Quiz 5 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-4 and Exam 1
2. Differentiate any function, including logarithmic, exponential, and inverse trigonometric functions.
3. Apply the fundamental theorem to compute definite integrals and differentiate functions defined using definite integrals.
4. Graph trigonometric functions (all 12 of them) and state their basic propoerties, differentiate them and integrate them.
5. Integrate using substitution, trigonometric substitution, integration by parts, and partial fractions. Integrate trigonometric and inverse trigonometric functions.
6. Compute improper integrals.
7. Determine limits using L'Hôpital's Rule.
8. Approximate definite integrals using the following Riemann sums: left, right, midpoint, trapezoid, and by Simpson's rule.

# Sample Quiz 5

1. a) Graph $f(x) = \tan^{-1} x$ and state its basic properties.

   b) Compute $\dfrac{d}{dx}\left(\tan^{-1} x\right)$ c) Compute $\displaystyle\int \tan^{-1} x dx$

2. Compute $\dfrac{dy}{dx}$ if $y = \displaystyle\int_1^{\ln x} \sqrt[3]{1+e^{2t}}dt$

3. Compute each of the following integrals.

   a) $\displaystyle\int \frac{x}{\sqrt{x^2+3}}dx$

   b) $\displaystyle\int \frac{1}{\sqrt{x^2+3}}dx$

   c) $\displaystyle\int \frac{x}{x^2(x-3)}dx$

   d) $\displaystyle\int \frac{3x^2-2x+2}{x^2-2x+x^3}dx$

   e) $\displaystyle\int \frac{1-2x}{x+x^3}dx$

   f) $\displaystyle\int \frac{1}{\sqrt{4x^2+25}}dx$

   g) $\displaystyle\int_{\pi/6}^{\pi/3} \sin(2\theta)\, d\theta$

   h) $\displaystyle\int \sec^3 x dx$

   i) $\displaystyle\int \sqrt{r^2-x^2}dx$

   j) $\displaystyle\int_6^{\infty} \frac{1}{(x-2)(x+2)}dx$

4. Compute each of the following limits.

   a) $\displaystyle\lim_{x\to 0} \frac{3x-\sin 3x}{x^3}$

   b) $\displaystyle\lim_{x\to\infty}\left(\left(1-\frac{2}{x}\right)^{5x}\right)$

   c) $\displaystyle\lim_{x\to\infty}\left(\left(1+\frac{1}{x^2}\right)^{5x}\right)$

   d) $\displaystyle\lim_{x\to\infty}\left(\left(1-\frac{1}{x^2}\right)^{-2x^3}\right)$

   e) $\displaystyle\lim_{x\to 0}(x\ln x)$

5. Consider the definite integral $\int_0^{12} x^2 dx$. Compute the **exact value** of the right-hand sum approximating this definite integral using a uniform partition with $n = 100$.

6. Consider the definite integral $\int_0^{2} \sqrt{\sin^4 x + 1} dx$. Let $P$ be a uniform permutation where $n = 6$. (That means 6 subintervals.) Compute an approximate value (up to four or more decimal places) of each of the following.

a) Left-sum on $P$.

b) Right-Sum on $P$

c) Using Trapezoids on $P$.

d) Midpoint-Sum on $P$ (approximate the function as constant $f(x_M)$ where $x_M$ is the midpoint of the subinterval.)

e) Using Simpson's rule.

f) Given that the value of the value of the definite integral is $2.41579359$, find the error in each case and express it as a percentage of the correct result. Which approximation is best?

# Answers

1. a) see handout b) $\frac{1}{1+x^2}$ c) $\int \tan^{-1} x dx = x \arctan x - \frac{1}{2}\ln\left(x^2+1\right) + C$

2. $\frac{1}{x}\sqrt[3]{x^2+1}$

3. a) $\sqrt{x^2+3}+C$ b) $\ln\left(x+\sqrt{x^2+3}\right)+C$ c) $\frac{1}{3}\left(\ln|x-3|-\ln|x|\right)+C$

   d) $\ln|x-1|+3\ln|x+2|-\ln|x|+C$ e) $\ln|x|-2\tan^{-1}x-\frac{1}{2}\ln\left(x^2+1\right)+C$

   f) $\frac{1}{2}\ln\left(2x+\sqrt{4x^2+25}\right)+C$ g) $\frac{1}{2}$ h) $\frac{1}{2}\left(\sec x\tan x+\ln|\sec x+\tan x|\right)+C$

   i) $\frac{r^2}{2}\sin^{-1}\left(\frac{x}{r}\right)+\frac{x\sqrt{r^2-x^2}}{2}+C$ j) $\frac{1}{4}\ln 2$

4. a) $\frac{9}{2}$ b) $e^{-10}$ c) 1 d) $\infty$ e) 0

5. $\frac{365\,418}{625} = 584.668\,8$

6. a) $2.361\,674$ b) $2.460\,857\,24$ c) $2.411\,265\,7$ d) $2.418\,08$ e) $2.416\,125\,3$

   f) Left sum's error is 2.24% Trapezoid's error: 1.874% Simpson's Rule's error: 0.014%

   Right sum's error is 1.8965% Midpoint's error: 0.095%

   So Simpson's Rule is the best one here.