Review Problems

1. Simplify each of the following.

   (a) $(5a-1)^2 =$

   (b) $\left(3x^5+4y\right)\left(3x^5-4y\right) =$

   (c) $\dfrac{3a-8}{8-3a} =$

   (d) $\dfrac{2x+1}{4x^2-1} =$

   (e) $\dfrac{ab-a-b+1}{b^2-1} =$

   (f) $\dfrac{5x-30}{x^2-36}\cdot\dfrac{3x+18}{5} =$

   (g) $\dfrac{3x}{x-2}-\dfrac{x+4}{x-2} =$

   (h) $2a^3\left(-2ab^2\right)^3 ab^7 =$

   (i) $\dfrac{(-2x)^2\, y^3}{2x^6y^2} =$

   (j) $(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right) =$

2. Factor completely each of the following:

   (a) $4a^2mn-15abm^2-6abmn+10a^2m^2 =$

   (b) $a^2x^3-b^2x-a^2x+b^2x^3 =$

   (c) $162a+162b-2ax^4-2bx^4 =$

3. Factor by grouping.

   (a) $x^2-6x+8 =$

   (b) $3a^2-5a-2 =$

   (c) $4b^2-b-5 =$

4. Solve each of the following equations. Make sure to check your solutions.

   (a) $\dfrac{2x+1}{5}-\dfrac{5-x}{2} = x-1$

   (b) $-3(2x-1)-(3-7x) = 2(x+1)-(x-1)$

   (c) $8x^3 = 50x^2$

   (d) $8p^3 = 50p$

   (e) $2-(3-x)(2x+5) = (x-1)(2x-1)$

   (f) $8a+2a^2 = 42$

5. Solve each of the following inequalities. Graph the solution set.

   (a) $\dfrac{3-4x}{3}-\dfrac{2x-3}{7} \leq -x+7$

(b) $\dfrac{3-2a}{7} > -1$

(c) $3(2x-3) - (5x+4) > -14$

6. Solve each of the following formulas.

   (a) $A = 2a - 3b$ for a.

   (b) $A = 2a - 3b$ for b.

   (c) $F = \dfrac{mMG}{d^2}$ for m.

   (d) $3x - 5y = 60$ for y.

7. Graph the straight lines $5x - 3y = 11$ and $y = -x - 9$ in the same coordinate system.

   (a) Use your graph to find the coordinates of the point where the lines intersect.

   (b) Use algebraic methods to check your answer for part a).

8. Word Problems.

   (a) A couch went on a 15% sale. The sale price is \$ 697. Find the original price.

   (b) The difference between two numbers is 7, their sum is 37. Find these numbers.

   (c) Ann and Betty are roommates. The monthly rent is \$ 950. The amount paid by Ann is \$ 310 less than twice the amount paid by Betty. How much do they each pay for rent?

   (d) The population of a town has decreased from 90 000 to 82 800. What percent of a change does this represent?

   (e) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft.

   (f) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is 84 ft^2.

Review Problems - Answers

1. Simplify each of the following.

   (a) $(5a-1)^2 = \ 25a^2 - 10a + 1$

   (b) $\left(3x^5+4y\right)\left(3x^5-4y\right) = \ 9x^{10} - 16y^2$

   (c) $\dfrac{3a-8}{8-3a} = \ -1$

   (d) $\dfrac{2x+1}{4x^2-1} = \ \dfrac{1}{2x-1}$

   (e) $\dfrac{ab-a-b+1}{b^2-1} = \ \dfrac{a-1}{b+1}$

   (f) $\dfrac{5x-30}{x^2-36}\cdot\dfrac{3x+18}{5} = \ 3$

   (g) $\dfrac{3x}{x-2} - \dfrac{x+4}{x-2} = \ 2$

   (h) $2a^3\left(-2ab^2\right)^3 ab^7 = \ -16a^7b^{13}$

   (i) $\dfrac{(-2x)^2y^3}{2x^6y^2} = \ \dfrac{2y}{x^4}$

   (j) $(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right) = \ x^6-y^6$

2. Factor completely each of the following:

   (a) $4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 = \ am\,(2n+5m)\,(2a-3b)$

   (b) $a^2x^3 - b^2x - a^2x + b^2x^3 = \ x\left(a^2+b^2\right)(x+1)(x-1)$

   (c) $162a + 162b - 2ax^4 - 2bx^4 = \ 2\left(9+x^2\right)(3+x)(3-x)(a+b)$

3. Factor by grouping.

   (a) $x^2 - 6x + 8 = \ (x-2)(x-4)$

   (b) $3a^2 - 5a - 2 = \ (a-2)(3a+1)$

   (c) $4b^2 - b - 5 = \ (4b-5)(b+1)$

4. Solve each of the following equations. Make sure to check your solutions.

   (a) $\dfrac{2x+1}{5} - \dfrac{5-x}{2} = x-1 \quad -13$

   (b) $-3(2x-1) - (3-7x) = 2(x+1) - (x-1)$ no solution

   (c) $8x^3 = 50x^2 \quad 0, \dfrac{25}{4}$

   (d) $8p^3 = 50p \quad -\dfrac{5}{2},\ 0,$ and $\dfrac{5}{2}$

   (e) $2-(3-x)(2x+5) = (x-1)(2x-1) \quad 7$

   (f) $8a + 2a^2 = 42 \quad 3, -7$

5. Solve each of the following inequalities. Graph the solution set.

   (a) $\dfrac{3-4x}{3} - \dfrac{2x-3}{7} \leq -x+7$ $\quad -9 \leq x$

   (b) $\dfrac{3-2a}{7} > -1$ $\quad 5 > a$

   (c) $3(2x-3)-(5x+4) > -14$ $\quad x > -1$

6. Solve each of the following formulas.

   (a) $A = 2a - 3b$ for a. $\quad a = \dfrac{A+3b}{2}$ or $\dfrac{A}{2}+\dfrac{3b}{2}$ or $\dfrac{1}{2}A+\dfrac{3}{2}b$

   (b) $A = 2a - 3b$ for b. $\quad b = \dfrac{2a-A}{3}$ or $\dfrac{2a}{3}-\dfrac{A}{3}$ or $\dfrac{2}{3}a-\dfrac{1}{3}A$

   (c) $F = \dfrac{mMG}{d^2}$ for m. $\quad m = \dfrac{Fd^2}{MG}$

   (d) $3x - 5y = 60$ for y. $\quad y = \dfrac{3x-60}{5}$ or $\dfrac{3x}{5}-12$ or $\dfrac{3}{5}x-12$

7. Graph the straight lines $5x - 3y = 11$ and $y = -x - 9$ in the same coordinate system.

   (a) Use your graph to find the coordinates of the point where the lines intersect. $(-2,-7)$

   (b) Use algebraic methods to check your answer for part a).
   Solution: Is the point $(-2,-7)$ on the line $5x - 3y = 11$?

   $$\text{LHS} = 5(-2) - 3(-7) = -10 - (-21) = -10 + 21 = 11 = \text{RHS} \implies \text{yes}$$

   Is the point $(-2,-7)$ on the line $y = -x - 9$?

   $$\text{RHS} = -(-2) - 9 = 2 - 9 = -7 = \text{LHS} \implies \text{yes}$$

8. Word Problems.

   (a) A couch went on a 15% sale. The sale price is \$ 697. Find the original price. \$ 820

   (b) The difference between two numbers is 7, their sum is 37. Find these numbers. 15 and 22

   (c) Ann and Betty are roommates. The monthly rent is \$ 950. The amount paid by Ann is \$ 310 less than twice the amount paid by Betty. How much do they each pay for rent? \$ 420 and \$ 530

   (d) The population of a town has decreased from 90 000 to 82 800. What percent of a change does this represent? 8% decrease

   (e) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft. 9 ft and 23 ft

   (f) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is 84 ft^2. 6 ft and 14 ft

Review Problems - Solutions

1. Simplify each of the following.

(a) $(5a-1)^2 = \quad 25a^2 - 10a + 1$

Solution: to square something means to write it down twice and multiply. Then it is a FOIL problem. F stand for first with first, O for outer terms, I for inner terms, and L for last terms.

$$\begin{aligned}(5a-1)^2 &= (5a-1)(5a-1)\\ &= \underbrace{25a^2}_{\text{F}}\ \underbrace{-5a}_{\text{O}}\ \underbrace{-5a}_{\text{I}}\ \underbrace{+1}_{\text{L}} \qquad \text{combine like terms}\\ &= 25a^2 - 10a + 1\end{aligned}$$

(b) $\left(3x^5+4y\right)\left(3x^5-4y\right) = \quad 9x^{10} - 16y^2$

$$\begin{aligned}\left(3x^5+4y\right)\left(3x^5-4y\right) &= \underbrace{9x^{10}}_{\text{F}}\ \underbrace{-12x^5}_{\text{O}}\ \underbrace{+12x^5}_{\text{I}}\ \underbrace{-16y^2}_{\text{L}} \qquad \text{combine like terms}\\ &= 9x^{10} - 16y^2\end{aligned}$$

The expressions $3x^5+4y$ and $3x^5-4y$ are called conjugates. Becuase of the same terms and alternating signs, O and I cacel out when "FOIL"-ing, giving us the difference of two squares.

(c) $\dfrac{3a-8}{8-3a} = \quad -1$

Solution: We need to notice that the numerator and denominator are opposites of each other. Indeed,

$$-1(8-3a) = -8+3a = 3a-8$$

Thus

$$\frac{3a-8}{8-3a} = \frac{-1(8-3a)}{8-3a} = -1$$

(d) $\dfrac{2x+1}{4x^2-1} = \quad \dfrac{1}{2x-1}$

Solution: We factor the denominator via the difference of squares theorem, and then cancel.

$$\frac{2x+1}{4x^2-1} = \frac{2x+1}{(2x+1)(2x-1)} = \frac{1}{2x-1}$$

(e) $\dfrac{ab-a-b+1}{b^2-1} = \quad \dfrac{a-1}{b+1}$

Solution: We will facotr both numerator and denominator and then cancel. The numerator can be factored by grouping

$$\begin{aligned}\underbrace{ab-a}\ \underbrace{-b+1} &= a(b-1) - 1(b-1)\\ &= (a-1)(b-1)\end{aligned}$$

The denominaotr factors by the difference of squares theorem.

$$b^2-1 = (b+1)(b-1)$$

Thus the fraction can be simplified as

$$\frac{ab-a-b+1}{b^2-1} = \frac{(a-1)(b-1)}{(b+1)(b-1)} = \frac{a-1}{b+1}$$

(f) $\dfrac{5x-30}{x^2-36} \cdot \dfrac{3x+18}{5} = \ 3$
Solution: we will factor whatever we can and then cancel.

$$\frac{5x-30}{x^2-36} \cdot \frac{3x+18}{5} = \frac{5(x-6)}{(x+6)(x-6)} \cdot \frac{3(x+6)}{5} = 3$$

(g) $\dfrac{3x}{x-2} - \dfrac{x+4}{x-2} = \ 2$
Solution: This is a subtraction of fractions. The denominators are the same, the only difficulty is that we are subtracting expressions instead of numbers. The second pair of parentheses is essential.

$$\frac{3x}{x-2} - \frac{x+4}{x-2} = \frac{(3x)-(x+4)}{x-2} = \frac{3x-x-4}{x-2} = \frac{2x-4}{x-2} = \frac{2(x-2)}{x-2} = 2$$

(h) $2a^3\left(-2ab^2\right)^3 ab^7 = \ -16a^7b^{13}$
Solution:

$$\begin{aligned}
2a^3\left(-2ab^2\right)^3 ab^7 &= && \text{use rule } (ab)^n = a^nb^n \\
2a^3(-2)^3a^3\left(b^2\right)^3 ab^7 &= && \text{use rule } (a^n)^m = a^{nm} \\
2a^3(-2)^3a^3b^6ab^7 &= && \text{multiplication is commutative and } (-2)^3 = -8 \\
2(-8)a^3a^3ab^6b^7 &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\
&= -16a^7b^{13}
\end{aligned}$$

(i) $\dfrac{(-2x)^2 y^3}{2x^6y^2} = \ \dfrac{2y}{x^4}$
Solution:

$$\begin{aligned}
\frac{(-2x)^2 y^3}{2x^6y^2} &= && \text{use rule } (ab)^n = a^nb^n \\
\frac{(-2)^2 x^2y^3}{2x^6y^2} &= \frac{4x^2y^3}{2x^6y^2} && \text{simplify numbers} \\
\frac{2x^2y^3}{x^6y^2} &= && \text{simplify } x\text{-terms} \\
\frac{2y^3}{x^4y^2} &= && \text{simplify } y\text{-terms} \\
&= \frac{2y}{x^4}
\end{aligned}$$

(j) $(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right) = \ x^6-y^6$
Solution: We apply the law of distributivity

$$(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right) =$$

$$\begin{aligned}
&= x\cdot x^5 + x\cdot x^4y + x\cdot x^3y^2 + x\cdot x^2y^3 + x\cdot xy^4 + x\cdot y^5 \\
&\qquad - y\cdot x^5 - y\cdot x^4y - y\cdot x^3y^2 - y\cdot x^2y^3 - y\cdot xy^4 - y\cdot y^5 \\
&= x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 \\
&\qquad - x^5y - x^4y^2 - x^3y^3 - x^2y^4 - xy^5 - y^6 \\
&= x^6 - y^6
\end{aligned}$$

2. Factor completely each of the following:

   (a) $4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 = \quad am(2n+5m)(2a-3b)$
   Solution:

$$\begin{aligned} 4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 &= \quad \text{the GCF is } am \\ am(4an - 15bm - 6bn + 10am) &= \quad \text{rearrange} \\ am\left(\underbrace{4an - 6bn}\ \underbrace{+10am - 15bm}\right) &= \\ am(2n(2a-3b) + 5m(2a-3b)) &= am(2n+5m)(2a-3b) \end{aligned}$$

   (b) $a^2x^3 - b^2x - a^2x + b^2x^3 = \quad x\left(a^2+b^2\right)(x+1)(x-1)$
   Solution:

$$\begin{aligned} a^2x^3 - b^2x - a^2x + b^2x^3 &= \quad \text{the GCF is } x \\ x\left(a^2x^2 - b^2 - a^2 + b^2x^2\right) &= \quad \text{rearrange} \\ x\left(\underbrace{a^2x^2 - a^2}\ \underbrace{+b^2x^2 - b^2}\right) &= \\ x\left(a^2\left(x^2-1\right) + b^2\left(x^2-1\right)\right) &= x\left(a^2+b^2\right)\left(x^2-1\right) \end{aligned}$$

   We are not done yet since $\left(x^2-1\right) = \left(x^2-1^2\right)$ further factors via the difference of squares theorem. Thus the answer is

$$\begin{aligned} x\left(a^2+b^2\right)\left(x^2-1\right) &= x\left(a^2+b^2\right)\left(x^2-1^2\right) \\ &= x\left(a^2+b^2\right)(x+1)(x-1) \end{aligned}$$

   (c) $162a + 162b - 2ax^4 - 2bx^4 = \quad 2\left(9+x^2\right)(3+x)(3-x)(a+b)$
   Solution:

$$\begin{aligned} 162a + 162b - 2ax^4 - 2bx^4 &= \quad \text{the GCF is } 2 \\ 2\left(\underbrace{81a + 81b}\ \underbrace{-ax^4 - bx^4}\right) &= 2\left(81(a+b) - x^4(a+b)\right) = 2\left(81 - x^4\right)(a+b) \end{aligned}$$

   We are not done yet, since $81 - x^4 = 9^2 - \left(x^2\right)^2$ further factors via the difference of squares theorem.

$$2\left(81-x^4\right)(a+b) = 2\left(9^2 - \left(x^2\right)^2\right)(a+b) = 2\left(9+x^2\right)\left(9-x^2\right)(a+b)$$

   One factor still further factors: $9 - x^2 = 3^2 - x^2 = (3+x)(3-x)$. Thus the final answer is

$$2\left(9+x^2\right)\left(9-x^2\right)(a+b) = 2\left(9+x^2\right)\left(3^2-x^2\right)(a+b) = 2\left(9+x^2\right)(3+x)(3-x)(a+b)$$

3. Factor by grouping.

   (a) $x^2 - 6x + 8 = \quad (x-2)(x-4)$
   Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= 8 && \text{1st coefficient times 3rd coefficient} \\ p+q &= -6 && \text{2nd coefficient} \end{aligned}$$

We start by expressing 8 as a product of two numbers. there are only two pairs, 1 with 8 and 2 with 4. Since the product pq is positive, p and q have to have the same sign. Since the sum $p+q$ is negative, they both have to be negative. We only need to consider -1 with -8 and -2 with -4. Clearly -2 with -4 work as p and q. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} x^2-6x+8 &= \underbrace{x^2-2x}\ \underbrace{-4x+8} \\ &= x(x-2)-4(x-2)=(x-2)(x-4) \end{aligned}$$

We check by multiplication:

$$(x-2)(x-4)=x^2-4x-2x+8=x^2-6x+8$$

Thus our result is correct.

(b) $3a^2-5a-2=$ $(a-2)(3a+1)$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= -6 && \text{1st coefficient times 3rd coefficient} \\ p+q &= -5 && \text{2nd coefficient} \end{aligned}$$

We start by expressing 6 as a product of two numbers. there are only two pairs, 1 with 6 and 2 with 3. Since the product pq is negative, one number must be positive, the other one must be positive.. Since the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -6 and 2 with -3. Clearly 1 with -6 work as p and q. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 3a^2-5a-2 &= \underbrace{3a^2+a}\ \underbrace{-6a-2} \\ &= a(3a+1)-2(3a+1)=(a-2)(3a+1) \end{aligned}$$

We check by multiplication:

$$(a-2)(3a+1)=3a^2+a-6a-2=3a^2-5a-2$$

Thus our result is correct.

(c) $4b^2-b-5=$ $(4b-5)(b+1)$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= -20 && \text{1st coefficient times 3rd coefficient} \\ p+q &= -1 && \text{2nd coefficient} \end{aligned}$$

We start by expressing 20 as a product of two numbers. the possible pairs are, 1 with 20, 2 with 10, and 4 with 5. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -20, 2 with -10, and 4 with -5. Clearly 4 with -5 work as p and q. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 4b^2-b-5 &= \underbrace{4b^2+4b}\ \underbrace{-5b-5} \\ &= 4b(b+1)-5(b+1)=(4b-5)(b+1) \end{aligned}$$

We check by multiplication:

$$(4b-5)(b+1) = 4b^2 + 4b - 5b - 5 = 4b^2 - b - 5$$

Thus our result is correct.

4. Solve each of the following equations. Make sure to check your solutions.

(a) $\dfrac{2x+1}{5} - \dfrac{5-x}{2} = x - 1$ $\quad -13$

(b) $-3(2x-1) - (3-7x) = 2(x+1) - (x-1)$ $\quad$ no solution

Solution: In case of subtracting algebraic expressions, we either subtract the opposite, or distribute -1. We will use the second method here.

$$\begin{aligned} -3(2x-1) - (3-7x) &= 2(x+1) - (x-1) \\ -3(2x-1) - 1(3-7x) &= 2(x+1) - 1(x-1) && \text{distribute} \\ -6x + 3 - 3 + 7x &= 2x + 2 - x + 1 && \text{combine like terms} \\ x &= x + 3 && \text{subtract } x \\ 0 &= 3 \end{aligned}$$

Since x disappeared from the equation, we are left with an unconditional statement, this time unconditionally false. There is no value of x that can make $0 = 3$ be true, and thus this equation has no solution. An equation of this type is called a contradiction.

(c) $8x^3 = 50x^2$ $\quad 0, \dfrac{25}{4}$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8x^3 &= 50x^2 && \text{subtract } 50x^2 \\ 8x^3 - 50x^2 &= 0 && \text{the GCF is } 2x^2 \\ 2x^2(4x-25) &= 0 \end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2x^2 = 0$ or $4x - 25 = 0$. We solve these equations for x.

$$\begin{aligned} 2x^2 &= 0 && \text{or} && 4x - 25 = 0 \\ 2 \cdot x \cdot x &= 0 && \text{or} && 4x = 25 \\ x &= 0 && \text{or} && x = \frac{25}{4} \end{aligned}$$

We check both solutions. If $x = 0$, then

$$\begin{aligned} \text{LHS} &= 8 \cdot 0^3 = 8 \cdot 0 = 0 \\ \text{RHS} &= 50 \cdot 0^2 = 50 \cdot 0 = 0 \quad \checkmark \end{aligned}$$

If $x = \dfrac{25}{4}$, then

$$\begin{aligned} \text{LHS} &= 8\left(\frac{25}{4}\right)^3 = \frac{8}{1} \cdot \frac{15\,625}{64} = \frac{15\,625}{8} \\ \text{RHS} &= 50\left(\frac{25}{4}\right)^2 = \frac{50}{1} \cdot \frac{625}{16} = \frac{15\,625}{8} \quad \checkmark \end{aligned}$$

Thus both solutions, 0 and $\dfrac{25}{4}$ are correct.

(d) $8p^3 = 50p$ $\quad -\frac{5}{2}$, 0, and $\frac{5}{2}$
Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8p^3 &= 50p && \text{subtract } 50p \\ 8p^3 - 50p &= 0 && \text{the GCF is } 2p \\ 2p\left(4p^2 - 25\right) &= 0 \\ 2p\left((2p)^2 - 5^2\right) &= 0 && \text{factor via difference of squares theorem} \\ 2p\,(2p+5)\,(2p-5) &= 0 \end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2p = 0$ or $2p+5 = 0$ or $2p - 5 = 0$. We solve these equations for p.

$$\begin{array}{lllllll} 2p+5 = 0 & \text{or} & 2p-5=0 & \text{or} & 2p=0 \\ 2p = -5 & \text{or} & 2p = 5 & \text{or} & p = 0 \\ p = -\frac{5}{2} & \text{or} & p = \frac{5}{2} \end{array}$$

We check all three solutions. If $p = -\frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8\left(-\frac{5}{2}\right)^3 = \frac{8}{1} \cdot \frac{-125}{8} = -125 \\ \text{RHS} &= 50\left(-\frac{5}{2}\right) = \frac{50}{1} \cdot \frac{-5}{2} = \frac{-250}{2} = -125 \quad \checkmark \end{aligned}$$

If $p = \frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8\left(\frac{5}{2}\right)^3 = \frac{8}{1} \cdot \frac{125}{8} = 125 \\ \text{RHS} &= 50\left(\frac{5}{2}\right) = \frac{50}{1} \cdot \frac{5}{2} = \frac{250}{2} = 125 \quad \checkmark \end{aligned}$$

and if $p = 0$, then

$$\begin{aligned} \text{LHS} &= 8 \cdot 0^3 = 8 \cdot 0 = 0 \\ \text{RHS} &= 50 \cdot 0 = 0 \quad \checkmark \end{aligned}$$

Thus all three solutions, $-\frac{5}{2}$, 0, and $\frac{5}{2}$ are correct.

(e) $2 - (3-x)(2x+5) = (x-1)(2x-1)$ $\quad 7$
Solution: We have to use the FOIL method on both sides to perform the multiplications. It is very important, however, to keep the expressions in a parentheses since we are dealing with

subtraction between *algebraic expressions.*

$$\begin{aligned}
2-(3-x)(2x+5) &= (x-1)(2x-1) && \text{FOIL}\\
2-\left(6x+15-2x^2-5x\right) &= 2x^2-x-2x+1 && \text{combine like terms}\\
2-\left(-2x^2+x+15\right) &= 2x^2-3x+1 && \text{perform subtraction}\\
2+2x^2-x-15 &= 2x^2-3x+1 && \text{combine like terms}\\
2x^2-x-13 &= 2x^2-3x+1 && \text{subtract } 2x^2 \quad \text{(the equation is linear!)}\\
-x-13 &= -3x+1 && \text{add } 3x\\
2x-13 &= 1 && \text{add } 13\\
2x &= 14 && \text{divide by } 2\\
x &= 7
\end{aligned}$$

We check: if $x=7$, then

$$\begin{aligned}
\text{LHS} &= 2-(3-7)(2(7)+5)=2-(-4)(14+5)=2-(-4)19=2-(-76)=78\\
\text{RHS} &= (7-1)(2(7)-1)=6(14-1)=6\cdot 13=78 \quad \checkmark
\end{aligned}$$

Thus our solution, 7 is correct.

(f) $8a+2a^2=42$ $3,-7$
Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned}
8a+2a^2 &= 42 && \text{subtract 42, rearrange}\\
2a^2+8a-42 &= 0 && \text{the GCF is } 2\\
2\left(a^2+4a-21\right) &= 0
\end{aligned}$$

We will factor $a^2+4a-21$ by grouping. First we conduct the "pq−game".

$$\begin{aligned}
pq &= -21 && \text{1st coefficient times 3rd coefficient}\\
p+q &= 4 && \text{2nd coefficient}
\end{aligned}$$

We start by expressing 21 as a product of two numbers. the only possible pairs are, 1 with 21 and 3 with 7. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p+q$ is positive, the negative sign has to be in front of the smaller number. We only need to consider -1 with 20, and -3 with 7. Clearly -3 with 7 work as p and q. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned}
2\left(a^2+4a-21\right) &= 0\\
2\left(\underbrace{a^2+7a}\ \ \underbrace{-3a-21}\right) &= 0\\
2\left(a(a+7)-3(a+7)\right) &= 0\\
2(a-3)(a+7) &= 0
\end{aligned}$$

Thus our equation is

$$2(a-3)(a+7)=0$$

We now apply the special zero property. If this product is zero, then either $2=0$ or $a-3=0$ or $a+7=0$. We solve these equations for a.

$$\begin{aligned} a-3 &= 0 \quad \text{or} \quad a+7=0 \quad \text{or} \quad 2=0 \\ a &= 3 \quad \text{or} \quad a=-7 \quad \text{or} \quad \text{no solution here} \end{aligned}$$

We check both solutions. If $a=3$, then

$$\text{LHS} = 8(3) + 2(3)^2 = 8\cdot 3 + 2\cdot 9 = 24 + 18 = 42 = \text{RHS} \quad \checkmark$$

If $a=-7$, then

$$\text{LHS} = 8(-7) + 2(-7)^2 = 8\cdot(-7) + 2\cdot 49 = -56 + 98 = 42 = \text{RHS} \quad \checkmark$$

Thus both solutions, -7 and 3 are correct.

5. Solve each of the following inequalities. Graph the solution set.

(a) $\dfrac{3-4x}{3} - \dfrac{2x-3}{7} \leq -x+7$ $\quad -9 \leq x$

(b) $\dfrac{3-2a}{7} > -1$ $\quad 5 > a$

(c) $3(2x-3) - (5x+4) > -14$ $\quad x > -1$

6. Solve each of the following formulas.

(a) $A = 2a - 3b$ for a. $\quad a = \dfrac{A+3b}{2}$ or $\dfrac{A}{2} + \dfrac{3b}{2}$ or $\dfrac{1}{2}A + \dfrac{3}{2}b$

Solution:

$$\begin{aligned} A &= 2a - 3b && \text{add } 3b \\ A + 3b &= 2a && \text{divide by 2} \\ \frac{A+3b}{2} &= a \end{aligned}$$

Thus $a = \dfrac{A+3b}{2}$ or $\dfrac{A}{2} + \dfrac{3b}{2}$ or $\dfrac{1}{2}A + \dfrac{3}{2}b$

(b) $A = 2a - 3b$ for b. $\quad b = \dfrac{2a-A}{3}$ or $\dfrac{2a}{3} - \dfrac{A}{3}$ or $\dfrac{2}{3}a - \dfrac{1}{3}A$

Solution:

$$\begin{aligned} A &= 2a - 3b && \text{add } 3b \\ A + 3b &= 2a && \text{subtract } A \\ 3b &= 2a - A && \text{divide by 3} \\ b &= \frac{2a-A}{3} \quad \text{or} \quad \frac{2a}{3} - \frac{A}{3} \quad \text{or} \quad \frac{2}{3}a - \frac{1}{3}A \end{aligned}$$

(c) $F = \dfrac{mMG}{d^2}$ for m. $\quad m = \dfrac{Fd^2}{MG}$

Solution:

$$\begin{aligned} F &= \frac{mMG}{d^2} && \text{multiply by } d^2 \\ Fd^2 &= mMG && \text{divide by } MG \\ \frac{Fd^2}{MG} &= m \end{aligned}$$

(d) $3x - 5y = 60$ for y. $y = \dfrac{3x-60}{5}$ or $\dfrac{3x}{5} - 12$ or $\dfrac{3}{5}x - 12$

Solution:

$$\begin{aligned} 3x - 5y &= 60 && \text{add } 5y \\ 3x &= 5y + 60 && \text{subtract } 60 \\ 3x - 60 &= 5y && \text{divide by } 5 \\ \frac{3x-60}{5} &= y \end{aligned}$$

The answer is $y = \dfrac{3x-60}{5}$ or $\dfrac{3x}{5} - 12$ or $\dfrac{3}{5}x - 12$

7. Graph the straight lines $5x - 3y = 11$ and $y = -x - 9$ in the same coordinate system.

(a) Use your graph to find the coordinates of the point where the lines intersect. $(-2, -7)$

Solution: We first graph the line $5x - 3y = 11$.

$$\begin{aligned} \text{If } x &= 1,\ y = ? && \text{substitute } x = 1 \text{ into the equation of the line} \\ 5(1) - 3y &= 11 && \text{solve for } y \\ 5 - 3y &= 11 && \text{subtract } 5 \\ -3y &= 6 && \text{divide by } -3 \\ y &= -2 && \Longrightarrow \text{ we found } (1, -2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 4,\ y = ? && \text{substitute } x = 4 \text{ into the equation of the line} \\ 5(4) - 3y &= 11 && \text{solve for } y \\ 20 - 3y &= 11 && \text{subtract } 20 \\ -3y &= -9 && \text{divide by } -3 \\ y &= 3 && \Longrightarrow \text{ we found } (4, 3) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 7,\ y = ? && \text{substitute } x = 7 \text{ into the equation of the line} \\ 5(7) - 3y &= 11 && \text{solve for } y \\ 35 - 3y &= 11 && \text{subtract } 35 \\ -3y &= -24 && \text{divide by } -3 \\ y &= 8 && \Longrightarrow \text{ we found } (7, 8) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -2,\ y = ? && \text{substitute } x = -2 \text{ into the equation of the line} \\ 5(-2) - 3y &= 11 && \text{solve for } y \\ -10 - 3y &= 11 && \text{add } 10 \\ -3y &= 21 && \text{divide by } -3 \\ y &= -7 && \Longrightarrow \text{ we found } (-2, -7) \end{aligned}$$

We graph the points $(1, -2)$, $(4, 3)$, $(7, 8)$, and $(-2, -7)$ and connect the points. (Green line.)

We now graph the line $y = -x - 9$.

$$\begin{aligned} \text{If } x &= 0,\ y = ? && \text{substitute } x = 0 \text{ into the equation of the line} \\ y &= -(0) - 9 = 0 - 9 = -9 && \Longrightarrow \text{ we found } (0, -9) \end{aligned}$$

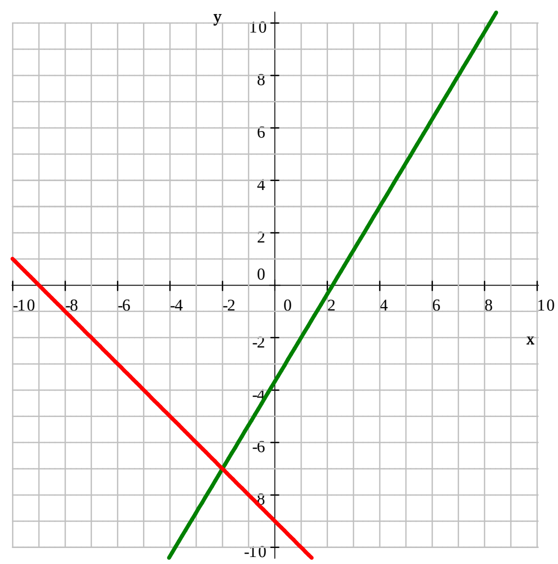
$$\begin{aligned} \text{If } x &= 2,\ y = ? && \text{substitute } x = 2 \text{ into the equation of the line} \\ y &= -(2) - 9 = -11 && \Longrightarrow \text{ we found } (2, -11) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -2,\ y = ? && \text{substitute } x = -2 \text{ into the equation of the line} \\ y &= -(-2) - 9 = 2 - 9 = -7 && \Longrightarrow \text{ we found } (-2, -7) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -4,\ y = ? && \text{substitute } x = -4 \text{ into the equation of the line} \\ y &= -(-4) - 9 = 4 - 9 = -5 && \Longrightarrow \text{ we found } (-4, -5) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -7,\ y = ? && \text{substitute } x = -7 \text{ into the equation of the line} \\ y &= -(-7) - 9 = 7 - 9 = -2 && \Longrightarrow \text{ we found } (-7, -2) \end{aligned}$$

We graph the points $(0, -9)$, $(2, -11)$, $(-2, -7)$, $(-4, -5)$, and $(-7, -2)$ and connect the points. (Red line.)



We read from the graph that the lines intersect at $(-2, -7)$

(b) Use algebraic methods to check your answer for part a).
Solution: Is the point $(-2, -7)$ on the line $5x - 3y = 11$?

$$\text{LHS} = 5(-2) - 3(-7) = -10 - (-21) = -10 + 21 = 11 = \text{RHS} \implies \text{yes}$$

Is the point $(-2, -7)$ on the line $y = -x - 9$?

$$\text{RHS} = -(-2) - 9 = 2 - 9 = -7 = \text{LHS} \implies \text{yes}$$

8. Word Problems.

(a) A couch went on a 15% sale. The sale price is \$ 697. Find the original price. \$ 820

(b) The difference between two numbers is 7, their sum is 37. Find these numbers. 15 and 22
Solution: Let us call the smaller number x. Then the larger number is $x+7$, since the difference between the two numbers is 7. The equation then expresses the sum of these numbers

$$\begin{aligned} \underbrace{x}_{\text{smaller number}} + \underbrace{x+7}_{\text{larger number}} &= 37 && \text{solve for } x \\ 2x+7 &= 37 && \text{subtract 7} \\ 2x &= 30 && \text{divide by 2} \\ x &= 15 \end{aligned}$$

Thus the smaller number, labeled x is 15. The larger number was labeled $x+7$, so it must be $15+7=22$. Thus the numbers are 15 and 22. We check: the difference between 22 and 15 is $22-(15)=7$, and their sum is indeed $22+15=37$. Thus our solution is correct.

(c) Ann and Betty are roommates. The monthly rent is \$ 950. The amount paid by Ann is \$ 310 less than twice the amount paid by Betty. How much do they each pay for rent? \$ 420 and \$ 530
Solution: Let x denote the amount that is paid by Betty. Then Ann must pay $2x-310$ per month. The equation expresses the monthly rent as the sum of the two payments

$$\begin{aligned} \underbrace{x}_{\text{smaller amount}} + \underbrace{2x-310}_{\text{larger amount}} &= 950 && \text{solve for } x \\ 3x-310 &= 950 && \text{add 310} \\ 2x &= 1260 && \text{divide by 3} \\ x &= 420 \end{aligned}$$

Thus the amount paid by Betty is \$ 420. The amount paid by Betty was labeled as $2x-310$, so Betty must pay $2(420)-310=840-310=530$. Thus the payments are \$ 420 and \$ 530. We check: \$ 530 = 2 (\$ 420) − \$ 310 and \$ 420 + \$ 530 = \$ 950. Thus our solution is correct.

(d) The population of a town has decreased from 90 000 to 82 800. What percent of a change does this represent? 8% decrease

(e) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft. 9 ft and 23 ft
Solution: Let us denote the shorter side by x. Then the other side is $3x-4$. The equation expresses the perimeter of the rectangle.

$$\begin{aligned} 2(x)+2(3x-4) &= 64 && \text{multiply out parentheses} \\ 2x+6x-8 &= 64 && \text{combine like terms} \\ 8x-8 &= 64 && \text{add} \\ 8x &= 72 && \text{divide by 8} \\ x &= 9 \end{aligned}$$

If the shorter side was denoted by x, we now know it is 9 ft. The longer side was denoted by $3x-4$, so it must be $3(9\text{ ft})-4\text{ ft}=23\text{ ft}$. Thus the sides of the rectangle are 9 ft and 23 ft. We check: $P=2(9\text{ ft})+2(23\text{ ft})=64\text{ ft}$ and $23\text{ ft}=3(9\text{ ft})-4\text{ ft}$. Thus our solution is correct.

(f) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is 84 ft^2. 6 ft and 14 ft

Solution: Let us denote the shorter side by x. Then the other side is $3x-4$. The equation expresses the area of the rectangle.

$$\begin{aligned} x(3x-4) &= 84 \quad \text{multiply out parentheses} \\ 3x^2-4x &= 84 \quad \text{subtract 84} \\ 3x^2-4x-84 &= 0 \end{aligned}$$

Because the equation is quadratic, we need to factor the left-hand side and then apply the zero property. We will factor by grouping. First we conduct the "$pq-$game". The sum of p and q has to be the linear coefficient (the number in front of x, with its sign), so it is -4. The product of p and q has to be the product of the other coefficients, $3(-84)=-252$.

$$\begin{aligned} pq &= -252 \\ p+q &= -4 \end{aligned}$$

Now we need to find p and q. Because the product is negative, we re looking for a positive and a negative number. Becuse the sum is negtive, the larger number must carry the negative sign. We enter $\sqrt{252}=15.874\,507\,87$ into the calculator and get a decimal:

$$\sqrt{252}=15.874....$$

So we start looking for factors of 252, starting at 15, and moving down. We soon find 14 and -18. These are our values for p and q. We use these numbers to express the linear term:

$$-4x=14x-18x$$

and factor by grouping.

$$\begin{aligned} 3x^2-4x-84 &= 0 \\ \underbrace{3x^2+14x}\underbrace{-18x-84} &= 0 \\ x(3x+14)-6(3x+14) &= 0 \\ (x-6)(3x+14) &= 0 \end{aligned}$$

We now apply the zero property. Either $x-11=0$ or $3x+14=0$. We solve both these equations for x.

$$\begin{aligned} x-6 &= 0 \\ x &= 6 \end{aligned}$$

and

$$\begin{aligned} 3x+14 &= 0 \\ 3x &= -14 \\ x &= -\frac{14}{3} \end{aligned}$$

Since distances can not be negative, the second solution for x, $-\dfrac{14}{3}$ is ruled out. Thus $x=6$. Then the longer side is $3(6 \text{ ft})-4 \text{ ft}=14 \text{ ft}$, and so the rectangle's sides are 6 ft and 14 ft long. We check: $6 \text{ ft}(14 \text{ ft})=84 \text{ ft}^2$ and $14 \text{ ft}=3(6 \text{ ft})-4 \text{ ft}$. Thus our solution is correct.