

Quiz 10 will cover the following material: (all handouts posted on the web site so far)

- All material for Quizzes 1-9 and Exams 1, 2
- State the axioms of real numbers.
- Proving statements about sequences using $\varepsilon - N$ proofs. Findin N given ε .
- Proving statements using induction.
- Proving statements about limits of sequences (see Sequences part 2)
- Finding limits of sequences defined recursively.

Sample Quiz 10

1. Let R be the region defined by $y = \cos x$ and $y = 0$ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
 - a) Find the volume of the object we obtain when rotating R to the line $y = -2$.
 - b) Find the volume of the object we obtain when rotating R to the line $x = -2$.
 - c) Find the center of mass of R using a uniform density δ .
2. Prove that if a sequence is convergent, then it is bounded.
3. Let $a_n = \frac{3n - 5}{n + 2}$
 - a) Find $\lim_{n \rightarrow \infty} a_n$
 - b) Find a value of N that works with $\varepsilon = 0.02$
 - c) Find a value of N that works with $\varepsilon = 0.0001$
 - d) Find a general expression for N in terms of ε .
4. Find the limit for each of the following sequences. If the limit does not exist, state so.
 - a) $a_1 = 3 \quad a_{n+1} = 2 + \frac{1}{a_n}$
 - b) $a_1 = 8 \quad a_{n+1} = \sqrt{a_n + 1}$
 - c) $a_1 = 5 \quad a_{n+1} = \frac{2}{3}a_n + 10$

Answers

1. a) $8\pi + \frac{1}{2}\pi^2$ b) 8π c) $(0, \frac{\pi}{8})$
2. see handout
3. a) 3 b) answers may vary, and any value $N \geq 548$ is correct
 c) answers may vary, and any value $N \geq 10998$ is correct
 d) answers may vary, but $\lceil \frac{11}{\varepsilon} - 2 \rceil$ or $\lceil \frac{11}{\varepsilon} \rceil$ or any greater integer are correct
4. a) $\sqrt{2} + 1$ b) $\frac{\sqrt{5} + 1}{2}$ c) 30