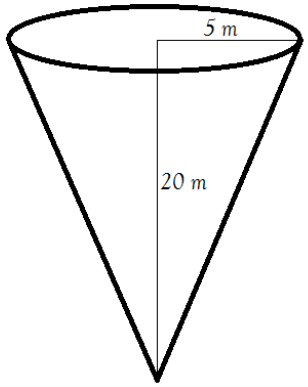
Quiz 8 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1-7 and Exam 1
2. Computing work.
3. Computing arc length.
4. Computing center of mass in one- and two-dimensional objects.

Sample Quiz 8

1. A tank, shaped like a straight cone is positioned with its circular base upward. It is full of water. Compute the work that is required to pump out all the water from the tank. Assume the following: at all times, the pipe is leveled at the surface of the water, and we are pumping out the water to the top of the tank. The density of water is $1000\frac{\text{kg}}{\text{m}^3}$ and the $g = 9.81\frac{\text{m}}{\text{s}^2}$



2. The gravitational force between two objects can be computed as $F_{gr} = \frac{m_1 m_2 G}{r^2}$ where m_1 and m_2 denote the mass of each of the two objects, r is the distance between their centers of mass, and G is the universal gravitational constant, $G = 6.67 \times 10^{-11}\frac{\text{N}\,\text{m}^2}{\text{kg}^2}$. We want to lift a spaceship, measuring 1000 kg, from the surface of the Earth to a galaxy far, far from ours. How much work does this require? (Assume that Earth is a sphere with radius 6370 km and mass of 5.97×10^{24} kg.)
3. Use integration to compute the volume of the torus we obtain when we rotate the circle $x^2 + y^2 = r$ about the line $x = R$ where $R > r$.
4. Compute the arc length of the graph of $f(x) = \ln(\sec x)$ on $\left[0, \frac{\pi}{4}\right]$.
5. A tank, shaped like a sphere with radius R (measured in meters) is full of water and is buried so that its center is h meters below the ground . Compute the work that is required to pump out all the water from the tank. Assume the following: at all times, the pipe is leveled at the surface of the water, and we are pumping out the water to the top of the tank. The density of water is $1000\frac{\text{kg}}{\text{m}^3}$ and the gravitational acceleration is $g \approx 10\frac{\text{m}}{\text{s}^2}$

6. Compute the center of mass of the unit (upper) semicircle centered at the origin. Assume a uniform density.

7. Compute the center of mass of R where R is the region bounded by $y = x^3$ and $y = \sqrt{x}$ between $x = 0$ and $x = 1$.

   a) Assume a uniform density.

   b) Assume a density of $\delta(x) = \sqrt{x}$

Answers

1. $8175000\pi(\text{ J}) \approx 25682519.943\,1\ (\text{ J})$
2. $62\,511\,617\ (\text{ J})$
3. $V = 2\pi^2 Rr^2$
4. $\ln\left(\sqrt{2}+1\right)$
5. $\frac{40\,000}{3}\pi R^3 h$
5. $\left(0, \frac{4}{3\pi}\right)$
6. a) $\left(\frac{12}{25}, \frac{3}{7}\right)$ b) $\left(\frac{6}{11}, \frac{12}{25}\right)$