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1. Simplify each of the following.

(a) $(5a - 1)^2 =$

Solution: to square something means to write it down twice and multiply. Then it is a FOIL problem. F stand for first with first, O for outer terms, I for inner terms, and L for last terms.

$$\begin{aligned} (5a - 1)^2 &= (5a - 1)(5a - 1) \\ &= \underbrace{25a^2}_F \quad \underbrace{-5a}_O \quad \underbrace{-5a}_I \quad \underbrace{+1}_L \quad \text{combine like terms} \\ &= 25a^2 - 10a + 1 \end{aligned}$$

(b) $(3x^5 + 4y)(3x^5 - 4y) =$

$$\begin{aligned} (3x^5 + 4y)(3x^5 - 4y) &= \underbrace{9x^{10}}_F \quad \underbrace{-12x^5}_O \quad \underbrace{+12x^5}_I \quad \underbrace{-16y^2}_L \quad \text{combine like terms} \\ &= 9x^{10} - 16y^2 \end{aligned}$$

The expressions $3x^5 + 4y$ and $3x^5 - 4y$ are called conjugates. Because of the same terms and alternating signs, O and I cancel out when "FOIL"-ing, giving us the difference of two squares.

(c) $\frac{3a - 8}{8 - 3a} =$

Solution: We need to notice that the numerator and denominator are opposites of each other. Indeed,

$$-1(8 - 3a) = -8 + 3a = 3a - 8$$

Thus

$$\frac{3a - 8}{8 - 3a} = \frac{-1(8 - 3a)}{8 - 3a} = -1$$

(d) $\frac{2x + 1}{4x^2 - 1} =$

Solution: We factor the denominator via the difference of squares theorem, and then cancel.

$$\frac{2x + 1}{4x^2 - 1} = \frac{2x + 1}{(2x + 1)(2x - 1)} = \frac{1}{2x - 1}$$

(e) $\frac{ab - a - b + 1}{b^2 - 1} =$

Solution: We will factor both numerator and denominator and then cancel. The numerator can be factored by grouping

$$\begin{aligned} \underbrace{ab - a} \quad \underbrace{-b + 1} &= a(b - 1) - 1(b - 1) \\ &= (a - 1)(b - 1) \end{aligned}$$

The denominator factors by the difference of squares theorem.

$$b^2 - 1 = (b + 1)(b - 1)$$

Thus the fraction can be simplified as

$$\frac{ab - a - b + 1}{b^2 - 1} = \frac{(a - 1)(b - 1)}{(b + 1)(b - 1)} = \frac{a - 1}{b + 1}$$

$$(f) \frac{5x - 30}{x^2 - 36} \cdot \frac{3x + 18}{5} =$$

Solution: we will factor whatever we can and then cancel.

$$\frac{5x - 30}{x^2 - 36} \cdot \frac{3x + 18}{5} = \frac{5(x - 6)}{(x + 6)(x - 6)} \cdot \frac{3(x + 6)}{5} = 3$$

$$(g) \frac{3x}{x - 2} - \frac{x + 4}{x - 2} =$$

Solution: This is a subtraction of fractions. The denominators are the same, the only difficulty is that we are subtracting expressions instead of numbers. The second pair of parentheses is essential.

$$\frac{3x}{x - 2} - \frac{x + 4}{x - 2} = \frac{(3x) - (x + 4)}{x - 2} = \frac{3x - x - 4}{x - 2} = \frac{2x - 4}{x - 2} = \frac{2(x - 2)}{x - 2} = 2$$

$$(h) 2^0 + (-2)^0 =$$

Solution: 2^0 and $(-2)^0$ are both equal to zero. Thus $2^0 + (-2)^0 = 2$.

$$(i) 2^3 \cdot (2^{-2})^{-2} =$$

Solution:

$$\begin{aligned} 2^3 \cdot (2^{-2})^{-2} &= && \text{use rule } (a^n)^m = a^{nm} \\ 2^3 \cdot 2^4 &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\ &= 2^7 = 128 \end{aligned}$$

$$(j) (2^3 \cdot 2^{-2})^{-2} =$$

Solution:

$$\begin{aligned} (2^3 \cdot 2^{-2})^{-2} &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\ (2^1)^{-2} &= && \text{use rule } (a^n)^m = a^{nm} \\ 2^{-2} &= && \text{use rule } a^{-n} = \frac{1}{a^n} \\ \frac{1}{2^2} &= \frac{1}{4} \end{aligned}$$

$$(k) 2a^3 (-2ab^{-2})^{-2} ab^0 =$$

Solution:

$$\begin{aligned} 2a^3 (-2ab^{-2})^{-2} ab^0 &= && \text{use rule } (ab)^n = a^n b^n \\ 2a^3 (-2)^{-2} a^{-2} (b^{-2})^{-2} ab^0 &= && \text{use rule } (a^n)^m = a^{nm} \\ 2a^3 (-2)^{-2} a^{-2} b^4 ab^0 &= && \text{use: } b^0 = 1 \text{ and multiplication is commutative} \\ 2(-2)^{-2} a^3 a^{-2} ab^4 &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\ 2(-2)^{-2} a^2 b^4 &= && \text{use rule } a^{-n} = \frac{1}{a^n} \\ 2 \left(\frac{1}{2^2} \right) a^2 b^4 &= \frac{2}{1} \cdot \frac{1}{4} \cdot \frac{a^2 b^4}{1} = \frac{a^2 b^4}{2} \end{aligned}$$

$$(l) \frac{(-2x)^2 y^{-3}}{2x^{-3}y^2} =$$

Solution:

$$\begin{aligned} \frac{(-2x)^2 y^{-3}}{2x^{-3}y^2} &= && \text{use rule } (ab)^n = a^n b^n \\ \frac{(-2)^2 x^2 y^{-3}}{2x^{-3}y^2} &= \frac{4x^2 y^{-3}}{2x^{-3}y^2} = \frac{2x^2 y^{-3}}{x^{-3}y^2} && \text{use rule } a^{-n} = \frac{1}{a^n} \\ \frac{2x^2 \cdot \frac{1}{y^3}}{\frac{1}{x^3} \cdot y^2} &= && \text{make everything a fraction} \\ \frac{2x^2 \cdot \frac{1}{y^3}}{\frac{1}{x^3} \cdot \frac{y^2}{1}} &= \frac{2x^2}{\frac{y^3}{x^3}} && \text{to divide is to multiply by the reciprocal} \\ \frac{2x^2}{\frac{1}{x^3} \cdot \frac{y^2}{1}} &= \frac{2x^2}{\frac{y^2}{x^3}} && \\ \frac{2x^2}{y^3} \cdot \frac{x^3}{y^2} &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\ &= \frac{2x^5}{y^5} \end{aligned}$$

$$(m) (x - y)(x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5) =$$

Solution: We apply the law of distributivity

$$\begin{aligned} (x - y)(x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5) &= \\ &= x \cdot x^5 + x \cdot x^4y + x \cdot x^3y^2 + x \cdot x^2y^3 + x \cdot xy^4 + x \cdot y^5 \\ &\quad - y \cdot x^5 - y \cdot x^4y - y \cdot x^3y^2 - y \cdot x^2y^3 - y \cdot xy^4 - y \cdot y^5 \\ &= x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 \\ &\quad - x^5y - x^4y^2 - x^3y^3 - x^2y^4 - xy^5 - y^6 \\ &= x^6 - y^6 \end{aligned}$$

2. Factor completely each of the following:

$$(a) 4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 =$$

Solution:

$$\begin{aligned} 4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 &= && \text{the GCF is } am \\ am(4an - 15bm - 6bn + 10am) &= && \text{rearrange} \\ am \left(\underbrace{4an - 6bn}_{2n(2a-3b)} + \underbrace{10am - 15bm}_{5m(2a-3b)} \right) &= \\ am(2n(2a-3b) + 5m(2a-3b)) &= && am(2n+5m)(2a-3b) \end{aligned}$$

(b) $a^2x^3 - b^2x - a^2x + b^2x^3 =$

Solution:

$$\begin{aligned} a^2x^3 - b^2x - a^2x + b^2x^3 &= && \text{the GCF is } x \\ x(a^2x^2 - b^2 - a^2 + b^2x^2) &= && \text{rearrange} \\ x\left(\underbrace{a^2x^2 - a^2} \quad \underbrace{+b^2x^2 - b^2}\right) &= && \\ x(a^2(x^2 - 1) + b^2(x^2 - 1)) &= && x(a^2 + b^2)(x^2 - 1) \end{aligned}$$

We are not done yet since $(x^2 - 1) = (x^2 - 1^2)$ further factors via the difference of squares theorem. Thus the answer is

$$\begin{aligned} x(a^2 + b^2)(x^2 - 1) &= x(a^2 + b^2)(x^2 - 1^2) \\ &= x(a^2 + b^2)(x + 1)(x - 1) \end{aligned}$$

(c) $162a + 162b - 2ax^4 - 2bx^4 =$

Solution:

$$\begin{aligned} 162a + 162b - 2ax^4 - 2bx^4 &= && \text{the GCF is } 2 \\ 2\left(\underbrace{81a + 81b} \quad \underbrace{-ax^4 - bx^4}\right) &= && \\ 2(81(a + b) - x^4(a + b)) &= && 2(81 - x^4)(a + b) \end{aligned}$$

We are not done yet, since $81 - x^4 = 9^2 - (x^2)^2$ further factors via the difference of squares theorem.

$$\begin{aligned} 2(81 - x^4)(a + b) &= 2(9^2 - (x^2)^2)(a + b) \\ &= 2(9 + x^2)(9 - x^2)(a + b) \end{aligned}$$

One factor still further factors: $9 - x^2 = 3^2 - x^2 = (3 + x)(3 - x)$. Thus the final answer is

$$\begin{aligned} &= 2(9 + x^2)(9 - x^2)(a + b) \\ &= 2(9 + x^2)(3^2 - x^2)(a + b) \\ &= 2(9 + x^2)(3 + x)(3 - x)(a + b) \end{aligned}$$

3. Factor by grouping.

(a) $x^2 - 6x + 8 =$

Solution: we will factor by grouping. First we conduct the "pq-game".

$$\begin{array}{ll} pq &= 8 && \text{1st coefficient times 3rd coefficient} \\ p + q &= -6 && \text{2nd coefficient} \end{array}$$

We start by expressing 8 as a product of two numbers. there are only two pairs, 1 with 8 and 2 with 4. Since the product pq is positive, p and q have to have the same sign. Since the sum $p + q$ is negative, they both have to be negative. We only need to consider -1 with -8 and

-2 with -4 . Clearly -2 with -4 work as p and q . We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned}x^2 - 6x + 8 &= \underbrace{x^2 - 2x}_{x(x-2)} \quad \underbrace{-4x + 8}_{-4(x-2)} \\ &= x(x-2) - 4(x-2) = (x-2)(x-4)\end{aligned}$$

We check by multiplication:

$$(x-2)(x-4) = x^2 - 4x - 2x + 8 = x^2 - 6x + 8$$

Thus our result is correct.

(b) $3a^2 - 5a - 2 =$

Solution: we will factor by grouping. First we conduct the "pq-game".

$$\begin{aligned}pq &= -6 && \text{1st coefficient times 3rd coefficient} \\ p+q &= -5 && \text{2nd coefficient}\end{aligned}$$

We start by expressing 6 as a product of two numbers. there are only two pairs, 1 with 6 and 2 with 3. Since the product pq is negative, one number must be positive, the other one must be positive.. Since the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -6 and 2 with -3 . Clearly 1 with -6 work as p and q . We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned}3a^2 - 5a - 2 &= \underbrace{3a^2 + a}_{a(3a+1)} \quad \underbrace{-6a - 2}_{-2(3a+1)} \\ &= a(3a+1) - 2(3a+1) = (a-2)(3a+1)\end{aligned}$$

We check by multiplication:

$$(a-2)(3a+1) = 3a^2 + a - 6a - 2 = 3a^2 - 5a - 2$$

Thus our result is correct.

(c) $4b^2 - b - 5 = (4b-5)(b+1)$

Solution: we will factor by grouping. First we conduct the "pq-game".

$$\begin{aligned}pq &= -20 && \text{1st coefficient times 3rd coefficient} \\ p+q &= -1 && \text{2nd coefficient}\end{aligned}$$

We start by expressing 20 as a product of two numbers. the possible pairs are, 1 with 20, 2 with 10, and 4 with 5. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -20 , 2 with -10 , and 4 with -5 . Clearly 4 with -5 work as p and q . We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned}4b^2 - b - 5 &= \underbrace{4b^2 + 4b}_{4b(b+1)} \quad \underbrace{-5b - 5}_{-5(b+1)} \\ &= 4b(b+1) - 5(b+1) = (4b-5)(b+1)\end{aligned}$$

We check by multiplication:

$$(4b-5)(b+1) = 4b^2 + 4b - 5b - 5 = 4b^2 - b - 5$$

Thus our result is correct.

4. Solve each of the following equations. Make sure to check your solutions.

(a) $\frac{2x+1}{5} - \frac{5-x}{2} = x-1$

Solution:

$$\begin{aligned} \frac{2x+1}{5} - \frac{5-x}{2} &= x-1 && \text{re-write everything as a fraction} \\ \frac{2x+1}{5} - \frac{5-x}{2} &= \frac{x-1}{1} && \text{common denominator is } 10 \\ \frac{2(2x+1)}{10} - \frac{5(5-x)}{10} &= \frac{10(x-1)}{10} && \text{multiply both sides by } 10 \\ 2(2x+1) - 5(5-x) &= 10(x-1) && \text{distribute} \\ 4x+2-25+5x &= 10x-10 && \text{combine like terms} \\ 9x-23 &= 10x-10 && \text{subtract } 9x \\ -23 &= x-10 && \text{add } 10 \\ -13 &= x \end{aligned}$$

We check: if $x = -13$, then

$$\begin{aligned} \text{LHS} &= \frac{2(-13)+1}{5} - \frac{5-(-13)}{2} = \frac{-26+1}{5} - \frac{5+13}{2} = \frac{-25}{5} - \frac{18}{2} = -5-9 = -14 \\ \text{RHS} &= -13-1 = -14 \quad \checkmark \end{aligned}$$

Thus our solution, -13 is correct.

(b) $-3(2x-1) - (3-7x) = 2(x+1) - (x-1)$

Solution: In case of subtracting algebraic expressions, we either subtract the opposite, or distribute -1 . We will use the second method here.

$$\begin{aligned} -3(2x-1) - (3-7x) &= 2(x+1) - (x-1) \\ -3(2x-1) - 1(3-7x) &= 2(x+1) - 1(x-1) && \text{distribute} \\ -6x+3-3+7x &= 2x+2-x+1 && \text{combine like terms} \\ x &= x+3 && \text{subtract } x \\ 0 &= 3 \end{aligned}$$

Since x disappeared from the equation, we are left with an unconditional statement, this time unconditionally false. There is no value of x that can make $0 = 3$ be true, and thus this equation has **no solution**. An equation of this type is called a contradiction.

(c) $8x^3 = 50x^2$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8x^3 &= 50x^2 && \text{subtract } 50x^2 \\ 8x^3 - 50x^2 &= 0 && \text{the GCF is } 2x^2 \\ 2x^2(4x-25) &= 0 \end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2x^2 = 0$ or $4x - 25 = 0$. We solve these equations for x .

$$\begin{array}{rcl} 2x^2 & = & 0 \\ 2 \cdot x \cdot x & = & 0 \\ x & = & 0 \end{array} \quad \text{or} \quad \begin{array}{rcl} 4x - 25 & = & 0 \\ 4x & = & 25 \\ x & = & \frac{25}{4} \end{array}$$

We check both solutions. If $x = 0$, then

$$\begin{array}{rcl} \text{LHS} & = & 8 \cdot 0^3 = 8 \cdot 0 = 0 \\ \text{RHS} & = & 50 \cdot 0^2 = 50 \cdot 0 = 0 \quad \checkmark \end{array}$$

If $x = \frac{25}{4}$, then

$$\begin{array}{rcl} \text{LHS} & = & 8 \left(\frac{25}{4} \right)^3 = \frac{8}{1} \cdot \frac{15\,625}{64} = \frac{15\,625}{8} \\ \text{RHS} & = & 50 \left(\frac{25}{4} \right)^2 = \frac{50}{1} \cdot \frac{625}{16} = \frac{15\,625}{8} \quad \checkmark \end{array}$$

Thus both solutions, 0 and $\frac{25}{4}$ are correct.

(d) $8p^3 = 50p$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{array}{rcl} 8p^3 & = & 50p & \text{subtract } 50p \\ 8p^3 - 50p & = & 0 & \text{the GCF is } 2p \\ 2p(4p^2 - 25) & = & 0 \\ 2p((2p)^2 - 5^2) & = & 0 & \text{factor via difference of squares theorem} \\ 2p(2p + 5)(2p - 5) & = & 0 \end{array}$$

We now apply the special zero property. If this product is zero, then either $2p = 0$ or $2p + 5 = 0$ or $2p - 5 = 0$. We solve these equations for p .

$$\begin{array}{rcl} 2p + 5 & = & 0 & \text{or} & 2p - 5 = 0 & \text{or} & 2p = 0 \\ 2p & = & -5 & \text{or} & 2p = 5 & \text{or} & p = 0 \\ p & = & -\frac{5}{2} & \text{or} & p = \frac{5}{2} & & \end{array}$$

We check all three solutions. If $p = -\frac{5}{2}$, then

$$\begin{array}{rcl} \text{LHS} & = & 8 \left(-\frac{5}{2} \right)^3 = \frac{8}{1} \cdot \frac{-125}{8} = -125 \\ \text{RHS} & = & 50 \left(-\frac{5}{2} \right) = \frac{50}{1} \cdot \frac{-5}{2} = \frac{-250}{2} = -125 \quad \checkmark \end{array}$$

If $p = \frac{5}{2}$, then

$$\text{LHS} = 8 \left(\frac{5}{2} \right)^3 = \frac{8}{1} \cdot \frac{125}{8} = 125$$

$$\text{RHS} = 50 \left(\frac{5}{2} \right) = \frac{50}{1} \cdot \frac{5}{2} = \frac{250}{2} = 125 \quad \checkmark$$

and if $p = 0$, then

$$\text{LHS} = 8 \cdot 0^3 = 8 \cdot 0 = 0$$

$$\text{RHS} = 50 \cdot 0 = 0 \quad \checkmark$$

Thus all three solutions, $-\frac{5}{2}$, 0 , and $\frac{5}{2}$ are correct.

(e) $2 - (3 - x)(2x + 5) = (x - 1)(2x - 1)$

Solution: We have to use the FOIL method on both sides to perform the multiplications. It is very important, however, to keep the expressions in a parentheses since we are dealing with subtraction between *algebraic expressions*.

$2 - (3 - x)(2x + 5) = (x - 1)(2x - 1)$	FOIL
$2 - (6x + 15 - 2x^2 - 5x) = 2x^2 - x - 2x + 1$	combine like terms
$2 - (-2x^2 + x + 15) = 2x^2 - 3x + 1$	perform subtraction
$2 + 2x^2 - x - 15 = 2x^2 - 3x + 1$	combine like terms
$2x^2 - x - 13 = 2x^2 - 3x + 1$	subtract $2x^2$ (the equation is linear!)
$-x - 13 = -3x + 1$	add $3x$
$2x - 13 = 1$	add 13
$2x = 14$	divide by 2
$x = 7$	

We check: if $x = 7$, then

$$\text{LHS} = 2 - (3 - 7)(2(7) + 5) = 2 - (-4)(14 + 5) = 2 - (-4)19 = 2 - (-76) = 78$$

$$\text{RHS} = (7 - 1)(2(7) - 1) = 6(14 - 1) = 6 \cdot 13 = 78 \quad \checkmark$$

Thus our solution, 7 is correct.

(f) $8a + 2a^2 = 42$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$8a + 2a^2 = 42$	subtract 42, rearrange
$2a^2 + 8a - 42 = 0$	the GCF is 2
$2(a^2 + 4a - 21) = 0$	

We will factor $a^2 + 4a - 21$ by grouping. First we conduct the "pq-game".

$pq = -21$	1st coefficient times 3rd coefficient
$p + q = 4$	2nd coefficient

We start by expressing 21 as a product of two numbers. the only possible pairs are, 1 with 21 and 3 with 7. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p + q$ is positive, the negative sign has to be in front of the smaller number. We only need to consider -1 with 20, and -3 with 7. Clearly -3 with 7 work as p and q . We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 2(a^2 + 4a - 21) &= 0 \\ 2\left(\underbrace{a^2 + 7a}_{a(a+7)} \quad \underbrace{-3a - 21}_{-3(a+7)}\right) &= 0 \\ 2(a(a+7) - 3(a+7)) &= 0 \\ 2(a-3)(a+7) &= 0 \end{aligned}$$

Thus our equation is

$$2(a-3)(a+7) = 0$$

We now apply the special zero property. If this product is zero, then either $2 = 0$ or $a - 3 = 0$ or $a + 7 = 0$. We solve these equations for a .

$$\begin{array}{llll} a - 3 = 0 & \text{or} & a + 7 = 0 & \text{or} & 2 = 0 \\ a = 3 & \text{or} & a = -7 & \text{or} & \text{no solution here} \end{array}$$

We check both solutions. If $a = 3$, then

$$\text{LHS} = 8(3) + 2(3)^2 = 8 \cdot 3 + 2 \cdot 9 = 24 + 18 = 42 = \text{RHS} \quad \checkmark$$

If $a = -7$, then

$$\text{LHS} = 8(-7) + 2(-7)^2 = 8 \cdot (-7) + 2 \cdot 49 = -56 + 98 = 42 = \text{RHS} \quad \checkmark$$

Thus both solutions, -7 and 3 are correct.

5. Solve each of the following inequalities. Graph the solution set.

$$(a) \frac{3-4x}{3} - \frac{2x-3}{7} \leq -x+7$$

Solution:

$$\begin{array}{ll} \frac{3-4x}{3} - \frac{2x-3}{7} \leq -x+7 & \text{re-write everything as a fraction} \\ \frac{3-4x}{3} - \frac{2x-3}{7} \leq \frac{-x+7}{1} & \text{common denominator is 21} \\ \frac{7(3-4x)}{21} - \frac{3(2x-3)}{21} \leq \frac{21(-x+7)}{21} & \text{multiply both sides by 21} \\ 7(3-4x) - 3(2x-3) \leq 21(-x+7) & \text{distribute} \\ 21 - 28x - 6x + 9 \leq -21x + 147 & \text{combine like terms} \\ -34x + 30 \leq -21x + 147 & \text{add } 34x \\ 30 \leq 13x + 147 & \text{subtract } 147 \\ -117 \leq 13x & \text{divide by } 13 \\ -9 \leq x & \end{array}$$

(b) $\frac{3-2a}{7} > -1$

Solution:

$$\begin{array}{ll} \frac{3-2a}{7} > -1 & \text{multiply both sides by 7} \\ 3-2a > -7 & \text{add } 2a \\ 3 > 2a-7 & \text{add 7} \\ 10 > 2a & \text{divide by 2} \\ 5 > a & \end{array}$$

(c) $3(2x-3) - (5x+4) > -14$

Solution:

$$\begin{array}{ll} 3(2x-3) - (5x+4) > -14 & \text{distribute} \\ 6x-9-5x-4 > -14 & \text{combine like terms} \\ x-13 > -14 & \text{add 13} \\ x > -1 & \end{array}$$

6. Solve each of the following formulas.

(a) $A = 2a - 3b$ for a .

Solution:

$$\begin{array}{ll} A = 2a - 3b & \text{add } 3b \\ A + 3b = 2a & \text{divide by 2} \\ \frac{A + 3b}{2} = a & \end{array}$$

Thus $a = \frac{A + 3b}{2}$ or $\frac{A}{2} + \frac{3b}{2}$ or $\frac{1}{2}A + \frac{3}{2}b$

(b) $A = 2a - 3b$ for b .

Solution:

$$\begin{array}{ll} A = 2a - 3b & \text{add } 3b \\ A + 3b = 2a & \text{subtract } A \\ 3b = 2a - A & \text{divide by 3} \\ b = \frac{2a - A}{3} & \text{or } \frac{2a}{3} - \frac{A}{3} \text{ or } \frac{2}{3}a - \frac{1}{3}A \end{array}$$

(c) $F = \frac{mMG}{d^2}$ for m .

Solution:

$$\begin{array}{ll} F = \frac{mMG}{d^2} & \text{multiply by } d^2 \\ Fd^2 = mMG & \text{divide by } MG \\ \frac{Fd^2}{MG} = m & \end{array}$$

(d) $3x - 5y = 60$ for y .

Solution:

$$\begin{array}{rcl}
 3x - 5y & = & 60 & \text{add } 5y \\
 3x & = & 5y + 60 & \text{subtract } 60 \\
 3x - 60 & = & 5y & \text{divide by } 5 \\
 \frac{3x - 60}{5} & = & y &
 \end{array}$$

The answer is $y = \frac{3x - 60}{5}$ or $\frac{3x}{5} - 12$ or $\frac{3}{5}x - 12$

7. Graph the straight lines $5x - 3y = 11$ and $y = -x - 9$ in the same coordinate system.

(a) Use your graph to find the coordinates of the point where the lines intersect.

Solution: We first graph the line $5x - 3y = 11$.

$$\begin{array}{rcl}
 \text{If } x = 1, y = ? & & \text{substitute } x = 1 \text{ into the equation of the line} \\
 5(1) - 3y & = & 11 & \text{solve for } y \\
 5 - 3y & = & 11 & \text{subtract } 5 \\
 -3y & = & 6 & \text{divide by } -3 \\
 y & = & -2 & \implies \text{we found } (1, -2)
 \end{array}$$

$$\begin{array}{rcl}
 \text{If } x = 4, y = ? & & \text{substitute } x = 4 \text{ into the equation of the line} \\
 5(4) - 3y & = & 11 & \text{solve for } y \\
 20 - 3y & = & 11 & \text{subtract } 20 \\
 -3y & = & -9 & \text{divide by } -3 \\
 y & = & 3 & \implies \text{we found } (4, 3)
 \end{array}$$

$$\begin{array}{rcl}
 \text{If } x = 7, y = ? & & \text{substitute } x = 7 \text{ into the equation of the line} \\
 5(7) - 3y & = & 11 & \text{solve for } y \\
 35 - 3y & = & 11 & \text{subtract } 35 \\
 -3y & = & -24 & \text{divide by } -3 \\
 y & = & 8 & \implies \text{we found } (7, 8)
 \end{array}$$

$$\begin{array}{rcl}
 \text{If } x = -2, y = ? & & \text{substitute } x = -2 \text{ into the equation of the line} \\
 5(-2) - 3y & = & 11 & \text{solve for } y \\
 -10 - 3y & = & 11 & \text{add } 10 \\
 -3y & = & 21 & \text{divide by } -3 \\
 y & = & -7 & \implies \text{we found } (-2, -7)
 \end{array}$$

We graph the points $(1, -2)$, $(4, 3)$, $(7, 8)$, and $(-2, -7)$ and connect the points. (Green line.)

We now graph the line $y = -x - 9$.

$$\begin{array}{ll} \text{If } x = 0, y = ? & \text{substitute } x = 0 \text{ into the equation of the line} \\ y = -(0) - 9 = 0 - 9 = -9 & \implies \text{we found } (0, -9) \end{array}$$

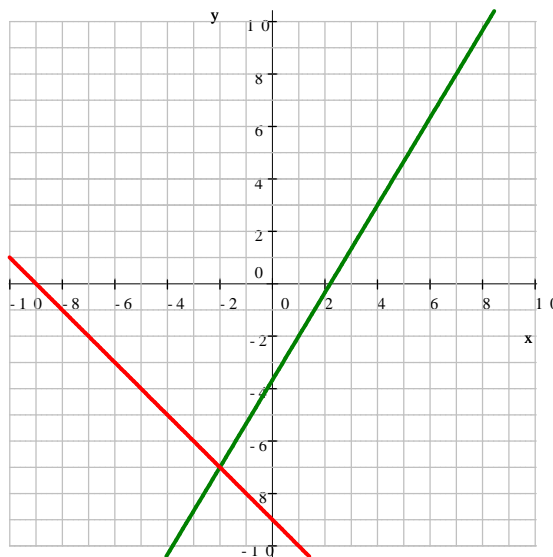
$$\begin{array}{ll} \text{If } x = 2, y = ? & \text{substitute } x = 2 \text{ into the equation of the line} \\ y = -(2) - 9 = -11 & \implies \text{we found } (2, -11) \end{array}$$

$$\begin{array}{ll} \text{If } x = -2, y = ? & \text{substitute } x = -2 \text{ into the equation of the line} \\ y = -(-2) - 9 = 2 - 9 = -7 & \implies \text{we found } (-2, -7) \end{array}$$

$$\begin{array}{ll} \text{If } x = -4, y = ? & \text{substitute } x = -4 \text{ into the equation of the line} \\ y = -(-4) - 9 = 4 - 9 = -5 & \implies \text{we found } (-4, -5) \end{array}$$

$$\begin{array}{ll} \text{If } x = -7, y = ? & \text{substitute } x = -7 \text{ into the equation of the line} \\ y = -(-7) - 9 = 7 - 9 = -2 & \implies \text{we found } (-7, -2) \end{array}$$

We graph the points $(0, -9)$, $(2, -11)$, $(-2, -7)$, $(-4, -5)$, and $(-7, -2)$ and connect the points. (Red line.)



We read from the graph that the lines intersect at $(-2, -7)$

(b) Use algebraic methods to check your answer for part a).

Solution: Is the point $(-2, -7)$ on the line $5x - 3y = 11$?

$$\text{LHS} = 5(-2) - 3(-7) = -10 - (-21) = -10 + 21 = 11 = \text{RHS} \implies \text{yes}$$

Is the point $(-2, -7)$ on the line $y = -x - 9$?

$$\text{RHS} = -(-2) - 9 = 2 - 9 = -7 = \text{LHS} \implies \text{yes}$$

8. Word Problems.

- (a) A couch went on a 15% sale. The sale price is \$ 697. Find the original price.

Solution: A 15% sale means that we have to pay 85% of the original price. The question is: 85% of what number is 697?

$$(\text{is}) = 697$$

$$F = \frac{85}{100}$$

$$(\text{of}) = x$$

We substitute these values into the formula $(\text{is}) = F \cdot (\text{of})$ and solve for x .

$$\begin{aligned} (\text{is}) &= F \cdot (\text{of}) \\ 697 &= \frac{85}{100} \cdot x && \text{divide both sides by } \frac{85}{100} \\ 697 \div \frac{85}{100} &= x \\ x &= \frac{697}{1} \cdot \frac{100}{85} = 820 \end{aligned}$$

Thus the original price is **\$ 820**

- (b) The difference between two numbers is 7, their sum is 37. Find these numbers.

Solution: Let us call the smaller number x . Then the larger number is $x + 7$, since the difference between the two numbers is 7. The equation then expresses the sum of these numbers

$$\begin{aligned} \underbrace{x}_{\text{smaller number}} + \underbrace{x+7}_{\text{larger number}} &= 37 && \text{solve for } x \\ 2x + 7 &= 37 && \text{subtract 7} \\ 2x &= 30 && \text{divide by 2} \\ x &= 15 \end{aligned}$$

Thus the smaller number, labeled x is 15. The larger number was labeled $x + 7$, so it must be $15 + 7 = 22$. Thus the numbers are **15 and 22**. We check: the difference between 22 and 15 is $22 - (15) = 7$, and their sum is indeed $22 + 15 = 37$. Thus our solution is correct.

- (c) Ann and Betty are roommates. The monthly rent is \$ 950. The amount paid by Ann is \$ 310 less than twice the amount paid by Betty. How much do they each pay for rent?

Solution: Let x denote the amount that is paid by Betty. Then Ann must pay $2x - 310$ per month. The equation expresses the monthly rent as the sum of the two payments

$$\begin{aligned} \underbrace{x}_{\text{smaller amount}} + \underbrace{2x-310}_{\text{larger amount}} &= 950 && \text{solve for } x \\ 3x - 310 &= 950 && \text{add 310} \\ 2x &= 1260 && \text{divide by 3} \\ x &= 420 \end{aligned}$$

Thus the amount paid by Betty is \$ 420. The amount paid by Betty was labeled as $2x - 310$, so Betty must pay $2(420) - 310 = 840 - 310 = 530$. Thus the payments are **\$ 420 and \$ 530**. We check: $\$ 530 = 2(\$ 420) - \$ 310$ and $\$ 420 + \$ 530 = \$ 950$. Thus our solution is correct.

- (d) The population of a town has decreased from 90 000 to 82 800. What percent of a change does this represent?

Solution: The decrease in the population is $90\,000 - 82\,800 = 7200$. The question is then:

7200 is what percent of 90 000?

(is) = 7200

$F = x$

(of) = 90 000

We substitute these values into the formula (is) = $F \cdot$ (of) and solve for x .

$$\begin{aligned} \text{(is)} &= F \cdot \text{(of)} \\ 7200 &= x \cdot 90\,000 && \text{divide both sides by 90 000} \\ \frac{7200}{90\,000} &= x \\ x &= \frac{8}{100} = 8\% \end{aligned}$$

Thus the change in population is **8%**.

- (e) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft.

Solution: Let us denote the shorter side by x . Then the other side is $3x - 4$. The equation expresses the perimeter of the rectangle.

$$\begin{aligned} 2(x) + 2(3x - 4) &= 64 && \text{multiply out parentheses} \\ 2x + 6x - 8 &= 64 && \text{combine like terms} \\ 8x - 8 &= 64 && \text{add} \\ 8x &= 72 && \text{divide by 8} \\ x &= 9 \end{aligned}$$

If the shorter side was denoted by x , we now know it is 9 ft. The longer side was denoted by $3x - 4$, so it must be $3(9 \text{ ft}) - 4 \text{ ft} = 23 \text{ ft}$. Thus the sides of the rectangle are **9 ft and 23 ft**. We check: $P = 2(9 \text{ ft}) + 2(23 \text{ ft}) = 64 \text{ ft}$ and $23 \text{ ft} = 3(9 \text{ ft}) - 4 \text{ ft}$. Thus our solution is correct.

- (f) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is 84 ft^2 .

Solution: Let us denote the shorter side by x . Then the other side is $3x - 4$. The equation expresses the area of the rectangle.

$$\begin{aligned} x(3x - 4) &= 84 && \text{multiply out parentheses} \\ 3x^2 - 4x &= 84 && \text{subtract 84} \\ 3x^2 - 4x - 84 &= 0 \end{aligned}$$

Because the equation is quadratic, we need to factor the left-hand side and then apply the zero property. We will factor by grouping. First we conduct the "pq-game". The sum of p and q has to be the linear coefficient (the number in front of x , with its sign), so it is -4 . The product of p and q has to be the product of the other coefficients, $3(-84) = -252$.

$$\begin{aligned} pq &= -252 \\ p + q &= -4 \end{aligned}$$

Now we need to find p and q . Because the product is negative, we're looking for a positive and a negative number. Because the sum is negative, the larger number must carry the negative sign. We enter $\sqrt{252} = 15.87450787$ into the calculator and get a decimal:

$$\sqrt{252} = 15.874\dots$$

So we start looking for factors of 252, starting at 15, and moving down. We soon find 14 and -18 . These are our values for p and q . We use these numbers to express the linear term:

$$-4x = 14x - 18x$$

and factor by grouping.

$$\begin{aligned} 3x^2 - 4x - 84 &= 0 \\ \underbrace{3x^2 + 14x - 18x - 84} &= 0 \\ x(3x + 14) - 6(3x + 14) &= 0 \\ (x - 6)(3x + 14) &= 0 \end{aligned}$$

We now apply the zero property. Either $x - 11 = 0$ or $3x + 14 = 0$. We solve both these equations for x .

$$\begin{aligned} x - 6 &= 0 \\ x &= 6 \end{aligned}$$

and

$$\begin{aligned} 3x + 14 &= 0 \\ 3x &= -14 \\ x &= -\frac{14}{3} \end{aligned}$$

Since distances can not be negative, the second solution for x , $-\frac{14}{3}$ is ruled out. Thus $x = 6$. Then the longer side is $3(6 \text{ ft}) - 4 \text{ ft} = 14 \text{ ft}$, and so the rectangle's sides are **6 ft and 14 ft** long. We check: $6 \text{ ft}(14 \text{ ft}) = 84 \text{ ft}^2$ and $14 \text{ ft} = 3(6 \text{ ft}) - 4 \text{ ft}$. Thus our solution is correct.