

Quiz 4 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-3
2. Differentiate any function, including logarithmic, exponential, and inverse trigonometric functions. Use implicit differentiation.
3. Apply the fundamental theorem to compute definite integrals and differentiate functions defined using definite integrals.
4. Graph hyperbolic functions (all 12 of them) and state their basic properties (domain, range, increasing/decreasing, even/odd, asymptotes) differentiate them and integrate them.
5. Integrate using substitution, trigonometric substitution, and integration by parts. Integrate trigonometric and inverse trigonometric functions.
6. Determine limits using L'Hôpital's Rule.

Sample Quiz 4

1. a) Graph $f(x) = \tanh^{-1} x$ and state its basic properties.
 b) Compute $\frac{d}{dx} (\tanh^{-1} x)$ c) Compute $\int \tanh x dx$ d) Compute $\int \sinh^{-1} x dx$
2. a) Compute $\frac{dy}{dx}$ if $y = \int_1^{\ln x} \sqrt[3]{1 + e^{2t}} dt$ b) Compute y' if $x^4 - y^4 = \cos^{-1}(xy)$
3. Compute each of the following integrals.
 - a) $\int \frac{x}{\sqrt{x^2 + 3}} dx$
 - b) $\int \frac{1}{\sqrt{x^2 + 3}} dx$
 - c) $\int \frac{x}{x^2(x-3)} dx$
 - d) $\int \frac{3x^2 - 2x + 2}{x^2 - 2x + x^3} dx$
 - e) $\int \frac{1 - 2x}{x + x^3} dx$
4. Compute each of the following limits.
 - a) $\lim_{x \rightarrow 0} \frac{3x - \sin 3x}{x^3}$
 - b) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^{5x}$
 - c) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)^{5x}$
 - d) $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x^2}\right)^{-2x^3}$
 - e) $\lim_{x \rightarrow 0} (x \ln x)$

Answers

1. a) see handout b) $\frac{1}{1-x^2}$ c) $\ln(e^x + e^{-x}) + C$ d) $x \sinh^{-1} x - \sqrt{x^2 + 1} + C$

2. a) $\frac{1}{x} \sqrt[3]{1+x^2}$ b) $\frac{4x^3 + \frac{y}{\sqrt{1-x^2y^2}}}{4y^3 - \frac{x}{\sqrt{1-x^2y^2}}} = \frac{4x^3 \sqrt{1-x^2y^2} + y}{4y^3 \sqrt{1-x^2y^2} - x}$

3. a) $\sqrt{x^2+3} + C$ b) $\ln(x + \sqrt{x^2+3}) + C$ c) $\int \frac{x}{x^2(x-3)} dx = \frac{1}{3} \ln \left| \frac{x-3}{x} \right| + C$

d) $\ln|x-1| + 3 \ln|x+2| - \ln|x| + C$ e) $\ln|x| - 2 \tan^{-1} x - \frac{1}{2} \ln(x^2+1) + C$

4. a) $\frac{9}{2}$ b) e^{-10} c) 1 d) ∞ e) 0