

1. Differentiate each of the following. Make sure to simplify your final answer.

a) $f(x) = 5^{-2x+1}$

g) $L(\theta) = \frac{\tan \theta - \cot \theta}{\tan \theta}$

m) $g(w) = \cos^{-1}(w^2 + 1)$

b) $m(y) = \log_7(\sqrt{y^6 + 4y^2 + 10})$

h) $v(t) = e^{2t} - e^{-2t}$

n) $f(y) = 2^{\log_3 y}$

c) $f(m) = \frac{1}{3}m^3 \ln m - \frac{1}{9}m^3$

i) $f(x) = \log_2(x^{10})$

o) $f(\theta) = \cot(5\theta)$

d) $m(p) = \sqrt{1-p^2}$

j) $g(x) = \tan^{-1}(5x)$

p) $g(x) = \sqrt{x + \sqrt{x}}$

e) $g(t) = e^{\sin t - \cos t}$

k) $h(\alpha) = \log_2(\tan 2\alpha)$

q) $f(\beta) = \csc(\beta^2 + 1)$

f) $f(a) = \frac{2^{3a-1}}{2^{-a+1}}$

l) $f(x) = \cos(\sin^{-1} x)$

r) $m(y) = \sec^{-1}(3y)$

s) $P(t) = \log_5(\csc t)$

2. Derive the formula for $\frac{d}{dx}(\sec^{-1} x)$.

3. Compute each of the following integrals.

a) $\int \frac{1}{(2a-7)^4} da$

d) $\int_0^1 \frac{1}{1+\theta^2} d\theta$

g) $\int \frac{1}{|x|\sqrt{x^2-1}} dx$

b) $\int \frac{1}{2a-7} da$

e) $\int \frac{1}{\sqrt{1-y^2}} dy$

h) $\int \csc x \cot x dx$

c) $\int_0^2 e^{-2t+1} dt$

f) $\int_0^3 2^x dx$

i) $\int_0^{\pi/4} \tan^2 x dx$

4. Use the Fundamental Theorem of Calculus to compute each of the following derivatives.

a) $\frac{d}{dx} \left(\int_1^x \sqrt{t} dt \right)$

c) $\frac{d}{dx} \left(\int_1^{9x} \sqrt{t} dt \right)$

e) $\frac{d}{dx} \left(\int_0^{x^2} \frac{1}{1+t^2} dt \right)$

b) $\frac{d}{dx} \left(\int_x^{16} \sqrt{t} dt \right)$

d) $\frac{d}{dx} \left(\int_0^x \frac{1}{1+t^2} dt \right)$

f) $\frac{d}{dx} \left[\left(\int_0^x \frac{1}{1+t^2} dt \right)^2 \right]$

Answers

1. a) $f'(x) = -2(\ln 5)5^{-2x+1}$ b) $m'(y) = \frac{3y^5 + 4y}{\ln 7(y^6 + 4y^2 + 10)}$ c) $f'(m) = m^2 \ln m$
d) $m'(p) = -\frac{p}{\sqrt{1-p^2}}$ e) $g'(t) = (\cos t + \sin t)e^{\sin t - \cos t}$ f) $f'(a) = (\ln 2)2^{4a}$
g) $L'(\theta) = \frac{2 \cos \theta}{\sin^3 \theta} = 2 \cot^3 \theta + 2 \cot \theta$ h) $v'(t) = 2e^{2t} + 2e^{-2t}$ i) $f'(x) = \frac{10}{x \ln 2}$
j) $g'(x) = \frac{5}{25x^2 + 1}$ k) $h'(\alpha) = \frac{2}{\ln 2}(\cot 2\alpha + \tan 2\alpha) = \frac{2 \tan^2 2\alpha + 2}{(\ln 2) \tan 2\alpha}$ l) $f'(x) = -\frac{x}{\sqrt{1-x^2}}$
m) $g'(w) = \frac{-2w}{\sqrt{1-(w^2+1)^2}}$ n) $f'(y) = \frac{\ln 2}{y \ln 3} 2^{\log_3 y}$ o) $f'(\theta) = -5 \csc^2 5\theta = -5 \cot^2 5\theta - 5$
p) $g'(x) = \frac{1}{2\sqrt{x+\sqrt{x}}}\left(\frac{1}{2\sqrt{x}} + 1\right)$ q) $f'(\beta) = -2\beta \frac{\cos(\beta^2+1)}{\sin^2(\beta^2+1)} = -2\beta \csc(\beta^2+1) \cot(\beta^2+1)$
r) $m'(y) = \frac{1}{|y|\sqrt{9y^2-1}}$ s) $P'(t) = -\frac{\cot t}{\ln 5}$
2. see handout on differentiating inverse trigonometric functions.
3. a) $\frac{-1}{6(2a-7)^3} + C$ b) $\frac{1}{2} \ln |2a-7| + C$ c) $\frac{1}{2}e - \frac{1}{2}e^{-3}$ d) $\frac{\pi}{4}$ e) $\sin^{-1} y + C$ f) $\frac{7}{\ln 2}$
g) $\sec^{-1} x + C$ h) $-\csc x + C$ i) $1 - \frac{\pi}{4}$
4. a) \sqrt{x} b) $-\sqrt{x}$ c) $27\sqrt{x}$ d) $\frac{1}{1+x^2}$ e) $\frac{2x}{1+x^4}$ f) $\frac{2 \tan^{-1} x}{1+x^2}$

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