Quiz 6 Information

Quiz 6 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-5 and Exam 1
2. Computing volumes using cross sections, the disk method and the washer method.

Sample Quiz 6

1. Compute the average value of $f(x) = xe^{-2x}$ on the interval $[-2, 2]$.
2. Compute the volume of the solid with a circular base with radius r if cross sections perpendicular to the base are
   a) isosceles right triangles with the hypotenuse lying on the base.
   b) isosceles right triangles with the shorter side on the base.
3. A pyramid has height H and a base that is a rectangle with sides a and b. Set up an integral expressing the volume of this pyramid and evaluate it.
4. a) A wedge is cut out of a circular cylinder of radius 8 units by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of $60°$ along a diameter of the cylinder. Compute the volume of the wedge.
   b) Redo part a) but this time the radius is R and the angle is α
5. Let R be the region determined by the graphs of $y = x^2$, $y = 4x$ and $y = 4$. Compute the volume of the object we obtain when we rotate R about
   a) the $x-$axis b) the $y-$axis
6. Approximate the integral $\int_0^1 \sqrt{x^4 + 1}dx$ using Simpsons rule with $n = 8$.

Answers

1. $\dfrac{1}{4}\displaystyle\int_{-2}^{2} xe^{-2x}dx = -\frac{5}{16}e^{-4} - \frac{3}{16}e^{4} \approx -10.\,242\,876\,77$

2. a) $\dfrac{4}{3}r^3$ b) $\dfrac{8}{3}r^3$

3. $V = \displaystyle\int_{0}^{H} \left(a\frac{x}{H}\right)\left(b\frac{x}{H}\right) dx = \frac{1}{3}Hab$

4. a) $\displaystyle\int_{-8}^{8} \frac{\sqrt{3}}{2}\left(\sqrt{64-x^2}\right)^2 dx = \frac{1024}{3}\sqrt{3}$ b) $\displaystyle\int_{-R}^{R} \frac{1}{2}\left(\sqrt{R^2-x^2}\right)^2 (\tan\alpha)\, dx = \frac{2}{3}R^3\tan\alpha$

5. a) $\displaystyle\int_{0}^{1} \pi\left((4x)^2 - \left(x^2\right)^2\right) dx + \int_{1}^{2} \pi\left(4^2 - \left(x^2\right)^2\right) dx = \frac{77}{15}\pi + \frac{49}{5}\pi = \frac{224}{15}\pi$

   b) $\displaystyle\int_{0}^{4} \pi\left(\left(\sqrt{y}\right)^2 - \left(\frac{y}{4}\right)^2\right) dy = \frac{20}{3}\pi$

6. $1.\,089\,429\,298\,985\,02$

