Quiz 9 will cover the following material: (all handouts posted on the web site so far)
All material for Quizzes 1-8 and Exams 1 and 2

Sample Quiz 9

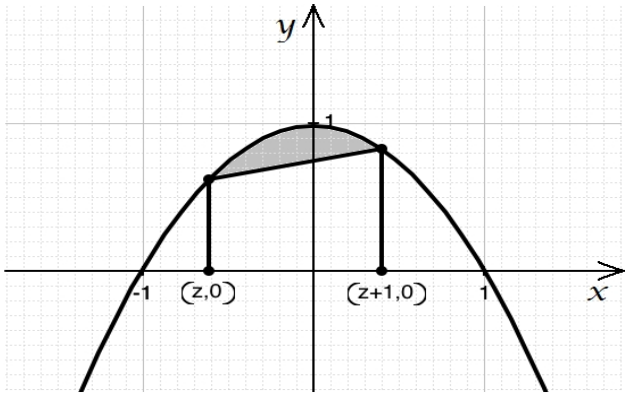
1. Compute $\frac{d}{dx}\left(\tanh^{-1} x\right)$ and $\int \tanh^{-1} x dx$.

2. Compute $\frac{d}{dx}\left(\int_{x^2}^{1} e^{-t^2} dt\right)$

3. Could an infinite object have a center of mass? If possible, compute both coordinates of the center of mass for the region determined by $y = 0$, $x = 1$, and $y = \frac{1}{x^3}$. (Hint: use improper integrals.)

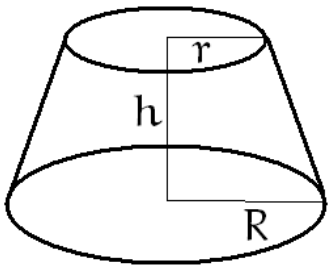
4. A tank, shaped like a sphere is sitting on the ground. It's radius is 10 m. It is full of water. Compute the work that is required to pump out all the water from the tank. Assume the following: at all times, the pipe is leveled at the surface of the water, and we are pumping out the water to the top of the tank. Assume that the density of water is $10\,000 \frac{\mathrm{N}}{\mathrm{m}^3}$.

5. Consider the parabola shown on the picture. It's equation is $y = 1 - x^2$. Let z be any real number with $z \in (-1, 0)$. Prove that the area of the shaded region is independent of the value of z.



6. Compute the average value of $f(x) = x \sin x$ on $[0, \pi]$.

7. Use an integral to compute the volume of the object shown on the picture.



8. Consider the sequence $a_n = \frac{3n^2 - 1}{n^2 + 2}$.

   a) What is $\lim_{n \to \infty} a_n$?

   b) Find the best $t \in \mathbb{N}$ so that for all $n > t$, $|a_n - L| < \frac{1}{100}$, where L is the limit you found in part a).

Answers

1. $\dfrac{d}{dx}\left(\tanh^{-1} x\right) = \dfrac{1}{1-x^2}$ $\displaystyle\int \tanh^{-1} x\,dx = \frac{1}{2}x\ln\left(\frac{x+1}{x-1}\right) + \frac{1}{2}\ln\left|x^2-1\right| + C$ 2. $-2xe^{-x^4}$

3. $\overline{x} = 2$ and $\overline{y} = \dfrac{1}{5}$ 4. $\dfrac{400\,000\,000\pi}{3}$ (J)

5. The area under the graph is $\displaystyle\int_z^{z+1}\left(1-x^2\right)dx = -z^2 - z + \frac{2}{3}$ and the area of trapezoid is $\dfrac{1}{2}\left(f(z) + f(z+1)\right) = -z^2 - z + \dfrac{1}{2}$. The difference is $\dfrac{1}{6}$, independent of z. 6. 1

7. $V = \dfrac{\pi h}{3}\left(R^2 + Rr + r^2\right)$ 8. a) 3 b) $t = 26$