Quiz 7 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-6 and Exam 1
2. Computing average value of an integrable function and areas between graphs.
3. Computing volumes using cross sections, the disk method, the washer method, and by cylindrical shells.
4. Graph and state the basic properties of hyperbolic functions as shown in class and the handouts. (Not the inverses yet.)
5. Differentiate and integrate hyperbolic functions as shown in class and in the handouts. (These include inverse hyperbolic functions as well.)

Sample Quiz 7

1. a) Graph $f(x) = \dfrac{1}{\cosh x}$ and state the basic properties of the function.

   b) Prove that $\sinh(2x) = 2\sinh x \cosh x$ c) Compute $\dfrac{d}{dx}\left(\tanh^{-1} x\right)$ d) Compute $\displaystyle\int \coth x dx$

2. Let R be the region bounded by $y = \sqrt{x}$, $x = 1$, $x = 9$, and $y = 0$. Compute the exact value of the volume of the object we obtain by rotating R about

   a) the $x-$axis b) the line $x = -3$ c) the $y-$axis d) the line $x = 12$

3. Suppose that $m > 0$. Let R be the region between $y = mx$ and $y = mx^2$ between $x = 0$ and $x = 1$. Compute the value of m if we know that $V_x = V_y$ where V_x is the volume of the object we obtain by rotating R about the $x-$axis and V_y is the volume of the object we obtain by rotating R about the $y-$axis.

4. Use integration to compute the volume of the torus we obtain when we rotate the circle $x^2 + y^2 = 4$ about the line $x = 6$.

5. Let R be the region bounded by the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ and the $x-$axis. Compute the volume of the object we obtain when rotating R about

   a) the $x-$axis b) the line $x = 7$

6. Compute the exact value of the average value of $f(x) = \sinh x$ on $[0, \ln 5]$.

7. Let R be the region bounded by the $x-$axis, $y = \sin^2 x$, $x = 0$ and $x = 2\pi$. Find the volume of the object we obtain when rotating R about the $x-$axis.

8. Can we compute the volume of the object we obtain when we rotate R about the $x-$axis, where R is the region determined by $y = 0$, $x = 0$, and $y = \dfrac{1}{\cosh x}$?

Answers

1. a) see handout b)

$$2\sinh x\cosh x = 2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right) = 2\left(\frac{(e^x)^2 - (e^{-x})^2}{4}\right) = \frac{e^{2x} - e^{-2x}}{2} = \sinh 2x$$

c) $\dfrac{1}{1-x^2}$ d) $\ln\left|\dfrac{e^x - e^{-x}}{2}\right| + C = \ln\left|e^{2x} - 1\right| - x + C$

2. a) 40π b) $\dfrac{1488}{5}\pi$ c) $\dfrac{968}{5}\pi$ d) $\dfrac{1112}{5}\pi$ 3. $m = \dfrac{5}{4}$ 4. $48\pi^2$

5. a) 24π b) $42\pi^2$ 6. $\dfrac{9}{16\ln 2}$ 7. $2\pi^3$ 8. π

