Quiz 11 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-10 and Exams 1, 2
2. Sequences, Limits of Sequences and proofs
3. Definition of series, convergence of series
4. Computing the sum of geometric series and telescoping series.
5. Determining convergence or divergence using the nth term test, or that the sequence of partial sums is bounded or not
6. Computing sums of series and a constant multiple of a series.

Sample Quiz 11

1. State the definition of a convergent series.
2. Prove that if $\lim\limits_{n\to\infty} a_n = A$ and $\lim\limits_{n\to\infty} b_n = B$, then $\lim\limits_{n\to\infty} (a_n + b_n) = A + B$
3. Compute the limit of each of the following sequences or state if it diverges. Justify your answer.

   a) $\displaystyle\lim_{n\to\infty}\left(1-\frac{4}{n}\right)^{3n}$ b) $\displaystyle\lim_{n\to\infty}\frac{2^n-4^n}{3^n}$ c) $a_1 = 8$ and $a_{n+1} = \sqrt{a_n + 20}$
4. Find the sum of each of the following geometric series or state if it diverges.

   a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n+(-1)^n}{3^{n+1}}$

   b) $\displaystyle\sum_{n=0}^{\infty}\frac{2^{2n-1}}{\pi^n}$

   c) $\displaystyle\sum_{n=1}^{\infty}\frac{3}{n(n+2)}$

   d) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+1}{n(n+1)}$

   e) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n-5^n}{4^n}$

   f) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n^2+3n+2}$

   g) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{2}{n}\right)^n$

   h) $\displaystyle\sum_{n=1}^{\infty}\frac{4n+4}{n^2(n+2)^2}$

   i) $\displaystyle\sum_{n=0}^{\infty}\ln\left(\frac{2n+4}{2n+2}\right)$

   j) $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n-1}}{e^n}$

   k) $\displaystyle\sum_{n=1}^{\infty}\frac{n-1}{n^2}$

   l) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{(n+2)!}$

   m) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{10n}$

   n) $\displaystyle\sum_{n=0}^{\infty}\frac{n!}{2^n}$

Answers

1. see handout Series 1   2. See handout Sequences Part 2

3. a) e^{-12} b) $-\infty$ c) 5

4. a) $-\dfrac{13}{60}$ (sum of two geometric series)   b) diverges because this is a geometric series with $r = \dfrac{4}{\pi} > 1$

   c) $\dfrac{9}{4}$ (telescoping sum)   d) diverges by the nth term test

   e) diverges because it is the difference between a convergent and a divergent series

   f) 1 (telescoping sum)   g) diverges by the nth term test   h) $\dfrac{5}{4}$ (telescoping sum)

   i) diverges (telescoping sum using $\ln\left(\dfrac{a}{b}\right) = \ln a - \ln b$)   j) $\dfrac{1}{e-2}$ (geometric series)

   k) diverges since $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)$ where $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

   l) $\dfrac{1}{2}$ (telescoping sum)   m) diverges (harmonic series, multiplied by a non-zero constant)

   n) diverges by nth term test

