Quiz 3 will cover the following (all handouts posted on the web site so far)

- All material for Quizzes 1 and 2
- Differentiate any function, including logarithmic, exponential, and inverse trigonometric functions.
- Apply the fundamental theorem to compute definite integrals and differentiate functions defined using definite integrals.
- Apply the fundamental theorem to estimate logarithms such as $\ln 2$
- Graph and state the basic properties of trigonometric functions, including inverse functions.
- Integrate exponential functions
- Integrate using substitution
- Integrate trigonometric functions
- Correctly and precisely **state**
  - the Fundamental Theorem of Calculus (both parts)
  - the definition of $\ln x$
- **Prove** each of the following
  - differentiation formulas of all trigonometric functions, including all inverse trigonometric functions, for example, derive the formula for $\frac{d}{dx}\left(\sec^{-1}(x)\right)$
  - For all $x, y > 0$, $\ln(x+y) = \ln x + \ln y$ and $\ln(x-y) = \ln\left(\frac{x}{y}\right)$
  - $\lim\limits_{x\to\infty} \ln x = \infty$

Sample Quiz 3

1. Use the definition of $\ln x = \int\limits_1^x \frac{1}{t} dt$ to prove that $\ln 3 < 1.3$
2. Graph $f(x) = \tan^{-1} x$ and state its properties.
3. Prove that if $\ln x = \int\limits_1^x \frac{1}{t} dt$, then $\ln x + \ln y = \ln(xy)$ for all positive x and y.
4. Differentiate each of the following:

   a) $f(x) = \cot x + \sec x$

   b) $m(x) = e^{\cos 5x}$

   c) $M(a) = \tan^{-1}\left(a^4\right)$

   d) $f(x) = \int\limits_{x^4}^0 \sqrt{t^2+1}\, dt$

   e) $g(x) = \frac{d}{dx}\left(\int\limits_0^{x^3} \frac{1}{\sqrt{1-t^2}} dt\right)^2$

   f) $f(x) = 3^{-2x+1}$

5. Compute each of the following integrals.

a) $\displaystyle\int \frac{x}{x^2+3}dx$

b) $\displaystyle\int \frac{1}{x^2+3}dx$

c) $\displaystyle\int_0^{\pi} \sqrt{1-\cos x}dx$

d) $\displaystyle\int_0^{\pi/4} \sqrt{1+\cos 2x}dx$

e) $\displaystyle\int \csc x dx$

f) $\displaystyle\int_0^{10} e^{5x}dx$

g) $\displaystyle\int \tan^3 x dx$

h) $\displaystyle\int_0^{\pi/4} \tan^3 x dx$

i) $\displaystyle\int_1^8 \frac{1}{\sqrt{3x+1}}$

j) $\displaystyle\int \sin(5x)\cos(11x)\,dx$

k) $\displaystyle\int \cos^3 x dx$

l) $\displaystyle\int \cos^4 x dx$

m) $\displaystyle\int \cos^5 x dx$

n) $\displaystyle\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)}dx$

o) $\displaystyle\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx$

Answers

1. proof: $\ln 3 = \displaystyle\int_1^3 \frac{1}{t}dt$. Consider the partition $\{1, 1.5, 2, 2.5, 3\} = \left\{\frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}\right\}$. A left Riemann sum on this partition overestimtes the integral and it is

$$\begin{aligned} R &= \frac{1}{2}\left(\frac{1}{1}\right)+\frac{1}{2}\left(\frac{1}{1.5}\right)+\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{2.5}\right) = \frac{1}{2}\left(\frac{1}{\frac{2}{2}}+\frac{1}{\frac{3}{2}}+\frac{1}{\frac{4}{2}}+\frac{1}{\frac{5}{2}}\right) \\ &= \frac{1}{2}\left(1+\frac{2}{3}+\frac{1}{2}+\frac{2}{5}\right) = \frac{1}{2}\left(\frac{30+20+15+12}{30}\right) = \frac{77}{60} \approx 1.283 \end{aligned}$$

and so $\ln 3 < 1.283 < 1.3$

2. see handout

3. see handout

4. a) $f'(x) = -\cot^2 x - 1 + \sec x \tan x$ b) $m'(x) = -5(\sin 5x)e^{\cos 5x}$ c) $M'(a) = \dfrac{4a^3}{a^8+1}$

d) $f'(x) = -4x^3\sqrt{x^8+1}$ e) $\dfrac{6x^2\sin^{-1}\left(x^3\right)}{\sqrt{1-x^6}}$ f) $f'(x) = -2\cdot(\ln 3)\cdot 3^{-2x+1}$

5. a) $\dfrac{1}{2}\ln\left(x^2+3\right)+C$ b) $\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)+C$ c) $2\sqrt{2}$ d) 1 e) $-\ln|\csc x+\cot x|+C$

f) $\dfrac{1}{5}e^{50}-\dfrac{1}{5}$ g) $\dfrac{1}{2}\tan^2 x+\ln|\cos x|+C$ h) $\dfrac{1}{2}-\dfrac{1}{2}\ln 2$ i) 2 j) $\dfrac{1}{12}\cos 6x-\dfrac{1}{32}\cos 16x+C$

k) $\sin x-\dfrac{\sin^3 x}{3}+C$ l) $\dfrac{3}{8}x+\dfrac{1}{4}\sin 2x+\dfrac{1}{32}\sin 4x+C$ m) $\sin x-\dfrac{2}{3}\sin^3 x+\dfrac{1}{5}\sin^5 x+C$

n) $2\ln\left(1+\sqrt{x}\right)+C$ o) $\dfrac{4}{3}\left(1+\sqrt{x}\right)\sqrt{1+\sqrt{x}}+C$

