1. Compute each of the following derivatives.

   a) $\dfrac{d}{dx}\left(\sinh^{-1} x\right)$ b) $\dfrac{d}{dx}\left(\displaystyle\int_{\cos x}^{\pi} e^{-t^2}dt\right)$

2. Compute each of the following integrals.

   a) $\displaystyle\int \frac{-2x^3+2x^2-x-2}{x^4+2x^3}dx$

   b) $\displaystyle\int (\sec x+\tan x)^2\,dx$

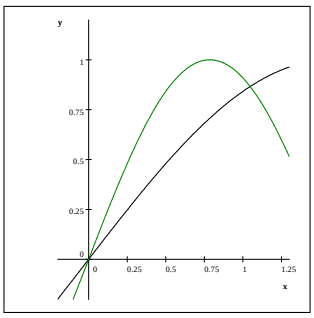
   c) $\displaystyle\int \sqrt{4-9x^2}dx$

   d) $\displaystyle\int_0^{\pi/4} \sec^2 x\tan^3 x\,dx$

   e) $\displaystyle\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

   f) $\displaystyle\int_0^{\infty} x^4e^{-x^5}dx$

3. Let R be the region bounded by the curves $y=\sin x$ and $y=\sin 2x$ between $x=0$ and $x=\dfrac{\pi}{3}$. Compute the volume of the object we obtain by rotating R about
   a) the $x-$axis
   b) the $y-$axis



4. Approximate the definite integral $\displaystyle\int_0^3 \sqrt[4]{x^3+2}dx$ by Simpson's rule with $n=6$.

5. Compute the volume of the object we obtain when rotating the circle $(x-3)^2+y^2=4$ about the $y-$axis.

6. Compute the length of the curve of $y=\dfrac{x^4}{8}+\dfrac{1}{4x^2}$ between $x=1$ and $x=2$.

7. How much work does it take to pump out all water from a tank that is shaped like a semi-sphere of radius 6 meters, and is full of water. Assume the water is pumped out from a pipe leveled at the surface of the water. Use density of $10\,000\dfrac{\mathrm{N}}{\mathrm{kg}^3}$.

8. Prove that a bounded increasing sequence is convergent.

9. Determine whether each of the given series converges absolutely, converges conditionally, or diverges.

   a) $\displaystyle\sum_{n=0}^{\infty} n^2\,3^{-n^2}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{3-n^2}{(n+3)^3}$ c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{n^n}$

10. Find a power series representation for the function $f(x)=\dfrac{1}{x+10}$ and determine the interval of convergence.

11. Compute the interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(x-5)^n}{3^n(n+1)}$

12. Find the sum of the infinite series. If the series diverges, state so.

a) $2^2 + 2^3 + \frac{1}{2}\cdot 2^4 + \frac{1}{6}\cdot 2^5 + \frac{1}{24}\cdot 2^6 + ...$

b) $\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + ...$

c) $1 - 3 + \frac{9}{2} - \frac{9}{2} + \frac{27}{8} - \frac{81}{40} + \frac{81}{80} - \frac{243}{560} + ...$

d) $\frac{x}{2} - \frac{x^3}{8\cdot 3!} + \frac{x^5}{32\cdot 5!} - \frac{x^7}{128\cdot 7!} - .....$

e) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - ....$

f) $\frac{1}{8} - \frac{1}{3}\cdot\frac{1}{32} + \frac{1}{5}\cdot\frac{1}{128} - \frac{1}{7}\cdot\frac{1}{512} + ...$

g) $\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \frac{1}{48} + ...$

Answers

1. a) $\frac{1}{\sqrt{x^2+1}}$ b) $\sin x e^{-\cos^2 x}$

2. a) $\ln x + \frac{1}{2x^2} - 3\ln|x+2| + C$ b) $2\tan x + 2\sec x - x + C$ c) $x\sqrt{1-\frac{9}{4}x^2} + \frac{2}{3}\sin^{-1}\left(\frac{3}{2}x\right) + C$ d) $\frac{1}{4}$

e) 4 f) $\frac{1}{5}$ 3. a) $\frac{3\sqrt{3}}{16}\pi$ b) $\frac{\pi^2}{2} - \frac{3\sqrt{3}}{4}\pi$ 4. 4.802 387 5. $24\pi^2$ 6. $\frac{33}{16}$ 7. $5400\,000\pi$ J

8. see handout Sequences 3 9. a) converges absolutely (ratio test)

b) diverges (sum of two series, one diverges by the comparison test) c) converges absolutely (ratio test)

10. $\frac{1}{10} - \frac{x}{100} + \frac{x^2}{1000} - \frac{x^3}{10\,000} + \frac{x^4}{100\,000} + ... = \sum_{n=0}^{\infty}\frac{1}{10}\left(-\frac{x}{10}\right)^n$ where $-10 < x < 10$ 11. $(2, 8]$

12. a) $4e^2$ b) $\frac{2}{5}$ c) $\frac{1}{e^3}$ d) $\sin\left(\frac{x}{2}\right)$ e) $\ln 2$ f) $\frac{1}{4}\arctan\left(\frac{1}{2}\right)$ g) $\sum_{n=1}^{\infty}\frac{1}{n(n+2)} = \frac{3}{4}$

