This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Simplify each of the following.

   a) $\dfrac{2^{-1}+5^{-1}\cdot 3}{2^{-1}-5^{-1}\cdot 3}$ b) $-\dfrac{2}{5}+\dfrac{1}{4}\left(-\dfrac{3}{2}\right)^2$ c) $1-\dfrac{2}{3-\dfrac{4}{5-\dfrac{1}{6}}}$ d) $\dfrac{1}{3}-\left(\left(-\dfrac{2}{3}\right)^2-\dfrac{5}{6}\right)$

2. Let $U=\{1,2,3,4,5,6,7,8,9,10\}$. Find each of the given sets.

   a) $\{x\in U : x\geq 8 \text{ and } x \text{ is even}\}$
   b) $\{x\in U : x\geq 8 \text{ or } x \text{ is even}\}$
   c) $\{x\in U : x<3 \text{ and } x>7\}$
   d) $\{x\in U : x<3 \text{ or } x>7\}$
   e) $\{x\in U : x\geq 4 \text{ and } x>6\}$
   f) $\{x\in U : x\geq 4 \text{ or } x>6\}$

3. Compute the prime factorization of each of the following.

   a) 2500 b) 72^{50}

4. Use the prime factorization to find the greatest common factor and least common multiple of 72 and 960.

5. Label each of the following statements as true or false.

   a) Every positive integer has an even number of factors.
   b) The product of two consecutive integers is always even.
   c) A degree three equation can have up to three different solutions.
   d) If we square an odd number and then subtract one, the result is always divisible by 4.
   e) There is no prime number that is divisible by 5.
   f) If n is a perfect square, then all exponents in the prime-factorization of n are even.
   g) If A and B are any sets such that $A\cup B=A$, then $A=B$.
   h) If A and B are any sets such that $A\cup B=A$, then $B\subseteq A$.

6. Simplify each of the following. Express your answer using only positive exponents.

   a) $x^6\cdot x^{-7}$ c) $\left(-x^6\right)\cdot\left(-x^{-7}\right)$ e) $\left(2xy^{-3}\right)^{-3}$ g) $\left(x^{-3}\right)^{-2}$ i) $\left(-5a^{-2}b\right)^2$

   b) $(-x)^6\cdot(-x)^{-7}$ d) $\left(x^6\right)^{-7}$ f) $\dfrac{\left(x^3\right)^{-8}}{x^3\cdot x^{-8}}$ h) $\sqrt{x^8}$ j) $\left(-5a^{-2}b\right)^3$

   k) $\dfrac{\left(2x^7y^{-3}\right)^{-2}\left(-2xy^{-2}x^4\right)^3}{\left(-x^3y^{-2}\right)^{-4}}$ l) $\dfrac{2a^{-3}\left(-2a^{-1}b\right)^3\left(-a^2b^5\right)^{-2}}{4b^{-5}\left(-2a^{-6}b\right)^2}$

7. Suppose that $x=2\,500\,000\,000$ and $y=0.000\,004$. Write each of the following in scientific notation.

   a) x b) y c) xy^2 d) x^2y^3 e) $\dfrac{1}{y}$ f) $\sqrt{y}$

8. Simplify each of the following.

   a) $(2x+3)+(5x-1)$
   b) $(2x+3)-(5x-1)$
   c) $-2(2x+3)-8(5x-1)$
   d) $(2x+3)(5x-1)$
   e) $2x+3(5x-1)$
   f) $(2x+3)5x-1$
   g) $(2(x+3)-5)(x-1)$
   h) $(2x+3)^2-(5x-1)^2$
   i) $(x-2)^3$

9. Completely factor each of the following.

   a) $3x^2 - 48x$

   b) $x(x-2) + 3(x-2)$

   c) $-5x^3y^3 + 10x^4y^2 - 25x^2y^4$

   d) $100x - 10x^2$

   e) $x^2 + 6x$

   f) $12x^2(y+1) - 8x(y+1) + 4x(y+1)$

   g) $15x^2 + 20x - 6x - 8$

   h) $5x^2 + 5x - x - 1$

10. Solve each of the following equations. Make sure to check your solutions.

    a) $2x - 5(x-3) = (x+1)^2 - (x-2)^2$

    b) $6x + x^2 = 0$

    c) $x(x+1)(3x-7)(x+5)^2 = 0$

    d) $-x(x-2) + 3(x-1)^2 = 3$

    e) $\frac{1}{2}(x-3) + \frac{1}{2}(x+1) = 3x - 1$

    f) $\frac{2}{3}(x-1) - \frac{1}{2}(x+5) = \frac{1}{6}(x-2)$

    g) $\frac{3}{4}x - \frac{2}{5} - \left(\frac{1}{2} - \frac{x}{4}\right) = x - \frac{9}{10}$

    h) $x^5 = 36x^4$

    i) $3x^2 + 2x - 6x - 4 = 0$

11. Solve each of the following inequalities.

    a) $\frac{2x-1}{3} - \frac{x-1}{2} \geq -x + 6$ b) $(x-3)^2 - (2x+1)^2 \geq 8 - 3x^2$ c) $-\frac{3}{5}x + \frac{1}{2} < \frac{2}{5}$

12. Expand each of the following.

    a) $(a-b)(a+b)$ b) $(a-b)\left(a^2+ab+b^2\right)$ c) $(a-b)\left(a^3+a^2b+ab^2+b^3\right)$

13. Write an equation that

    a) is of degree 3 and 2, -5, and 3 are its only solutions.

    b) is of degree 5 and 2, -5, and 3 are its only solutions.

    c) is of degree 3 and 2, -5, 0, and 3 are its only solutions.

    d) is of degree 10 and -1 is its only solution.

14. In a class of 30, students discuss their summer activities. 20 students report that they traveled to other cities or countries in the summer. 18 students report that they visited local beaches or waterparks. If 7 students claimed to do none of the above, how many students did both traveling and waterparks? (Hint: draw a Venn diagram!)

15. $A(-3,-1)$, $B(-5,4)$, and $C(5,8)$ are three vertices of a rectangle. Find the fourth vertex of the rectangle.

16. One side of a rectangle is ten feet shorter than five times another side. Find the sides of the rectangle if we also know that its perimeter is 52 ft.

17. If we increase the side of a square by 2 units, its area will increase by 12 unit^2. How long are the sides of the original square?

18. If we square a number, we get four times the opposite of twice the number. Find this number.

19. The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost \$11, adult tickets cost \$19. The number of children ticket purchased was five more than four times the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of \$685 dollars?

Answers

1. a) -11 b) $\frac{13}{80}$ c) $\frac{5}{63}$ d) $\frac{13}{18}$
2. a) $\{8,10\}$ b) $\{2,4,6,8,9,10\}$ c) $\varnothing$ d) $\{1,2,8,9,10\}$ e) $\{7,8,9,10\}$ f) $\{4,5,6,7,8,9,10\}$
3. a) $2^2 \cdot 5^4$ b) $2^{150} \cdot 3^{100}$
4. 24 and 2880
5. a) false b) true c) true d) true e) false f) true g) false h) true
6. a) $\frac{1}{x}$ b) $-\frac{1}{x}$ c) $\frac{1}{x}$ d) $\frac{1}{x^{42}}$ e) $\frac{y^9}{8x^3}$ f) $\frac{1}{x^{19}}$ g) x^6 h) x^4 i) $\frac{25b^2}{a^4}$ j) $-\frac{125b^3}{a^6}$ k) $\frac{-2x^{13}}{y^8}$ l) $-\frac{a^2}{b^4}$
7. a) $2.5 \cdot 10^9$ b) $4 \cdot 10^{-6}$ c) $4 \cdot 10^{-2}$ d) $4 \cdot 10^2$ e) $2.5 \cdot 10^5$ f) $2 \cdot 10^{-3}$
8. a) $7x+2$ b) $-3x+4$ c) $-44x+2$ d) $10x^2+13x-3$ e) $17x-3$ f) $10x^2+15x-1$ g) $2x^2-x-1$ h) $-21x^2+22x+8$ i) $x^3-6x^2+12x-8$
9. a) $3x\,(x-16)$ b) $(x+3)\,(x-2)$ c) $5x^2y^2\left(-xy+2x^2-5y^2\right)$ d) $10x\,(-x+10)$ e) $x\,(x+6)$ f) $4x\,(3x-1)\,(y+1)$ g) $(5x-2)\,(3x+4)$ h) $(5x-1)\,(x+1)$
10. a) 2 b) $0,-6$ c) $0,-1,-5,\frac{7}{3}$ d) $0,2$ e) 0 f) There is no solution. g) All numbers are solution. h) $0,36$ i) $-\frac{2}{3},2$
11. a) $[5,\infty)$ b) $(-\infty,0]$ c) $\left(\frac{1}{6},\infty\right)$
12. a) a^2-b^2 b) a^3-b^3 c) a^4-b^4
13. a) $(x-2)\,(x+5)\,(x-3)=0$ b) answers may vary $(x-2)^2\,(x+5)^2\,(x-3)=0$ c) not possible d) $(x+1)^{10}=0$
14. 15
15. $(7,3)$
16. 6 ft by 20 ft
17. 2 units
18. $0,-8$
19. 45 chidren tickets and 10 adult tickets

