Math 98 **Midterm Review** Solutions

1. Perform the operations as indicated.

(a) $\sqrt{(-1)^4-6\left(2^2-(-3)^2\right)-(-1)^3+10\div 5\cdot 2}=$ 6

Solution: We apply order of operations. Parentheses first.

$$\begin{aligned} &= \sqrt{(-1)^4-6\left(2^2-(-3)^2\right)-(-1)^3+10\div 5\cdot 2} = \sqrt{(-1)^4-6\,(4-9)-(-1)^3+10\div 5\cdot 2} \\ &= \sqrt{(-1)^4-6\,(-5)-(-1)^3+10\div 5\cdot 2} \end{aligned}$$

Now exponents:

$$\sqrt{(-1)^4-6\,(-5)-(-1)^3+10\div 5\cdot 2} = \sqrt{1-6\,(-5)-(-1)+10\div 5\cdot 2}$$

Now multiplications, divisions, left to right.

$$\sqrt{1-6\,(-5)-(-1)+10\div 5\cdot 2} = \sqrt{1-(-30)-(-1)+2\cdot 2} = \sqrt{1-(-30)-(-1)+4}$$

Now additions, subtractions, left to right.

$$\sqrt{1-(-30)-(-1)+4} = \sqrt{1+30-(-1)+4} = \sqrt{31-(-1)+4} = \sqrt{31+1+4} = \sqrt{32+4} = \sqrt{36}$$

Now we take the square root. (The fact that this operation is last is because the long square root is a case of an "invisible parentheses")

$$\sqrt{36}=6$$

Thus the solution is 6.

(b) $\dfrac{-3^2-(-3)^2-16\div(-2)\cdot(-2)+(-2)^2}{|(-4)\,(-7)-(-2)|}=$ -1

Solution: We apply order of operations. The big bar is an "invisible parentheses". It means that we have to completely work out the numerator and the denominator and then apply the division. In the numerator, there is no parentheses, so we start with exponents, left to right. Notice that $-3^2=-9$ and not 9.

$$-3^2-(-3)^2-16\div(-2)\cdot(-2)+(-2)^2 = -9-9-16\div(-2)\cdot(-2)+4$$

We now perform all multiplications and divisions, left to right.

$$-9-9-16\div(-2)\cdot(-2)+4 = -9-9-(-8)\cdot(-2)+4 = -9-9-16+4$$

Now we perform all additions, subtractins, left to right.

$$-9-9-16+4 = -18-16+4 = -34+4 = -30$$

Now the denominator. It has a parentheses, since the absolute value sign also functions as parentheses. We start with the multiplication.

$$|(-4)\,(-7)-(-2)| = |28-(-2)| = |28+2| = |30| = 30$$

The answer is thus $\dfrac{-30}{30}=-1$

(c) $\dfrac{(-1)^2-\left(-\dfrac{1}{2}\right)^2}{5\dfrac{5}{8}}+\dfrac{1}{5}= \quad \dfrac{1}{3}$

Solution: We apply order of operations. We will keep all negative signs in the numerator. We start with exponents, left to right. Every step will be shown.

$$\frac{(-1)^2-\left(\frac{-1}{2}\right)^2}{5\frac{5}{8}}+\frac{1}{5} = \qquad (-1)^2=-1\,(-1)=1$$

$$\frac{1-\left(\frac{-1}{2}\right)^2}{5\frac{5}{8}}+\frac{1}{5} = \qquad \left(\frac{-1}{2}\right)^2=\frac{-1}{2}\left(\frac{-1}{2}\right)=\frac{1}{4}$$

$$\frac{1-\frac{1}{4}}{5\frac{5}{8}}+\frac{1}{5} =$$

Subtraction in the numerator is next.

$$\frac{1-\frac{1}{4}}{5\frac{5}{8}}+\frac{1}{5} = \qquad 1-\frac{1}{4}=\frac{1}{1}-\frac{1}{4}=\frac{4}{4}-\frac{1}{4}=\frac{3}{4}$$

$$\frac{\frac{3}{4}}{5\frac{5}{8}}+\frac{1}{5} =$$

Converting the mixed number to improper fraction is an addition:

$$\frac{\frac{3}{4}}{5\frac{5}{8}}+\frac{1}{5} = \qquad 5+\frac{5}{8}=\frac{5}{1}+\frac{5}{8}=\frac{40}{8}+\frac{5}{8}=\frac{45}{8}$$

$$\frac{\frac{3}{4}}{\frac{45}{8}}+\frac{1}{5} =$$ To divide is to multiply by the reciprocal.

$$\frac{\frac{3}{4}}{\frac{45}{8}}=\frac{3}{4}\cdot\frac{8}{45}=\frac{24}{180}=\frac{2}{15}$$

$$=\frac{2}{15}+\frac{1}{5}=\frac{2}{15}+\frac{3}{15}=\frac{5}{15}=\frac{1}{3}$$

(d) $\frac{2}{3}-\frac{3}{5}\left(-\frac{1}{3}\right)^2 = \frac{3}{5}$

Solution: We apply the order of operations agreement. We start with the exponentiation.

$$\begin{aligned}
\frac{2}{3}-\frac{3}{5}\left(-\frac{1}{3}\right)^2 &= \qquad \left(-\frac{1}{3}\right)^2=\frac{-1}{3}\cdot\frac{-1}{3}=\frac{1}{9}\\
\frac{2}{3}-\frac{3}{5}\cdot\frac{1}{9} &= \qquad \text{multiplication: } \frac{3}{5}\cdot\frac{1}{9}=\frac{3\cdot 1}{5\cdot 9}=\frac{3}{45}=\frac{1}{15}\\
\frac{2}{3}-\frac{1}{15} &= \qquad \text{common denominator is 15}\\
\frac{2\cdot 5}{3\cdot 5}-\frac{1}{15} &=\\
\frac{10}{15}-\frac{1}{15} &= \frac{9}{15}=\frac{\not{3}\cdot 3}{\not{3}\cdot 5}=\frac{3}{5}
\end{aligned}$$

(e) $\left||2-3^3|-4^2\right| = 9$

Solution: one trick here is to understand how the absolute value signs are paired. The first two can not be a pair, since there is nothin between them. Thus they must be the beginning of two different pairs. Then, as always, the first one to open is the last one to close.

$$\begin{aligned}
\left||2-3^3|-4^2\right| &= \qquad \text{exponent in innermost parentheses}\\
\left||2-27|-4^2\right| &= \qquad \text{subtraction in innermost parentheses}\\
\left||-25|-4^2\right| &= \qquad \text{the absolute value of } -25 \text{ is } 25\\
\left|25-4^2\right| &= \qquad \text{exponent}\\
|25-16| &= \qquad \text{subtraction}\\
|9| &= \qquad \text{the absolute value of 9 is 9}\\
&= 9
\end{aligned}$$

(f) $-|-5| = -5$

Solution: Two negatives do not always make a positive. This reads: the opposite of the absolute value of -5. Since the absolute value of -5 is 5, we have the opposite of 5, which is -5.

2. Simplify each of the following.

(a) $(2x^5)(x^4) = 2x^9$

Solution: Let us first recall the rules of exponents.

$$\begin{aligned}
&1. & a^n\cdot a^m &= a^{n+m}\\
&2. & \frac{a^n}{a^m} &= a^{n-m}\\
&3. & (a^n)^m &= a^{nm}\\
&4. & (ab)^n &= a^n b^n\\
&5. & \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n}
\end{aligned}$$

$$(2x^5)(x^4)=2x^5x^4=2x^9 \quad \text{by rule 1}$$

(b) $\left(2x^5\right)^4 = \ 16x^{20}$

Solution:

$$\begin{aligned}\left(2x^5\right)^4 &= 2^4\left(x^5\right)^4 && \text{by rule } 4\\ &= 16x^{20} && \text{by rule } 3\end{aligned}$$

(c) $(-xy)^2\left(-xy^2\right)^3 = \ -x^5y^8$

Solution:

$$(-xy)^2\left(-xy^2\right)^3 =$$

$$\begin{aligned}&= (-1xy)^2\left(-1xy^2\right)^3 && \text{the 1's will help with signs}\\ &= (-1)^2x^2y^2(-1)^3x^3\left(y^2\right)^3 && \text{by rule 4}\\ &= 1\cdot x^2y^2(-1)x^3y^6 && \text{by rule 3}\\ &= 1(-1)x^2x^3y^2y^6 && \text{since multiplication is commutative}\\ &= -1x^5y^8 && \text{by rule 1}\end{aligned}$$

(d) $\dfrac{(2ab)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2} = \ -2a^5$

Solution:

$$\frac{(2ab)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2} =$$

$$\begin{aligned}&= \frac{(2ab)^3\left(-3a^2b\right)^2}{-1b\left(6ab^2\right)^2} && \text{the 1 will help with signs}\\ &= \frac{2^3a^3b^3(-3)^2\left(a^2\right)^2b^2}{-1\cdot b\cdot 6^2\cdot a^2\left(b^2\right)^2} && \text{by rule 4}\\ &= \frac{8a^3b^3\cdot 9\cdot a^4b^2}{-1\cdot b\cdot 36\cdot a^2b^4} && \text{by rule 3}\\ &= \frac{8\cdot 9\cdot a^3a^4b^3b^2}{-1\cdot 36\cdot a^2\cdot b\cdot b^4} && \text{since multiplication is commutative}\\ &= \frac{72a^7b^5}{-36a^2b^5} && \text{by rule 1}\\ &= \frac{-2a^7b^5}{a^2b^5} && \text{simplify among numbers: } \frac{72}{-36}=\frac{-72}{36}=\frac{-2}{1}\\ &= \frac{-2a^7}{a^2} && \text{cancel out } b^5\\ &= \frac{-2a^5}{1} && \text{by rule 2}\\ &= -2a^5\end{aligned}$$

3. Evaluate $15 - \left|-x - x^2 + 5\right|$ if

   (a) $x = 0$ 10

   Solution: We first re-write the expression replacing the variables with parentheses. Then we copy the value of x into the parenthes. Then we work out the order of operations problem.

$$\begin{aligned} 15 - \left|-x - x^2 + 5\right| &= \\ 15 - \left|-(\ \ ) - (\ \ )^2 + 5\right| &= \\ 15 - \left|-(0) - (0)^2 + 5\right| &= && \text{exponent} \\ 15 - |0 - 0 + 5| &= && \text{subtraction} \\ 15 - |0 + 5| &= && \text{addition} \\ 15 - |5| &= && \text{absolute value} \\ 15 - 5 &= 10 \end{aligned}$$

   (b) $x = 2$ 14

   Solution: We first re-write the expression replacing the variables with parentheses. Then we copy the value of x into the parenthes. Then we work out the order of operations problem.

$$\begin{aligned} 15 - \left|-x - x^2 + 5\right| &= \\ 15 - \left|-(\ \ ) - (\ \ )^2 + 5\right| &= \\ 15 - \left|-(2) - (2)^2 + 5\right| &= && \text{exponent} \\ 15 - |-2 - 4 + 5| &= && \text{subtraction} \\ 15 - |-6 + 5| &= && \text{addition} \\ 15 - |-1| &= && \text{absolute value} \\ 15 - 1 &= 14 \end{aligned}$$

   (c) $x = -2$ 12

   Solution: We first re-write the expression replacing the variables with parentheses. Then we copy the value of x into the parenthes. Then we work out the order of operations problem.

$$\begin{aligned} 15 - \left|-x - x^2 + 5\right| &= \\ 15 - \left|-(\ \ ) - (\ \ )^2 + 5\right| &= \\ 15 - \left|-(-2) - (-2)^2 + 5\right| &= && \text{exponent} \\ 15 - |2 - 4 + 5| &= && \text{subtraction} \\ 15 - |-2 + 5| &= && \text{addition} \\ 15 - |3| &= && \text{absolute value} \\ 15 - 3 &= 12 \end{aligned}$$

(d) $x=\frac{1}{2}$ $\frac{43}{4}$

Solution: We first re-write the expression replacing the variables with parentheses. Then we copy the value of x into the parenthes. Then we work out the order of operations problem.

$$\begin{aligned}
15-\left|-x-x^2+5\right| &= \\
15-\left|-(\ \ )-(\ \ )^2+5\right| &= \\
15-\left|-\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^2+5\right| &= \qquad \text{exponent: } \left(\frac{1}{2}\right)^2=\frac{1}{2}\cdot\frac{1}{2}=\frac{1\cdot 1}{2\cdot 2}=\frac{1}{4} \\
15-\left|-\frac{1}{2}-\frac{1}{4}+5\right| &= \qquad \text{subtraction: } -\frac{1}{2}-\frac{1}{4}=\frac{-2}{4}-\frac{1}{4}=\frac{-1-2}{4}=\frac{-3}{4} \\
15-\left|-\frac{3}{4}+5\right| &=
\end{aligned}$$

We will now perform the addition $-\frac{3}{4}+5$:

$$\begin{aligned}
-\frac{3}{4}+5 &= \frac{-3}{4}+\frac{5}{1} \quad \text{the common denominator is 4} \\
&= \frac{-3}{4}+\frac{5\cdot 4}{1\cdot 4} \\
&= \frac{-3}{4}+\frac{20}{4}=\frac{-3+20}{4}=\frac{17}{4}
\end{aligned}$$

So now we have

$$\begin{aligned}
15-\left|\frac{17}{4}\right| &= \qquad \text{absolute value} \\
15-\frac{17}{4} &= \\
\frac{15}{1}-\frac{17}{4} &= \qquad \text{common denominator is } 4 \\
\frac{15\cdot 4}{1\cdot 4}-\frac{17}{4} &= \\
\frac{60}{4}-\frac{17}{4} &= \frac{60-17}{4}=\frac{43}{4}
\end{aligned}$$

4. Evaluate $\dfrac{3ab+2a^2-2b^2}{a+2b}$ if

(a) $a=2$ and $b=-3$ 7

Solution: We need to plug in $a=2$ and $b=-3$ into the expression given and then evaluate it by applying order of operations.

$$\begin{aligned}
\frac{3ab+2a^2-2b^2}{a+2b} &= \frac{3(2)(-3)+2(2)^2-2(-3)^2}{(2)+2(-3)} = \qquad \text{exponents} \\
&= \frac{3(2)(-3)+2\cdot 4-2\cdot 9}{(2)+2(-3)}
\end{aligned}$$

Now we perform all multiplications and divisions, left to right

$$\frac{3(2)(-3)+2\cdot 4-2\cdot 9}{(2)+2(-3)} = \frac{6(-3)+2\cdot 4-2\cdot 9}{(2)+2(-3)} = \frac{-18+2\cdot 4-2\cdot 9}{(2)+2(-3)} = $$
$$= \frac{-18+8-2\cdot 9}{(2)+2(-3)} = \frac{-18+8-18}{(2)+2(-3)} = \frac{-18+8-18}{(2)+(-6)}$$

Now we perform all additions and subtractions, left to right. Finally we divide.

$$\frac{-18+8-18}{(2)+(-6)} = \frac{-10-18}{(2)+(-6)} = \frac{-28}{(2)+(-6)} = \frac{-28}{-4} = 7$$

(b) $a=-1$ and $b=-2$ 0

Solution: We need to plug in $a=-1$ and $b=-2$ into the expression given and then evaluate it by applying order of operations. Since there is no parentheses, we start with exponents.

$$\frac{3ab+2a^2-2b^2}{a+2b} = \frac{3(-1)(-2)+2(-1)^2-2(-2)^2}{(-1)+2(-2)} = \frac{3(-1)(-2)+2\cdot 1-2\cdot 4}{(-1)+2(-2)}$$

Now we perform all multiplications and divisions, left to right (we individually show them)

$$\frac{3(-1)(-2)+2\cdot 1-2\cdot 4}{(-1)+2(-2)} = \frac{(-3)(-2)+2\cdot 1-2\cdot 4}{(-1)+2(-2)} = \frac{6+2\cdot 1-2\cdot 4}{(-1)+2(-2)} = $$
$$= \frac{6+2-2\cdot 4}{(-1)+2(-2)} = \frac{6+2-8}{(-1)+2(-2)} = \frac{6+2-8}{(-1)+(-4)}$$

Now we perform all additions and subtractions, left to right. Finally we divide.

$$\frac{6+2-8}{(-1)+(-4)} = \frac{8-8}{-1+(-4)} = \frac{0}{-1+(-4)} = \frac{0}{-5} = 0$$

(c) $a=-6$ and $b=3$. undefined

Solution: We need to plug in $a=-6$ and $b=3$ into the expression given and then evaluate it by applying order of operations. Since there is no parentheses, we start with exponents.

$$\frac{3ab+2a^2-2b^2}{a+2b} = \frac{3(-6)(3)+2(-6)^2-2(3)^2}{(-6)+2(3)} = \frac{3(-6)(3)+2\cdot 36-2\cdot 9}{(-6)+2(3)}$$

Now we perform all multiplications and divisions, left to right (we individually show them)

$$\frac{3(-6)(3)+2\cdot 36-2\cdot 9}{(-6)+2(3)} = \frac{(-18)(3)+2\cdot 36-2\cdot 9}{(-6)+2(3)} = \frac{-54+2\cdot 36-2\cdot 9}{(-6)+2(3)} = $$
$$= \frac{-54+72-2\cdot 9}{(-6)+2(3)} = \frac{-54+72-18}{(-6)+2(3)} = \frac{-54+72-18}{(-6)+6}$$

Now we perform all additions and subtractions, left to right. Finally we divide (IF WE CAN).

$$\frac{-54+72-18}{(-6)+6} = \frac{18-18}{(-6)+6} = \frac{0}{0} = \text{undefined}$$

since division by 0 is not allowed. The answer is: undefined

(d) $a = -\frac{1}{2}$ and $b = \frac{3}{4}$ $-\frac{7}{4}$

Solution: We need to plug in $a = -\frac{1}{2}$ and $b = \frac{3}{4}$ into the expression given and then evaluate it by applying order of operations.

$$\frac{3ab + 2a^2 - 2b^2}{a + 2b} = \frac{3\left(-\frac{1}{2}\right)\left(\frac{3}{4}\right) + 2\left(-\frac{1}{2}\right)^2 - 2\left(\frac{3}{4}\right)^2}{\left(-\frac{1}{2}\right) + 2\left(\frac{3}{4}\right)}$$

Since there is no parentheses, we start with exponents. We proceed left to right. Keep the negative signs in the numerator.

$$\frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right) + 2\left(\frac{-1}{2}\right)^2 - 2\left(\frac{3}{4}\right)^2}{\left(\frac{-1}{2}\right) + 2\left(\frac{3}{4}\right)} = \qquad \left(\frac{-1}{2}\right)^2 = \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right) = \frac{1}{4}$$

$$\frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right) - 2\left(\frac{3}{4}\right)^2}{\left(\frac{-1}{2}\right) + 2\left(\frac{3}{4}\right)} = \qquad \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}$$

$$\frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right) - 2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right) + 2\left(\frac{3}{4}\right)} =$$

Now we perform all multiplications and divisions, left to right (we individually show them)

$$\frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right) - 2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right) + 2\left(\frac{3}{4}\right)} = \qquad 3\left(-\frac{1}{2}\right) = \frac{3}{1} \cdot \frac{-1}{2} = \frac{-3}{2}$$

$$\frac{\frac{-3}{2}\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right) - 2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right) + 2\left(\frac{3}{4}\right)} =$$

$$\frac{\frac{-3}{2}\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right) - 2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right) + 2\left(\frac{3}{4}\right)} = \qquad \frac{-3}{2}\left(\frac{3}{4}\right) = \frac{-9}{8}$$

$$\frac{\frac{-9}{8} + 2\left(\frac{1}{4}\right) - 2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right) + 2\left(\frac{3}{4}\right)} = \qquad 2\left(\frac{1}{4}\right) = \frac{2}{1} \cdot \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{\frac{-9}{8}+\frac{1}{2}-2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad 2\left(\frac{9}{16}\right)=\frac{2}{1}\cdot\frac{9}{16}=\frac{18}{16}=\frac{9}{8}$$

$$\frac{\frac{-9}{8}+\frac{1}{2}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad 2\left(\frac{3}{4}\right)=\frac{2}{1}\cdot\frac{3}{4}=\frac{6}{4}=\frac{3}{2}$$

$$= \frac{\frac{-9}{8}+\frac{1}{2}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+\frac{3}{2}}$$

Now we perform all additions and subtractions, left to right.

$$\frac{\frac{-9}{8}+\frac{1}{2}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+\frac{3}{2}} = \qquad \frac{-9}{8}+\frac{1}{2}=\frac{-9}{8}+\frac{4}{8}=\frac{-9+4}{8}=\frac{-5}{8}$$

$$\frac{\frac{-5}{8}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+\frac{3}{2}} = \qquad \frac{-5}{8}-\frac{9}{8}=\frac{-5-9}{8}=\frac{-14}{8}=\frac{-7}{4}$$

$$\frac{\frac{-7}{4}}{\left(\frac{-1}{2}\right)+\frac{3}{2}} = \qquad \left(\frac{-1}{2}\right)+\frac{3}{2}=\frac{-1+3}{2}=\frac{2}{2}=1$$

$$= \frac{\left(\frac{-7}{4}\right)}{1} = -\frac{7}{4}$$

5. Consider the equation $-x^2+2x^3+3=-4x(x-2)$. For each of the numbers given, determine whether it is a solution of the equation or not.

(a) $x=-2$ $\quad -17 \neq -32 \implies$ no

Solution: We evaluate both sides with $x=-2$

$$\begin{aligned}
\text{LHS} &= -x^2+2x^3+3 = -(\ \ )^2+2(\ \ )^3+3 = -(-2)^2+2(-2)^3+3 \\
&= -4+2(-8)+3 = -4-16+3 = -20+3 = -17 \\
\text{RHS} &= -4x(x-2) = -4(\ \ )((\ \ )-2) = -4(-2)((-2)-2) \\
&= -4(-2)(-4) = 8(-4) = -32
\end{aligned}$$

Since $-17 \neq -32$, the number -2 is not a solution.

(b) $x = -\frac{1}{2}$ $\quad \frac{5}{2} \neq -5 \implies$ no

Solution: We evaluate both sides with $x = -\frac{1}{2}$

$$\begin{aligned}
\text{LHS} &= -x^2 + 2x^3 + 3 = -(\ )^2 + 2(\ )^3 + 3 \\
&= -\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)^3 + 3 \\
&= -\frac{1}{4} + 2\left(-\frac{1}{2}\right)^3 + 3 \\
&= -\frac{1}{4} + 2\left(-\frac{1}{8}\right) + 3 \\
&= -\frac{1}{4} + \left(-\frac{1}{4}\right) + 3 \\
&= -\frac{1}{2} + 3 \\
&= \frac{5}{2}
\end{aligned}$$

$$\left(\frac{-1}{2}\right)^2 = \frac{-1}{2} \cdot \frac{-1}{2} = \frac{1}{4}$$

$$\left(\frac{-1}{2}\right)^3 = \frac{-1}{2} \cdot \frac{-1}{2} \cdot \frac{-1}{2} = \frac{-1}{8}$$

$$2 \cdot \frac{-1}{8} = \frac{2}{1} \cdot \frac{-1}{8} = \frac{-2}{8} = \frac{-1}{4}$$

$$\frac{-1}{4} + \frac{-1}{4} = \frac{-2}{4} = \frac{-1}{2}$$

$$\frac{-1}{2} + \frac{3}{1} = \frac{-1}{2} + \frac{6}{2} = \frac{-1+6}{2} = \frac{5}{2}$$

$$\begin{aligned}
\text{RHS} &= -4x(x-2) = -4(\ )((\ ) - 2) \\
&= -4\left(-\frac{1}{2}\right)\left(\left(-\frac{1}{2}\right) - 2\right) \\
&= -4\left(\frac{-1}{2}\right)\left(\frac{-5}{2}\right) \\
&= 2\left(\frac{-5}{2}\right) = \frac{2}{1} \cdot \frac{-5}{2} = \frac{-10}{2} = -5
\end{aligned}$$

$$\frac{-1}{2} - 2 = \frac{-1}{2} - \frac{2}{1} = \frac{-1}{2} - \frac{4}{2} = \frac{-1-4}{2} = \frac{-5}{2}$$

$$-4\left(\frac{-1}{2}\right) = \frac{-4}{1} \cdot \frac{-1}{2} = \frac{4}{2} = 2$$

Since $\frac{5}{2} \neq -5$, the number $-\frac{1}{2}$ is not a solution.

(c) $x = \frac{1}{2}$ $\quad 3 = 3 \implies$ yes

Solution: We evaluate both sides with $x = \frac{1}{2}$

$$\begin{aligned}
\text{LHS} &= -x^2 + 2x^3 + 3 = -(\ )^2 + 2(\ )^3 + 3 \\
&= -\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^3 + 3 \\
&= -\frac{1}{4} + 2\left(\frac{1}{2}\right)^3 + 3 \\
&= -\frac{1}{4} + 2\left(\frac{1}{8}\right) + 3 \\
&= -\frac{1}{4} + \frac{1}{4} + 3 = 0 + 3 = 3
\end{aligned}$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$2 \cdot \frac{1}{8} = \frac{2}{1} \cdot \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\begin{aligned} \text{RHS} &= -4x(x-2) = -4(\ \ )((\ \ )-2) \\ &= -4\left(\frac{1}{2}\right)\left(\left(\frac{1}{2}\right)-2\right) \\ &= -4\left(\frac{1}{2}\right)\left(\frac{-3}{2}\right) \\ &= -2\left(\frac{-3}{2}\right) = \frac{-2}{1}\cdot\frac{-3}{2} = \frac{6}{2} = 3 \end{aligned}$$

$$\frac{1}{2}-2 = \frac{1}{2}-\frac{2}{1} = \frac{1}{2}-\frac{4}{2} = \frac{1-4}{2} = \frac{-3}{2}$$

$$-4\left(\frac{1}{2}\right) = \frac{-4}{1}\cdot\frac{1}{2} = -\frac{4}{2} = -2$$

Since $3 = 3$, the number $\frac{1}{2}$ is a solution.

(d) $x = -3$ $\quad -60 = -60 \implies$ yes

Solution: We evaluate both sides with $x = -3$

$$\begin{aligned} \text{LHS} &= -x^2+2x^3+3 = -(\ \ )^2+2(\ \ )^3+3 = -(-3)^2+2(-3)^3+3 \\ &= -9+2(-27)+3 = -9-54+3 = -63+3 = -60 \\ \text{RHS} &= -4x(x-2) = -4(\ \ )((\ \ )-2) = -4(-3)((-3)-2) \\ &= -4(-3)(-5) = 12(-5) = -60 \end{aligned}$$

Since $-60 = -60$, the number -3 is a solution.

6. Solve each of the following equations. Make sure to check your solutions.

(a) $\frac{2x-7}{3} = -1$ $\quad 2$

Solution: We apply all operations to both sides.

$$\begin{aligned} \frac{2x-7}{3} &= -1 && \text{multiply by } 3 \\ 2x-7 &= -3 && \text{add } 7 \\ 2x &= 4 && \text{divide by } 2 \\ x &= 2 \end{aligned}$$

We check:

$$\text{LHS} = \frac{2(2)-7}{3} = \frac{4-7}{3} = \frac{-3}{3} = -1 = \text{RHS}$$

Thus our solution, $x = 2$ is correct.

(b) $\frac{x+8}{3} = -2$ $\quad -14$

Solution: We apply all operations to both sides.

$$\begin{aligned} \frac{x+8}{3} &= -2 && \text{multiply by } 3 \\ x+8 &= -6 && \text{subtract } 8 \\ x &= -14 \end{aligned}$$

We check:

$$\text{LHS} = \frac{-14+8}{3} = \frac{-6}{3} = -2 = \text{RHS}$$

Thus our solution, $x = -14$ is correct.

(c) $\frac{x}{3}+8=-2$ -30

Solution: We apply all operations to both sides.

$$\begin{aligned} \frac{x}{3}+8 &= -2 && \text{subtract } 8 \\ \frac{x}{3} &= -10 && \text{multiply by } 3 \\ x &= -30 \end{aligned}$$

We check:

$$\text{LHS} = \frac{-30}{3}+8 = -10+8 = -2 = \text{RHS}$$

Thus our solution, $x=-30$ is correct.

(d) $\frac{1}{5}x-\frac{2}{3}=\frac{26}{15}$ 12

Solution:

$$\begin{aligned} \frac{1}{5}x-\frac{2}{3} &= \frac{26}{15} && \text{add } \frac{2}{3} \text{ to both sides} && \frac{26}{15}+\frac{2}{3}=\frac{26}{15}+\frac{2\cdot 5}{3\cdot 5}= \\ \frac{1}{5}x &= \frac{12}{5} && \text{divide by } \frac{1}{5} && \frac{26}{15}+\frac{10}{15}=\frac{36}{15}=\frac{\not{3}\cdot 12}{\not{3}\cdot 5}=\frac{12}{5} \\ x &= 12 && && \frac{\frac{12}{5}}{\frac{1}{5}}=\frac{12}{5}\cdot\frac{5}{1}=\frac{12\cdot\not{5}}{1\cdot\not{5}}=\frac{12}{1}=12 \end{aligned}$$

We check: if $x=12$, then

$$\begin{aligned} \text{LHS} &= \frac{1}{5}\cdot 12-\frac{2}{3}=\frac{1}{5}\cdot\frac{12}{1}-\frac{2}{3}=\frac{12}{5}-\frac{2}{3}=\frac{12\cdot 3}{5\cdot 3}-\frac{2\cdot 5}{3\cdot 5}=\frac{36}{15}-\frac{10}{15}=\frac{26}{15} \\ \text{RHS} &= \frac{26}{15} \end{aligned}$$

Thus our solution, $x=12$ is correct.

(e) $\frac{3}{8}x+\left(1\frac{4}{5}\right)=\frac{3}{10}$ -4

Solution: this is a very simple equation, much like $2x+1=7$, only the numbers are fractions. But the principles and operations regarding equations are the same.

$$\begin{aligned} \frac{3}{8}x+\left(1\frac{4}{5}\right) &= \frac{3}{10} && \text{convert mixed number to improper fraction} \\ \frac{3}{8}x+\frac{9}{5} &= \frac{3}{10} && \text{subtract } \frac{9}{5} \text{ from both sides; } \frac{3}{10}-\frac{9}{5}=\frac{3}{10}-\frac{18}{10}=\frac{3-18}{10}=\frac{-15}{10}=\frac{-3}{2} \\ \frac{3}{8}x &= \frac{-3}{2} && \text{divide both sides by } \frac{3}{8} \\ x &= -4 && \left(\frac{-3}{2}\right)\div\left(\frac{3}{8}\right)=\frac{-3}{2}\cdot\frac{8}{3}=\frac{-24}{6}=-4 \end{aligned}$$

We check:

$$\begin{aligned} \text{RHS} &= \frac{3}{8}(-4)+\left(1\frac{4}{5}\right)=\frac{3}{8}\cdot\frac{-4}{1}+\frac{9}{5}=\frac{-12}{8}+\frac{9}{5}=\frac{-3}{2}+\frac{9}{5}=\frac{-15}{10}+\frac{18}{10}=\frac{3}{10} \\ \text{LHS} &= \frac{3}{10} \end{aligned}$$

Thus our solution, -4 is correct.

7. Word Problems.

(a) A TV is priced at \$ 600. How much would it cost if it went on a 15% sale? \$ 510
Solution:
Method 1. The new price is

$$\$\ 600 - (15\% \text{ of } \$\ 600)$$

We now need to compute 15% of \$ 600. Since 15% is the same as $\frac{15}{100}$,

$$\begin{aligned} \frac{1}{100} &\text{ of } \$\ 600 \text{ is } \$\ 6 \\ \frac{15}{100} &\text{ of } \$\ 600 \text{ is } \$\ 90 \end{aligned}$$

Thus the sale price is $\$\ 600 - \$\ 90 = \$\ 510$.
Method 2. Since the new price is

$$\begin{aligned} \$\ 600 - (15\% \text{ of } \$\ 600) &= \\ (100\% \text{ of } \$\ 600) - (15\% \text{ of } \$\ 600) &= \\ (100\% - 15\%) \text{ of } \$\ 600 &= 85\% \text{ of } \$\ 600 \end{aligned}$$

We now need to compute 85% of \$ 600. Since 85% is the same as $\frac{85}{100}$,

$$\begin{aligned} \frac{1}{100} &\text{ of } \$\ 600 \text{ is } \$\ 6 \\ \frac{85}{100} &\text{ of } \$\ 600 \text{ is } \$\ 510 \end{aligned}$$

Thus the sale price is \$ 510.

(b) We have placed \$ 5000 in a bank account with an annual interest rate of 6%. How much money do we have in the account after one year? \$ 5300
Solution:
Method 1: We will have

$$\$\ 5000 + \text{interest} = \$\ 5000 + (6\% \text{ of } \$\ 5000)$$

We now need to compute 6% of \$ 5000. 6% is the same as $\frac{6}{100}$ of \$ 5000

$$\begin{aligned} \frac{1}{100} &\text{ of } \$\ 5000 \text{ is } \$\ 50 \\ \frac{6}{100} &\text{ of } \$\ 5000 \text{ is } \$\ 300 \end{aligned}$$

Thus we have $\$\ 5000 + \$\ 300 = \$\ 5300$.
Method 2. We will have the principal and the interest

$$\begin{aligned} \$\ 5000 + (6\% \text{ of } \$\ 5000) &= \\ (100\% \text{ of } \$\ 5000) + (6\% \text{ of } \$\ 5000) &= \\ (100\% + 6\%) \text{ of } \$\ 5000 &= 106\% \text{ of } \$\ 5000 \end{aligned}$$

We now need to compute 106% of \$ 5000. Since 106% is the same as $\frac{106}{100}$,

$$\frac{1}{100} \text{ of } \$\,5000 \text{ is } \$\,50$$
$$\frac{106}{100} \text{ of } \$\,5000 \text{ is } \$\,5300$$

Thus we have \$ 5300.

(c) Ann took four exams. Her scores on the first three exams were 63, 76, and 68. How many points did she earn on the fourth exam if her average is 71? 77

Solution: Let x denote the score of Ann's fourth exam. The equation will express the average.

$$\begin{aligned} \frac{63+76+68+x}{4} &= 71 && \text{simplify left-hand side by adding the three scores} \\ \frac{x+207}{4} &= 71 && \text{multiply by 4} \\ x+207 &= 284 && \text{subtract 207} \\ x &= 77 \end{aligned}$$

We check: the average of the four exams is

$$\text{Average} = \frac{63+76+68+77}{4} = 71$$

Thus our solution, 77 is correct.

(d) If we multiply a number by -2 and add 7, the result is 25. Find this number. -9

Solution: Let x denote the number. Then the problem translate to

$$\begin{aligned} -2\cdot x+7 &= 25 && \text{solve for } x \\ -2x+7 &= 25 && \text{subtract 7} \\ -2x &= 18 && \text{divide by } -2 \\ x &= -9 \end{aligned}$$

Thus the number is 9. We check: if we multiply our number, -9 by -2, we get 18. Then we add 7, we indeed get 25.

(e) If we subtract 5 from the opposite of a number, we get -1. Find this number. -4

Solution: Let x denote the number. Then the problem translate to

$$\begin{aligned} -x-5 &= -1 && \text{solve for } x, \text{ add 5} \\ -x &= 4 && \text{multiply by } -1 \\ x &= -4 \end{aligned}$$

Thus the number is -4. We check: if we subtract 5 from the opposite of -4, we get $-(-4)-5 = 4-5 = -1$. Thus our solution is correct.

(f) Three times a number is 5 more than 16. Find this number. 7

Solution: Let x denote the number. Since 21 is the number that is 5 more than 16, we have

$$\begin{aligned} 3x &= 21 && \text{solve for } x, \text{ divide by 3} \\ x &= 7 \end{aligned}$$

Thus the number is 7. We check: three times 7 is 21 which is indeed 5 more than 16. Thus our solution is correct.

(g) The product of 3 and the opposite of a number is -63. Find this number. 21

Solution: Let x denote the number. The problem translates to

$$\begin{aligned} 3(-x) &= -63 && \text{solve for } x \\ -3x &= -63 && \text{divide by } -3 \\ x &= 21 \end{aligned}$$

Thus the number is 21. We check: three times the opposite of 21 is $3(-21) = -63$. Thus our solution is correct.

8. Consider the equations $2x - y = 2$ and $y = -x + 7$.

(a) Graph these lines in the same coordinate system. Use your graph to find the coordinates where the points intersect.

We first graph the line $2x - y = 2$.

$$\begin{aligned} \text{If } x &= 0,\ y = ? && \text{substitute } x = 0 \text{ into the equation of the line} \\ 2(0) - y &= 2 && \text{solve for } y \\ 0 - y &= 2 \\ -y &= 2 && \text{multiply by } -1 \\ y &= -2 && \Longrightarrow \text{ we found } (0, -2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 1,\ y = ? && \text{substitute } x = 1 \text{ into the equation of the line} \\ 2(1) - y &= 2 && \text{solve for } y \\ 2 - y &= 2 && \text{subtract } 2 \\ -y &= 0 && \text{multiply by } -1 \\ y &= 0 && \Longrightarrow \text{ we found } (1, 0) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 2,\ y = ? && \text{substitute } x = 2 \text{ into the equation of the line} \\ 2(2) - y &= 2 && \text{solve for } y \\ 4 - y &= 2 && \text{subtract } 4 \\ -y &= -2 && \text{multiply by } -1 \\ y &= 2 && \Longrightarrow \text{ we found } (2, 2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 3,\ y = ? && \text{substitute } x = 3 \text{ into the equation of the line} \\ 2(3) - y &= 2 && \text{solve for } y \\ 6 - y &= 2 && \text{subtract } 6 \\ -y &= -4 && \text{multiply by } -1 \\ y &= 4 && \Longrightarrow \text{ we found } (3, 4) \end{aligned}$$

We graph the points $(0, -2)$, $(1, 0)$, $(2, 2)$, $(3, 4)$, and connect the points. (Green line.)

We now graph the line $y = -x + 7$.

$$\begin{aligned} \text{If } x &= 0,\ y = ? \quad \text{substitute } x = 0 \text{ into the equation of the line} \\ y &= -(0) + 7 = 0 + 7 = 7 \quad \Longrightarrow \quad \text{we found } (0, 7) \end{aligned}$$

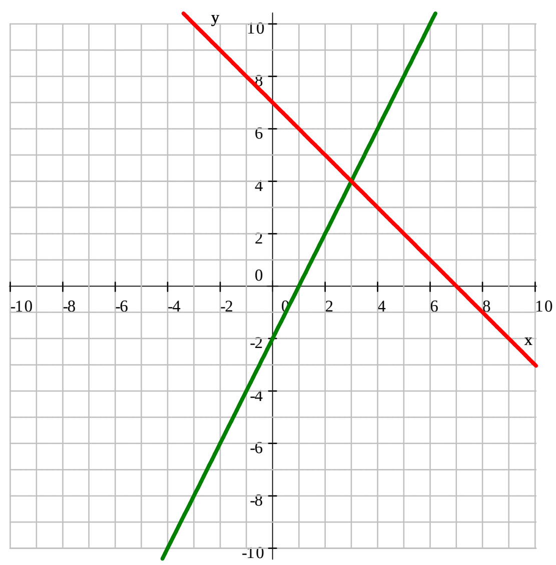
$$\begin{aligned} \text{If } x &= 2,\ y = ? \quad \text{substitute } x = 2 \text{ into the equation of the line} \\ y &= -(2) + 7 = 5 \quad \Longrightarrow \quad \text{we found } (2, 5) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 4,\ y = ? \quad \text{substitute } x = 4 \text{ into the equation of the line} \\ y &= -(4) + 7 = 3 \quad \Longrightarrow \quad \text{we found } (4, 3) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 6,\ y = ? \quad \text{substitute } x = 6 \text{ into the equation of the line} \\ y &= -(6) + 7 = 1 \quad \Longrightarrow \quad \text{we found } (6, 1) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 7,\ y = ? \quad \text{substitute } x = 7 \text{ into the equation of the line} \\ y &= -(7) + 7 = 0 \quad \Longrightarrow \quad \text{we found } (7, 0) \end{aligned}$$

We graph the points $(0, 7)$, $(2, 5)$, $(4, 3)$, $(6, 1)$, and $(7, 0)$ and connect the points. (Red line.)



We read from the graph that the lines intersect at $(3, 4)$.

(b) Use algebraic methods to check your answer for part a).

Solution: Is the point $(3, 4)$ on the line $2x - y = 2$?

$$\text{LHS} = 2x - y = 2(3) - (4) = 6 - 4 = 2 = \text{RHS} \quad \Longrightarrow \quad \text{yes}$$

Is the point $(3, 4)$ on the line $y = -x + 7$?

$$\begin{aligned} \text{LHS} &= y = 4 \\ \text{RHS} - x + 7 &= -3 + 7 = 4 = \text{LHS} \quad \Longrightarrow \quad \text{yes} \end{aligned}$$