparison test: $\dfrac{1}{(\ln n)^2} > \dfrac{1}{n}$ for all $n \geq 2$

   d) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^3+5}$ converges by the comparison test: $\dfrac{n}{n^3+5} < \dfrac{n}{n^3} = \dfrac{1}{n^2}$

   e) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^3-5}$ converges by the comparison test: $\dfrac{n}{n^3-5} < \dfrac{n}{n^3 - \frac{n^3}{2}} = \dfrac{2}{n^2}$

   f) $\displaystyle\sum_{n=0}^{\infty} \frac{n^n}{n!}$ diverges by the ratio test

$$\begin{aligned}
\rho &= \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \lim_{n\to\infty} \frac{(n+1)(n+1)^n}{(n+1)\,n!} \cdot \frac{n!}{n^n} \\
&= \lim_{n\to\infty} \frac{(n+1)^n}{n^n} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e > 1
\end{aligned}$$