

Quiz 6 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-5 and Exam 1
2. Applying definite integrals to area under the graph, area between two graphs. Computing the average value of an integrable function.
3. Computing volumes using cross sections and the disk method.

## Sample Quiz 6

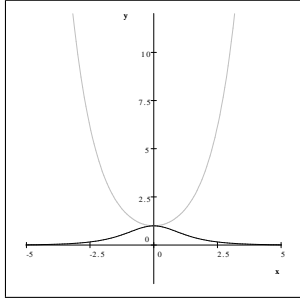
1. Find the area between the graphs of  $f(x) = \sqrt{x-2}$  and  $g(x) = \frac{1}{2}x - 1$ .
2. Compute the average value of  $f(x) = xe^{-2x}$  on the interval  $[-2, 2]$ .
3. a) Graph  $\cosh x$  and then use your graph to sketch the graph of  $\frac{1}{\cosh x}$ .  
b) Compute the integral  $\int \frac{1}{\cosh x} dx$ .  
c) Compute the improper integral  $\int_0^{\infty} \frac{1}{\cosh x} dx$ .
4. Compute the volume of the solid with a circular base with radius  $r$  if cross sections perpendicular to the base are
  - a) isosceles right triangles with the hypotenuse lying on the base.
  - b) isosceles right triangles with the shorter side on the base.
5. A pyramid has height  $H$  and a base that is a rectangle with sides  $a$  and  $b$ . Set up an integral expressing the volume of this pyramid and evaluate it.
6. a) A wedge is cut out of a circular cylinder of radius 8 units by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of  $60^\circ$  along a diameter of the cylinder. Compute the volume of the wedge.  
b) Redo part a) but this time the radius is  $R$  and the angle is  $\alpha$
7. a) Let  $R$  be the region determined by the graphs of  $y = x^3$ ,  $y = 0$  and  $x = 2$ . Compute the volume of the object we obtain when we rotate  $R$  about the  $x$ -axis.  
b) Let  $R$  be the region determined by the graphs of  $y = x^3$ ,  $x = 0$  and  $y = 8$ . Compute the volume of the object we obtain when we rotate  $R$  about the  $y$ -axis.
8. Approximate the integral  $\int_0^1 \sqrt{x^4 + 1} dx$  using Simpsons rule with  $n = 8$ .

## Answers

1.  $\frac{4}{3}$

2.  $\frac{1}{4} \int_{-2}^2 x e^{-2x} dx = -\frac{5}{16} e^{-4} - \frac{3}{16} e^4 \approx -10.24287677$

3. a)      b)  $2 \arctan(e^x) + C$       c)  $\frac{\pi}{2}$



4. a)  $\frac{4}{3} r^3$       b)  $\frac{8}{3} r^3$

5.  $V = \int_0^H \left(a \frac{x}{H}\right) \left(b \frac{x}{H}\right) dx = \frac{1}{3} Hab$

6. a)  $\int_{-8}^8 \frac{\sqrt{3}}{2} (\sqrt{64 - x^2})^2 dx = \frac{1024}{3} \sqrt{3}$

b)  $\int_{-R}^R \frac{1}{2} (\sqrt{R^2 - x^2})^2 (\tan \alpha) dx = \frac{2}{3} R^3 \tan \alpha$

7. a)  $\frac{128}{7} \pi$       b)  $\frac{96}{5} \pi$

8. 1.0894293