1. Integrate each of the following.

a) $\displaystyle\int \frac{1}{x^2+4x+5}dx$

b) $\displaystyle\int \frac{1}{x^2+4x-5}dx$

c) $\displaystyle\int \frac{e^x}{\sqrt{1-e^x}}dx$

d) $\displaystyle\int \frac{x}{\sqrt{1+x^2}}dx$

e) $\displaystyle\int \frac{1}{\sqrt{1+x^2}}dx$

f) $\displaystyle\int \sin^4 x \cos^3 x dx$

g) $\displaystyle\int_0^{\pi} \sin^2 2x dx$

h) $\displaystyle\int_1^{e} x \ln x dx$

i) $\displaystyle\int_0^{1} \frac{1}{\sqrt[3]{x}}dx$

j) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt[3]{x}}dx$

k) $\displaystyle\int \frac{x^2+x+1}{x^3+x}dx$

l) $\displaystyle\int \frac{\ln(\ln x)}{x\ln x}dx$

2. Compute $\displaystyle\int_0^2 x^3\sqrt{4-x^2}dx$.

3. Let R be the region bounded by the graphs of $y=\dfrac{1}{x^2+x}$ and $y=0$ between $x=1$ and $x=2$.

a) Compute the area of R.

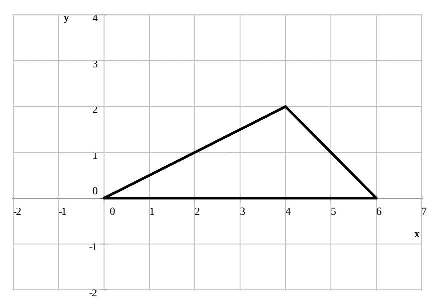
b) Compute the volume of the object we obtain by rotating R about the $x-$axis.

c) Compute the volume of the object we obtain when we rotate R about the $y-$axis.

4. a) Find the volume of the object that has this shape as its base, and cross sections perpendicular to the base are squares.

b) Find the volume of the object that has this shape as its base, and cross sections perpendicular to the base are equilateral triangles.

c) Find the center of mass of this object. Assume a uniform density.



5. The base of a solid is a circular disk with radius 3. Find the volume of the solid if parallel cross-sections perpendicular to the base are isosceles right triangles with one of the two equal sides lying along the base.

6. Compute the volume of the torus we obtain by rotating the region R about the line $x=-2$, where R is the region inside the circle $(x-3)^2+y^2=4$.

7. A spherical tank of radius R is resting on the ground. How much work is needed to pump it full of a liquid with density δ? Assume that we use a pipe that is always leveled at the surface of the water, and the water is initially at the ground level.

8. A 12 meter long chain weighs 36 Newtons and hangs over the edge of a 20 meter tall building. How much work is done in pulling the chain to the top of the building?

9. a) Give a definition of $\displaystyle\lim_{n\to\infty} a_n = L$. b) Give a definition of $\displaystyle\sum_{n=0}^{\infty} a_n$ converges.

10. Prove that a non-decreasing bounded sequence is convergent.

11. Give an $\varepsilon - N$ proof for the limit $\lim\limits_{n\to\infty} a_n$ where $a_n = \dfrac{3n-1}{2n+5}$.

12. Find each of the following limits.

a) $\lim\limits_{n\to\infty} \dfrac{n!}{2^n}$ b) $\lim\limits_{n\to\infty} \dfrac{n!}{n^n}$ c) $\lim\limits_{n\to\infty} n \sin \dfrac{\pi}{n}$ d) $\lim\limits_{n\to\infty} \left(\dfrac{n}{n+2}\right)^n$

13. In each case, determine whether the given series converges absolutely, converges conditionally, or diverges. Justify your conclusion.

a) $\sum\limits_{n=2}^{\infty} \dfrac{1}{n \ln n}$ b) $\sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{n \ln n}$ c) $\sum\limits_{n=1}^{\infty} \dfrac{\cos n}{n^2+1}$ d) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n n}{n^2+1}$ e) $\sum\limits_{n=1}^{\infty} \dfrac{n!}{5^n}$

14. a) Compute the Taylor polynomial of order seven about $x = 0$ generated by the function $f(x) = x \ln(1+x)$.

b) Use the polynomial in part a) to compute $f^{(7)}(0)$.

15. a) Find the first three terms of the Maclaurin series for $f(x) = \dfrac{x}{\sqrt{1+x}}$.

b) Find the first five terms of the power series representation for $\tan^{-1} x^2$.

16. Let $f(x) = x^3 \sin x$. Compute the 100th derivative evaluated at zero, i.e. $f^{(100)}(0)$.

17. Compute the interval of convergence in each of the following.

a) $\sum\limits_{n=1}^{\infty} \dfrac{x^{3n}}{n8^n}$ b) $\sum\limits_{n=1}^{\infty} \dfrac{(2x-1)^n}{n^2 \cdot 2^n}$ c) $\sum\limits_{n=1}^{\infty} \dfrac{(-5)^n x^{2n}}{n^n}$

18. Find the values of p for which the series $\sum\limits_{n=2}^{\infty} \dfrac{1}{n(\ln n)^p}$ is convergent.

19. Compute each of the following sums.

a) $\sum\limits_{n=0}^{\infty} \dfrac{1}{2^n \cdot n!}$ b) $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^2+n}$ c) $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$

20. A ball is dropped from 100 feet. Every time it hits the ground, it rebounds to $\dfrac{1}{3}$ of its previous height. Find the total distance the ball travels.

21. a) Find the function to which the following series converges $\sum\limits_{n=1}^{\infty} nx^n$.

b) Use your result from part a) to evaluate

$$\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{5}{32} + \frac{3}{32} + \ldots.. = \sum_{n=1}^{\infty} n \left(\frac{1}{2}\right)^n$$

22. Use a power series to evaluate the limit $\lim\limits_{x\to 0} \dfrac{24 - 12x^2 + x^4 - 24\cos x}{x^6}$

Answers

1.) a) $\tan^{-1}(x+2)+C$ b) $\frac{1}{6}\ln|x-1|-\frac{1}{6}\ln|x+5|+C$ c) $-2\sqrt{1-e^x}+C$ d) $\sqrt{x^2+1}+C$

e) $\sinh^{-1}x+C$ or $\ln\left|\sqrt{x^2+1}+x\right|+C$ f) $\frac{1}{5}\sin^5 x-\frac{1}{7}\sin^7 x+C$ g) $\frac{\pi}{2}$ h) $\frac{1}{4}\left(e^2+1\right)$

i) $\frac{3}{2}$ j) ∞ k) $\tan^{-1}x+\ln|x|+C$ l) $\frac{1}{2}\ln^2(\ln x)+C$ 2.) $\frac{64}{15}$

3.) a) $\ln\left(\frac{4}{3}\right)$ b) $\left(\frac{2}{3}+\ln\frac{9}{16}\right)\pi$ c) $2\pi(\ln 3-\ln 2)$ 4.) a) 8 b) $2\sqrt{3}$ c) $\left(\frac{10}{3},\frac{5}{3}\right)$

5.) 72 6.) $40\pi^2$ 7.) $\frac{4}{3}\pi R^4 g\delta$ 8.) 216 N

9.) a) For every $\varepsilon>0$, there exists an integer N so that for all $n\in\mathbb{N}$, if $n>N$, then $|a_n-L|<\varepsilon$.

b) The sequence $s_n=\sum_{k=1}^{n}a_k$ of partial sums converges.

10.) see handout 11) $L=\lim_{n\to\infty}a_n=\frac{3}{2}$ $|a_n-L|=\frac{17}{4n+10}<\varepsilon$ any value $N\geq\left\lceil\frac{\frac{17}{\varepsilon}-10}{4}\right\rceil$ will work

12.) a) ∞ b) 0 c) π d) $\frac{1}{e^2}$

13.) a) diverges by the integral test

b) converges because of Leibniz's test but the series of absolute values diverges - see part a, so it converges conditionally.

c) converges absolutely by the comparison and integral test: $\left|\frac{\cos n}{n^2+1}\right|\leq\frac{1}{n^2+1}\leq\frac{1}{n^2}$ and $\frac{1}{n^2}$ converges by the integral test

d) converges by Leibniz's test. However, it does not converge absolutely $|a_n|=\left|\frac{(-1)^n n}{n^2+1}\right|=\frac{n}{n^2+1}$ and $\frac{n}{n^2+1}>\frac{n}{n^2+n}=\frac{1}{n+1}$. By the comparison test, $\sum|a_n|$ diverges

e) diverges by the ratio test $\rho=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{n+1}{5}=\infty$

14.) a) $x^2-\frac{1}{2}x^3+\frac{1}{3}x^4-\frac{1}{4}x^5+\frac{1}{5}x^6-\frac{1}{6}x^7$ b) -840

15.) a) $x-\frac{1}{2}x^2+\frac{3}{8}x^3-\ldots\ldots$ b) $x^2-\frac{1}{3}x^6+\frac{1}{5}x^{10}-\frac{1}{7}x^{14}+\frac{1}{9}x^{18}\ldots..$ 16.) 970 200

17.) a) $[-2,2)$ b) $\left[-\frac{1}{2},\frac{3}{2}\right]$ c) $\mathbb{R}$ 18.) $p>1$ 19.) a) $\sqrt{e}$ b) 1 c) $\frac{\sqrt{2}}{2}$ 20.) 200 feet

21.) a) $f(x)=\frac{x}{(x-1)^2}$ b) 2 22.) $\frac{1}{30}$

