This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$. Find each of the following sets.

   a) $A = \{x \in U : x \text{ is divisible by } 3 \text{ or } x \text{ is odd}\}$
   b) $B = \{x \in U : x > 6 \text{ is divisible by } 3 \text{ and } x \text{ is odd}\}$
   c) $C = \{x \in U : x \text{ is divisible by } 4\}$
   d) $A \cap C$
   e) $B \cup C$

2. Perform each of the given operations.

   a) $(-\infty, 5) \cup (-\infty, 10]$ c) $(3, 8) \cap [4, 11]$ e) $(-\infty, 9) \cup [2, \infty)$ g) $(-\infty, 1] \cap (3, \infty)$
   b) $(-\infty, 5) \cap (-\infty, 10]$ d) $(3, 8) \cup [4, 11]$ f) $(-\infty, 9) \cap [2, \infty)$ h) $(-\infty, 1] \cup (3, \infty)$

3. There are 50 students in the senior class. 35 students are taking English, 28 are taking French, and 15 are taking both English and French. How many students are taking neither English, nor French?

4. Label each of the following statements as true or false.

   a) If n is an integer such that n^2 is divisible by 20, then n^2 is divisible by 400.
   b) If n is an integer greater than 1, then all exponents in the prime-factorization of n^2 are even.
   c) If integer n is divisible by 4 and by 6, then it is also divisible by 24.
   d) The square of an odd number is always odd.
   e) If A and B are sets such that $A \subseteq B$, then $A \cup B = A$.
   f) If A and B are sets such that $A \subseteq B$, then $A \cup B = B$.
   g) There is no even prime number.

5. Find the prime factorization for each of the following numbers.

   a) $10!$ b) 2016^{100} c) 2009 d) 219 615

   Note: $10!$ (read as ten factorial is notation for $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$)

6. Find the last digit of $7^{99} + 7^{100} + 7^{101} + 7^{102}$.

7. Suppose that $x = 12\,500\,000$ and $y = 45\,000\,000\,000\,000$.

   a) Express x and y using scientific notation.
   b) Compute each of the following. Present your answer using scientific notation.

      i) xy ii) x^2 iii) $\frac{y}{x}$ iv) $\sqrt{\frac{10y}{x}}$

8. Consider the expression $\frac{-2x^2 + 3x - 1}{-2x + 1}$. Compute the value of the expression with each of the values of x given.

   a) $x = 5$ b) $x = -2$ c) $x = \frac{1}{3}$ d) $x = -\frac{1}{2}$ e) $x = \frac{1}{2}$ f) 1

9. Consider the rule $(a^n)^m = a^{nm}$. Simplify $\left(x^{10}\right)^2$. Based on this result, what is $\sqrt{x^{20}}$?

10. Simplify each of the given expressions.

    a) $\frac{6^{2x+1}}{4^{x-1} \cdot 3^{2x+1}}$ b) $\frac{\left(-2x^4y\right)^3 \left(-xy^2\right)}{\left(-2xy^2x^3\right)^2}$ c) $-(-x)\left(-x^2\right)(-x)^3$

11. Simplify each of the given expressions.

    a) $(5x - 2)(-x + 8)$ c) $(3x - 1)^3$ e) $(-2x + 5)^2 - (x - 3)(3x + 1)$ g) $\left(3x^5 + 2\right)\left(3x^5 - 2\right)$
    b) $(3x - 1)^2$ d) $\left(2x^4 + 1\right)^2 - \left(2x^4 - 1\right)^2$ f) $\left(x^2 - 2\right)\left(x^2 + 1\right)\left(x^2 - 5\right)$

12. Solve each of the following equations.

a) $5x-3=-38$

b) $\frac{2}{3}x-\frac{3}{4}=-\frac{5}{12}$

c) $\frac{x-3}{7}=-2$

d) $\frac{x-\frac{5}{6}}{-\frac{3}{8}}=\frac{4}{9}$

e) $3(x+8)=-15$

f) $\frac{3}{8}\left(x-\frac{2}{5}\right)=-\frac{3}{2}$

g) $3(2x-5)-2(5x+3)=3x$

h) $\frac{1}{2}\left(6x-\frac{2}{3}\right)-\frac{5}{6}\left(12x+\frac{1}{2}\right)=-\frac{31}{4}$

i) $\frac{3}{4}x-\frac{1}{2}\left(\frac{2}{3}x-\frac{3}{5}\right)=\frac{1}{20}$

j) $(2x-3)^2-2x(x-5)=3-(x-1)(-2x+3)$

k) $(x+3)^2-(x-1)^2=8(x-1)$

l) $\frac{\frac{2x-5}{3}+1}{-2}-1=5$

13. Solve each of the following inequalities. Present your answer using interval notation.

a) $3(2x-5)-4(5x-3)\geq 3(2x-1)$

b) $\frac{3x+2}{5}-\frac{5x-2}{4}>3-x$

c) $(2x+1)^2-3x(x-2)<x^2+8x+9$

d) $-\frac{2}{3}x+\frac{4}{5}\leq\frac{7}{15}$

14. If we increase the side of a square by 2 units, its area will increase by 20 unit^2. How long are the sides of this square?

15. The sum of three consecutive multiples of 5 is 105. Find the value of these numbers.

16. The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost \$6, adult tickets cost \$20. The number of children ticket purchased was three more than twice the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of \$274 dollars?

17. One side of a rectangle is five meters shorter than seven times the other side. Find the length of the shorter side if we also know that the perimeter of the rectangle is 278 meters.

Answers

1. a) $\{1, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$ b) $\{3, 9, 15\}$ c) $\{4, 8, 12\}$ d) $\{12\}$ e) $\{3, 4, 8, 9, 12, 15\}$
2. a) $(-\infty, 5)$ b) $(-\infty, 10]$ c) $[4, 8)$ d) $(3, 11]$ e) $(-\infty, \infty)$ f) $[2, 9)$ g) $\varnothing$ h) $(-\infty, 1] \cup (3, \infty)$
3. 2
4. a) false b) true c) false d) true e) false f) true g) false
5. a) $2^8 \cdot 3^4 \cdot 5^2 \cdot 7$ b) $2^{500} \cdot 3^{200} \cdot 7^{100}$ c) $7^2 \cdot 41$ d) $3 \cdot 5 \cdot 11^4$
6. 0
7. a) $x = 1.25 \cdot 10^7$ and $y = 4.5 \cdot 10^{13}$
   b) i) $5.625 \cdot 10^{20}$ ii) $1.5625 \cdot 10^{14}$ iii) $3.6 \cdot 10^6$ iv) $6 \cdot 10^3$
8. a) 4 b) -3 c) $-\frac{2}{3}$ d) $-\frac{3}{2}$ e) undefined f) 0
9. x^{10}
10. a) 48 b) $2x^5y$ c) x^6
11. a) $-5x^2 + 42x - 16$ b) $9x^2 - 6x + 1$ c) $27x^3 - 27x^2 + 9x - 1$ d) $8x^4$ e) $x^2 - 12x + 28$
    f) $x^6 - 6x^4 + 3x^2 + 10$ g) $9x^{10} - 4$
12. a) -7 b) $\frac{1}{2}$ c) -11 d) $\frac{2}{3}$ e) -13 f) $-\frac{18}{5}$ g) -3 h) 1 i) $-\frac{3}{5}$ j) -1 k) no solution l) -17
13. a) $(-\infty, 0]$ b) $(6, \infty)$ c) $(-\infty, 4)$ d) $\left[\frac{1}{2}, \infty\right)$
14. 4 units
15. $30, 35, 40$
16. 19 children tickets and 8 adult tickets
17. $18\,\mathrm{m}$ by $121\,\mathrm{m}$

