1. Perform the operations as indicated.

(a) $\frac{3}{4}\cdot 6-5\cdot\frac{5}{2}=-8$
Solution: We apply order of operations.

$$\begin{aligned}\frac{3}{4}\cdot 6-5\cdot\frac{5}{2} &= \\ \frac{9}{2}-5\cdot\frac{5}{2} &= \\ \frac{9}{2}-\frac{25}{2} &= \\ \frac{9-25}{2} &= \\ \frac{-16}{2} &= -8\end{aligned}$$

$$\frac{3}{4}\cdot 6=\frac{3}{4}\cdot\frac{6}{1}=\frac{18}{4}=\frac{9}{2}$$
$$5\cdot\frac{5}{2}=\frac{5}{1}\cdot\frac{5}{2}=\frac{25}{2}$$

(b) $\dfrac{\frac{5}{6}-\frac{5}{4}}{\frac{2}{3}\cdot\frac{5}{8}}= -1$
Solution: We apply order of operations. First we perform the subtraction in the numerator. The common denominator is 12.

$$\frac{5}{6}-\frac{5}{4}=\frac{10}{12}-\frac{15}{12}=\frac{10-15}{12}=\frac{-5}{12}=-\frac{5}{12}$$

Multiplication in the denominator: we simply multiply the numerator by the numerator, the denominator by the denominator, and then simplify.

$$\frac{2}{3}\cdot\frac{5}{8}=\frac{2\cdot 5}{3\cdot 8}=\frac{10}{24}=\frac{5}{12}$$

Thus we have so far

$$\frac{\frac{5}{6}-\frac{5}{4}}{\frac{2}{3}\cdot\frac{5}{8}}=\frac{\frac{-5}{12}}{\frac{5}{12}}$$

To divide is to multiply by the reciprocal

$$\frac{\frac{5}{6}-\frac{5}{4}}{\frac{2}{3}\cdot\frac{5}{8}}=\frac{\frac{-5}{12}}{\frac{5}{12}}=\frac{-5}{12}\cdot\frac{12}{5}=\frac{-60}{60}=-1$$

2. Simplify each of the following.

(a) $-2a^3\left(-2a^4\right)^2=-8a^{11}$
Solution:

$$\begin{aligned}-2a^3\left(-2a^4\right)^2 &= && \text{use rule } (ab)^n=a^nb^n \\ -2a^3(-2)^2\left(a^4\right)^2 &= && \text{use rule } (a^n)^m=a^{nm} \\ -2a^3(4)\,a^8 &= && \text{use rule } a^n\cdot a^m=a^{n+m} \\ &= -8a^{11}\end{aligned}$$

(b) $2a^3\left(-2ab^2\right)^3 ab^2 = \quad -16a^7b^8$
Solution:

$$\begin{aligned}
2a^3\left(-2ab^2\right)^3 ab^2 &= && \text{use rule } (ab)^n = a^nb^n\\
2a^3(-2)^3a^3\left(b^2\right)^3ab^2 &= && \text{use rule } (a^n)^m = a^{nm}\\
2a^3(-8)a^3b^6ab^2 &= && \text{use that multiplication is commutative}\\
2(-8)a^3a^3ab^6b^2 &= && \text{use rule } a^n\cdot a^m = a^{n+m}\\
&= -16a^7b^8
\end{aligned}$$

(c) $\dfrac{(-2x)^2y^3}{2x^3y^2} = \quad \dfrac{2y}{x}$
Solution:

$$\begin{aligned}
\frac{(-2x)^2y^3}{2x^3y^2} &= && \text{use rule } (ab)^n = a^nb^n\\
\frac{(-2)^2x^2y^3}{2x^3y^2} &= \\
\frac{4x^2y^3}{2x^3y^2} &= \frac{2x^2y^3}{x^3y^2} && \text{simplify by canceling out } x^2\\
\frac{2y^3}{xy^2} &= && \text{simplify by canceling out } y^2\\
&= \frac{2y}{x}
\end{aligned}$$

(d) $3(x-2)-2(5x-2) = \quad -7x-2$
Solution: We first apply the distibutive law and then combine like terms

$$\begin{aligned}
3(x-2)-2(5x-2) &= && \text{distribute}\\
3x-6-10x+4 &= && \text{combine like terms}\\
&= -7x-2
\end{aligned}$$

(e) $(x+3)(5x-3) = \quad 5x^2+12x-9$
Solution: We multiply two-term polynomials by two-term polynomials by the 'FOIL' method. F stand for first with first, O for outer terms, I for inner terms, and L for last terms.

$$\begin{aligned}
(x+3)(5x-3) &= \underbrace{5x^2}_{\text{F}}\ \underbrace{-3x}_{\text{O}}\ \underbrace{+15x}_{\text{I}}\ \underbrace{-9}_{\text{L}} && \text{combine like terms}\\
&= 5x^2+12x-9
\end{aligned}$$

(f) $(5a-1)^2 = \quad 25a^2-10a+1$
Solution: to square something means to write it down twice and multiply. Then it is a FOIL problem. F stand for first with first, O for outer terms, I for inner terms, and L for last terms.

$$\begin{aligned}
(5a-1)^2 &= (5a-1)(5a-1)\\
&= \underbrace{25a^2}_{\text{F}}\ \underbrace{-5a}_{\text{O}}\ \underbrace{-5a}_{\text{I}}\ \underbrace{+1}_{\text{L}} && \text{combine like terms}\\
&= 25a^2-10a+1
\end{aligned}$$

(g) $\left(3x^5+4y\right)\left(3x^5-4y\right)=$ $9x^{10}-16y^2$

$$\begin{aligned}\left(3x^5+4y\right)\left(3x^5-4y\right) &= \underbrace{9x^{10}}_{\text{F}}\ \underbrace{-12x^5}_{\text{O}}\ \underbrace{+12x^5}_{\text{I}}\ \underbrace{-16y^2}_{\text{L}} \qquad \text{combine like terms}\\ &= 9x^{10}-16y^2\end{aligned}$$

The expressions $3x^5+4y$ and $3x^5-4y$ are called conjugates. Becuase of the same terms and alternating signs, O and I cacel out when "FOIL"-ing, giving us the difference of two squares.

(h) $(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right)=$ x^6-y^6
Solution: We apply the law of distributivity

$$(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right)=$$

$$\begin{aligned} &= x\cdot x^5+x\cdot x^4y+x\cdot x^3y^2+x\cdot x^2y^3+x\cdot xy^4+x\cdot y^5\\ &\qquad -y\cdot x^5-y\cdot x^4y-y\cdot x^3y^2-y\cdot x^2y^3-y\cdot xy^4-y\cdot y^5\\ &= x^6+x^5y+x^4y^2+x^3y^3+x^2y^4+xy^5\\ &\qquad -x^5y-x^4y^2-x^3y^3-x^2y^4-xy^5-y^6\\ &= x^6-y^6\end{aligned}$$

3. Solve each of the following equations. Make sure to check your solutions.

(a) $\left(\frac{3}{8}\right)x+\frac{3}{2}=-\frac{3}{4}$ -6
Solution: this is a very simple equation, much like $2x+1=7$, only the numbers are fractions. But the principles and operations regarding equations are the same.

$$\begin{aligned}\left(\frac{3}{8}\right)x+\frac{3}{2} &= -\frac{3}{4} && \text{subtract } \frac{3}{2} \text{ from both sides; } -\frac{3}{4}-\frac{3}{2}=\frac{-3}{4}-\frac{6}{4}=\frac{-3-6}{4}=\frac{-9}{4}\\ \frac{3}{8}x &= \frac{-9}{4} && \text{divide both sides by } \frac{3}{8}\\ x &= -6 && \left(\frac{-9}{4}\right)\div\left(\frac{3}{8}\right)=\frac{-9}{4}\cdot\frac{8}{3}=\frac{-72}{12}=-6\end{aligned}$$

We check:

$$\text{RHS}=\frac{3}{8}(-6)+\frac{3}{2}=\frac{3}{8}\cdot\frac{-6}{1}+\frac{3}{2}=\frac{-18}{8}+\frac{3}{2}=\frac{-9}{4}+\frac{6}{4}=\frac{-9+6}{4}=-\frac{3}{4}=\text{LHS}$$

Thus our solution, -6 is correct.

(b) $3(2x-3)-(5x+4)=-14$ -1
Solution:

$$\begin{aligned}3(2x-3)-(5x+4) &= -14 && \text{distribute}\\ 6x-9-5x-4 &= -14 && \text{combine like terms}\\ x-13 &= -14 && \text{add 13}\\ x &= -1\end{aligned}$$

We check: if $x=-1$, then

$$\begin{aligned}\text{LHS} &= 3(2x-3)-(5x+4)=3(2(-1)-3)-(5(-1)+4)\\ &= 3(-2-3)-(-5+4)=3(-5)-(-1)=-15+1=-14=\text{RHS}\end{aligned}$$

(c) $3w-5=5(w-2)$ $\frac{5}{2}$
Solution: we first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 3w-5 &= 5(w-2) \\ 3w-5 &= 5w-10 && \text{subtract } 3w \text{ from both sides} \\ -5 &= 2w-10 && \text{add } 10 \text{ to both sides} \\ 5 &= 2w && \text{divide both sides by } 2 \\ \frac{5}{2} &= w \end{aligned}$$

We check: if $w=\frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 3w-5 = 3\left(\frac{5}{2}\right)-5 = \frac{3}{1}\cdot\frac{5}{2}-5 = \frac{15}{2}-5 = \frac{15}{2}-\frac{10}{2} = \frac{5}{2} \\ \text{RHS} &= 5(w-2) = 5\left(\frac{5}{2}-2\right) = 5\left(\frac{5}{2}-\frac{4}{2}\right) = 5\left(\frac{1}{2}\right) = \frac{5}{1}\cdot\frac{1}{2} = \frac{5}{2} \end{aligned}$$

Thus our solution, $w=\frac{5}{2}$ is correct

(d) $8(x-3)-3(5-2x)=x$ 3
Solution:

$$\begin{aligned} 8(x-3)-3(5-2x) &= x && \text{multiply out parentheses} \\ 8x-24-15+6x &= x && \text{combine like terms} \\ 14x-39 &= x && \text{subtract } x \\ 13x-39 &= 0 && \text{add } 39 \\ 13x &= 39 && \text{divide by } 13 \\ x &= 3 \end{aligned}$$

We check our solution by evaluating the left hand side and the right hand side of the original equation with $x=3$.

$$\begin{aligned} \text{RHS} &= 8(3-3)-3(5-2(3)) = 8\cdot 0-3(5-6) = 8\cdot 0-3(-1) = \\ &= 0-(-3) = 3 \\ \text{LHS} &= 3 \end{aligned}$$

Since the left-hand side equals to the right-hand side, our solution $x=3$ is correct.

(e) $7(j-5)+8=2(j+5)+5j$ no solution
Solution:

$$\begin{aligned} 7(j-5)+8 &= 2(j+5)+5j && \text{distribute on both sides} \\ 7j-35+8 &= 2j+10+5j && \text{combine like terms} \\ 7j-27 &= 7j+10 && \text{subtract } 7j \\ -27 &= 10 \end{aligned}$$

There is no value of j that could make the statement $-27=10$ true. Thus there is no solution of this equation. An equation of this type is called a contradiction.

(f) $-3(2x-5)-(3-7x)=2(x+1)-(x-10)$ all numbers are solution
Solution: In case of subtracting algebraic expressions, we either subtract the opposite, or distribute -1. We will use the second method here.

$$\begin{aligned} -3(2x-5)-(3-7x) &= 2(x+1)-(x-10) & \\ -3(2x-5)-1(3-7x) &= 2(x+1)-1(x-10) & \text{distribute} \\ -6x+15-3+7x &= 2x+2-x+10 & \text{combine like terms} \\ x+12 &= x+12 & \text{subtract } x \\ 12 &= 12 & \end{aligned}$$

Since x disappeared from the equation, we are left with an unconditional statement, this time unconditionally true. All values of x will make $12=12$ be true, and thus all numbers are solution of this equation. . An equation of this type is called an identity.

(g) $\dfrac{3x-1}{4}+\dfrac{8-4x}{3}=-3-x$ -13
Solution:

$$\begin{aligned} \frac{3x-1}{4}+\frac{8-4x}{3} &= -3-x & \text{we express the left hand side as a fraction} \\ \frac{3x-1}{4}+\frac{8-4x}{3} &= \frac{-3-x}{1} & \text{express fractions with common denominator} \\ \frac{3(3x-1)}{12}+\frac{4(8-4x)}{12} &= \frac{12(-3-x)}{12} & \text{multiply both sides by 12} \\ 3(3x-1)+4(8-4x) &= 12(-3-x) & \text{mulitply out parentheses (distribute)} \\ 9x-3+32-16x &= -36-12x & \text{combine like terms} \\ -7x+29 &= -12x-36 & \text{add } 12x \text{ to both sides} \\ 5x+29 &= -36 & \text{subtract 29 from both sides} \\ 5x &= -65 & \text{divide by 5} \\ x &= -13 & \end{aligned}$$

We check our solution by evaluating the left hand side and the right hand side of the original equation with $x=-13$.

$$\begin{aligned} \text{RHS} &= \frac{3(-13)-1}{4}+\frac{8-4(-13)}{3}=\frac{-39-1}{4}+\frac{8-(-52)}{3}=\frac{-40}{4}+\frac{8+52}{3}= \\ &= \frac{-40}{4}+\frac{60}{3}=-10+20=10 \\ \text{LHS} &= -3-(-13)=-3+13=10 \end{aligned}$$

Since the left-hand side equals to the right-hand side, our solution $x=-13$ is correct.

(h) $\dfrac{3x-2}{5}+\dfrac{x+4}{3}=\dfrac{14(x+1)}{15}$ all numbers are solution
Solution:

$$\begin{aligned} \frac{3x-2}{5}+\frac{x+4}{3} &= \frac{14(x+1)}{15} & \text{express fractions with common denominator} \\ \frac{3(3x-2)}{15}+\frac{5(x+4)}{15} &= \frac{14(x+1)}{15} & \text{multiply both sides by 15} \\ 3(3x-2)+5(x+4) &= 14(x+1) & \text{mulitply out parentheses (distribute)} \\ 9x-6+5x+20 &= 14x+14 & \text{combine like terms} \\ 14x+14 &= 14x+14 & \end{aligned}$$

Since the left hand side and the right hand side are identical, every number will work if substituted. Thus this is an identity, all numbers are solution(s).

4. Intercepts.

   (a) Find the x-intercept of the line given by $5x - y = -10$. $(-2, 0)$
   Solution: We substitute $y = 0$ into the equation of the line and solve for x.

$$\begin{aligned} \text{If } y &= 0, \ x = ? \\ 5x - 0 &= -10 \\ 5x &= -10 \qquad \text{divide both sides by 5} \\ x &= -2 \qquad \Longrightarrow \quad \text{we found the point} \quad (-2, 0) \end{aligned}$$

   Thus the $y-$intercept is $(-2, 0)$.

   (b) What is the y-intercept of the line with equation $3x + 2y = 30$? $(0, 15)$
   Solution: The $y-$intercept of a graph is the point where it intercects the $y-$axis. We can find this point by substituting $x = 0$ into the equation of the line and solve for y.

$$\begin{aligned} \text{If } x &= 0, \ y = ? \\ 3(0) + 2y &= 30 \\ 0 + 2y &= 30 \\ 2y &= 30 \qquad \text{divide both sides by 2} \\ y &= 15 \qquad \Longrightarrow \quad \text{we found the point} \quad (0, 15) \end{aligned}$$

   Thus the $y-$intercept is $(0, 15)$.

5. Graph the lines $4x - 5y = 10$ and $y = 5 - \dfrac{x+1}{2}$ in the same coordinate system. Use your graph to find the coordinates of the point where the lines intersect. $(5, 2)$

   Solution: we find points by picking a value for x, and then substitute that value into the equation of the line and solve for y. Finally, when we have a few points, we graph them and connect the points.

$$\begin{aligned} \text{If } x &= 0, \ y = ? \\ 4(0) - 5y &= 10 \\ 0 - 5y &= 10 \\ -5y &= 10 \qquad \text{divide both sides by } -5 \\ y &= -2 \qquad \Longrightarrow \quad \text{we found the point} \quad (0, -2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 5, \ y = ? \\ 4(5) - 5y &= 10 \\ 20 - 5y &= 10 \qquad \text{subtract 20 from both sides} \\ -5y &= -10 \qquad \text{divide both sides by } -5 \\ y &= 2 \qquad \Longrightarrow \quad \text{we found the point} \quad (5, 2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -5, \; y = ? \\ 4(-5) - 5y &= 10 \\ -20 - 5y &= 10 \quad \text{add } 20 \text{ to both sides} \\ -5y &= 30 \quad \text{divide both sides by } -5 \\ y &= -6 \implies \text{we found the point } (-5,-6) \end{aligned}$$

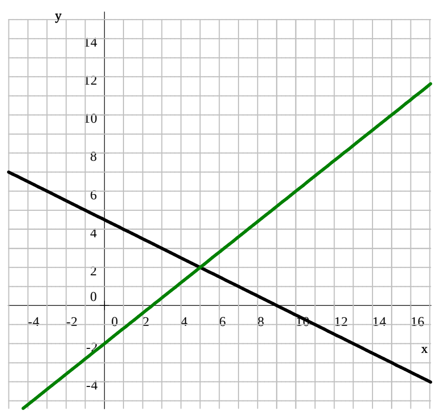
We now graph the other line, $y = 5 - \dfrac{x+1}{2}$. We find points by picking a value for x, and then substitute that value into the equation of the line and solve for y. Finally, once we have a few points, we graph and connect them.

$$\begin{aligned} \text{If } x &= 1, \; y = ? \\ y &= 5 - \frac{1+1}{2} \\ y &= 5 - \frac{2}{2} \\ y &= 5 - 1 \\ y &= 4 \implies \text{we found the point } (1,4) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 3, \; y = ? \\ y &= 5 - \frac{3+1}{2} \\ y &= 5 - \frac{4}{2} \\ y &= 5 - 2 \\ y &= 3 \implies \text{we found the point } (3,3) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 5, \; y = ? \\ y &= 5 - \frac{5+1}{2} \\ y &= 5 - \frac{6}{2} \\ y &= 5 - 3 \\ y &= 2 \implies \text{we found the point } (5,2) \end{aligned}$$

We are now ready to graph. Clearly, the lines intersect at the point $(5, 2)$.



6. Word Problems.

(a) A rectangle has a width which is seven inches less than its length. The perimeter is 46 inches. Find the sides. 8 in and 15 in

Solution: Let us call the shorter side (the width) by x. Then the longer side, the length must be $x+7$. We obtain the equation by expressing the perimeter.

$$\begin{aligned} 2x+2(x+7) &= 46 && \text{distribute} \\ 2x+2x+14 &= 46 && \text{combine like terms} \\ 4x+14 &= 46 && \text{subtract 14 from both sides} \\ 4x &= 32 && \text{divide both sides by 4} \\ x &= 8 \end{aligned}$$

We denoted the width by x, thus the width is 8 in long. The length is then $x+7=8+7=15$ in long.

(b) A couch went on a 15% sale. The sale price is \$ 697. Find the original price. \$ 820

Solution: A 15% sale means that we have to pay 85% of the original price. The question is: 85% of what number is 697?

(is) $= 697$

$F = \dfrac{85}{100}$

(of) $= x$

We substitute these values into the formula (is) $= F \cdot$ (of) and solve for x.

$$\begin{aligned} \text{(is)} &= F\cdot\text{(of)} \\ 697 &= \frac{85}{100}\cdot x && \text{divide bith sides by } \frac{85}{100} \\ 697 \div \frac{85}{100} &= x \\ x &= \frac{697}{1}\cdot\frac{100}{85} = 820 \end{aligned}$$

Thus the original price is \$ 820.

(c) The difference between two numbers is 7, their sum is 37. Find these numbers. 15 and 22

Solution: Let us call the smaller number x. Then the larger number is $x+7$, since the difference between the two numbers is 7. The equation then expresses the sum of these numbers

$$\begin{aligned} \underbrace{x}_{\text{smaller number}} + \underbrace{x+7}_{\text{larger number}} &= 37 && \text{solve for } x \\ 2x+7 &= 37 && \text{subtract 7} \\ 2x &= 30 && \text{divide by 2} \\ x &= 15 \end{aligned}$$

Thus the smaller number, labeled x is 15. The larger number was labeled $x+7$, so it must be $15+7=22$. Thus the numbers are 15 and 22. We check: the difference between 22 and 15 is $22-(15)=7$, and their sum is indeed $22+15=37$. Thus our solution is correct.

(d) A certain triangle's longest side is one centimeter less than six times the shortest side. The other side is five times the shortest side. The perimeter is thirty-five centimeters. Find the length of the longest side. 17 cm

Solution: Let x denote the shortest side. Then the longest side is $6x-1$, and the other side is $5x$. We obtain the equation by expressing the perimeter of the triangle. Then we solve for x.

$$\begin{aligned}
\underbrace{x}_{\text{shortest side}} + \underbrace{6x-1}_{\text{longest side}} + \underbrace{5x}_{\text{other side}} &= 35 && \text{combine like terms} \\
12x - 1 &= 35 && \text{add } 1 \text{ to both sides} \\
12x &= 36 && \text{divide both sides by } 12 \\
x &= 3
\end{aligned}$$

Now we know that $x=3$. Since the longest side was denoted by $6x-1$, it must be $6(3)-1 = 18-1 = 17$ cm long.

(e) Ann and Betty are roommates. The monthly rent is \$ 950. The amount paid by Ann is \$ 310 less than twice the amount paid by Betty. How much do they each pay for rent? \$ 420 and \$ 530

Solution: Let x denote the amount that is paid by Betty. Then Ann must pay $2x-310$ per month. The equation expresses the monthly rent as the sum of the two payments

$$\begin{aligned}
\underbrace{x}_{\text{smaller amount}} + \underbrace{2x-310}_{\text{larger amount}} &= 950 && \text{solve for } x \\
3x - 310 &= 950 && \text{add } 310 \\
2x &= 1260 && \text{divide by } 3 \\
x &= 420
\end{aligned}$$

Thus the amount paid by Betty is \$ 420. The amount paid by Betty was labeled as $2x-310$, so Betty must pay $2(420)-310 = 840-310 = 530$. Thus the payments are \$ 420 and \$ 530. We check: $\$\,530 = 2(\$\,420) - \$\,310$ and $\$\,420 + \$\,530 = \$\,950$. Thus our solution is correct.

(f) The population of a town has decreased from 90 000 to 82 800. What percent of a change does this represent? 8% decrease

Solution: The decrease in the population is $90\,000 - 82\,800 = 7200$. The question is then:

7200is what percent of 90 000?

(is) $= 7200$
$F = x$
(of) $= 90\,000$

We substitute these values into the formula (is) $= F \cdot$ (of) and solve for x.

$$\begin{aligned}
(\text{is}) &= F \cdot (\text{of}) \\
7200 &= x \cdot 90\,000 && \text{divide bith sides by } 90\,000 \\
\frac{7200}{90\,000} &= x \\
x &= \frac{8}{100} = 8\%
\end{aligned}$$

Thus the change in population is 8%.

(g) A bank teller has 47 more five-dollar bills than ten-dollar bills. The total value of the money is \$1000. How much of each denomination of bill does he have? 51 ten-dollar bills and 98 five-dollar bills

Solution: Let us denote the number of ten-dollar bills by x. Then we have $x+47$ many five-dollar bills. The equation expresses the value of the bills.

$$\begin{aligned}
\underbrace{10x}_{\text{amount in 10-bills}} + \underbrace{5(x+47)}_{\text{amount in 5-bills}} &= 1000 && \text{distribute} \\
10x + 5x + 235 &= 1000 && \text{combine like terms} \\
15x + 235 &= 1000 && \text{subtract } 235 \\
15x &= 765 && \text{divide by } 15 \\
x &= 51
\end{aligned}$$

Thus we have 51 tens and $51 + 47 = 98$ fives. We check: $98 - 51 = 47$ and $51(10) + 98(5) = 1000$. Thus our solution; 51 ten-dollar bills and 98 five-dollar bills; is correct.

(h) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft. 9 ft and 23 ft

Solution: Let us denote the shorter side by x. Then the other side is $3x - 4$. The equation expresses the perimeter of the rectangle.

$$\begin{aligned}
2(x) + 2(3x-4) &= 64 && \text{multiply out parentheses} \\
2x + 6x - 8 &= 64 && \text{combine like terms} \\
8x - 8 &= 64 && \text{add} \\
8x &= 72 && \text{divide by } 8 \\
x &= 9
\end{aligned}$$

If the shorter side was denoted by x, we now know it is 9 ft. The longer side was denoted by $3x-4$, so it must be $3(9 \text{ ft}) - 4 \text{ ft} = 23$ ft. Thus the sides of the rectangle are 9 ft and 23 ft. We check: $P = 2(9 \text{ ft}) + 2(23 \text{ ft}) = 64$ ft and $23 \text{ ft} = 3(9 \text{ ft}) - 4$ ft. Thus our solution is correct.

(i) Mary bought four less than three times the number of books that Jose did. Together they bought sixteen books. How many did Jose buy? 5

Solution: Let us denote the number of books bought by Jose by x. Then Mary baught $3x - 4$ many books. We obtain the equation by expressing the total number of books. Then we solve for x.

$$\begin{aligned}
x + 3x - 4 &= 16 && \text{combine like terms} \\
4x - 4 &= 16 && \text{add 4 to both sides} \\
4x &= 20 && \text{divide both sides by 4} \\
x &= 5
\end{aligned}$$

Thus Jose bought 5 books.