Quiz 11 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-9 and Exams 1, 2
2. Sequences, Limits of Sequences and proofs
3. Definition of series, convergence of series
4. Computing the sum of geometric series and telescoping series.
5. Determining convergence or divergence using the nth term test, the integral test, and the comparison test.

Sample Quiz 11

1. Compute the limit of each of the following sequences or state if it diverges. Justify your answer.

   a) $\lim\limits_{n\to\infty}\left(1+\dfrac{2}{\ln n}\right)^n$ b) $\lim\limits_{n\to\infty}\dfrac{3^n-\sin n}{3^n}$

2. Find the sum of each of the following geometric series or state if it diverges.

   a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^n}{5^{n+1}}$ b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^{2n+1}}{5^{n+1}}$ c) $\displaystyle\sum_{n=0}^{\infty}\frac{\pi^{n-1}}{e^{n+1}}$

3. Find the sum of each of the following telescoping sum or state if it diverges.

   a) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n^2+3n+2}$ b) $\displaystyle\sum_{n=1}^{\infty}\frac{4n+4}{n^2\,(n+2)^2}$ c) $\displaystyle\sum_{n=0}^{\infty}\ln\left(\frac{2n+4}{2n+2}\right)$

4. In case of each of the following series given, determine if it is convergent or divergent. State the test you used.

   a) $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n-1}}{e^n}$ b) $\displaystyle\sum_{n=0}^{\infty}\frac{n}{n^2+1}$ c) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{(n+2)!}$ d) $\displaystyle\sum_{n=0}^{\infty}ne^{-n^2}$ e) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{10n+1}$ f) $\displaystyle\sum_{n=0}^{\infty}\frac{3^n}{n^3}$

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Answers

1. a) $\displaystyle\lim_{n\to\infty}\left(1+\frac{2}{\ln n}\right)^n$ diverges to infinity

proof: $\displaystyle\left(1+\frac{2}{\ln n}\right)^n = e^{\ln\left(1+\frac{2}{\ln n}\right)^n} = e^{n\ln\left(1+\frac{2}{\ln n}\right)} = e^{\lim\limits_{n\to\infty} n\ln\left(1+\frac{2}{\ln n}\right)}$

Consider now the exponent, $\displaystyle\lim_{n\to\infty} n\ln\left(1+\frac{2}{\ln n}\right)$. We apply l'Hopital's Rule.

$$\lim_{n\to\infty} n\ln\left(1+\frac{2}{\ln n}\right) = \lim_{n\to\infty}\frac{\ln\left(1+\frac{2}{\ln n}\right)}{\frac{1}{n}} = \lim_{n\to\infty}\frac{\frac{1}{1+\frac{2}{\ln n}}\cdot 2\left(-\ln n\right)^{-2}\cdot\frac{1}{n}}{-\frac{1}{n^2}} = \lim_{n\to\infty}\frac{2n^2}{\left(1+\frac{2}{\ln n}\right)n\left(\ln n\right)^2}$$

$$= \lim_{n\to\infty}\frac{2n}{\left(\left(\ln n\right)^2+2\ln n\right)} = \lim_{n\to\infty}\frac{2}{\left(2\left(\ln n\right)\frac{1}{n}+\frac{2}{n}\right)} = \lim_{n\to\infty}\frac{2n}{\left(2\ln n+2\right)} = \lim_{n\to\infty}\frac{2}{\frac{2}{n}}$$

$$= \lim_{n\to\infty} n = \infty$$

b) $\displaystyle\lim_{n\to\infty}\frac{3^n-\sin n}{3^n}$ converges to 1 by the Sandwich rule

$$\frac{3^n-1}{3^n} \le \frac{3^n-\sin n}{3^n} \le \frac{3^n+1}{3^n} \quad \text{and}$$

$$\lim_{n\to\infty}\frac{3^n-1}{3^n} = \lim_{n\to\infty}\frac{1-\frac{1}{3^n}}{1} = \frac{1-0}{1} = 1 \quad\text{and}\quad \lim_{n\to\infty}\frac{3^n+1}{3^n} = \lim_{n\to\infty}\frac{1+\frac{1}{3^n}}{1} = \frac{1+0}{1} = 1$$

2. a) $\frac{1}{7}$ b) -2 c) diverges

3. a) 1 b) $\frac{5}{4}$ c) diverges

4. a) converges - geometric series with $r=\frac{2}{e}$, where $\frac{2}{e}$ is between -1 and 1

b) diverges by the integral test or by the comparison test: $\displaystyle\frac{n}{n^2+1} \ge \frac{n}{n^2+n^2} = \frac{1}{2n}$

c) converges - $a_n = \dfrac{1}{(n+1)(n+2)}$ and then either telescoping sums or the comparison test: $\displaystyle\frac{1}{(n+1)(n+2)} = \frac{1}{n^2+3n+2} \le \frac{1}{n^2}$

d) converges by the integral test

e) diverges by the integral test or by the comparison test $\displaystyle\frac{1}{10n+1} \ge \frac{1}{10n+10n} = \frac{1}{20}\cdot\frac{1}{n}$

f) diverges by the nth term test

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