This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Solve each of the following system of equations.

a) $\begin{cases} 2x - y = -1 \\ 5x - 2y = 2 \end{cases}$ b) $\begin{cases} 2x - 5y = -9 \\ x - y = -3 \end{cases}$ c) $\begin{cases} 3x + 5y = -20 \\ \frac{1}{3}x - \frac{1}{2}y = 2 \end{cases}$

d) $\begin{cases} (x-4)^2 + (y+2)^2 + 2 = (x-5)^2 + (y+1)^2 \\ x + 2y = -3 \end{cases}$

2. Label each of the following as true or false.

a) The product of two prime numbers is never a prime number.

b) The sum of two prime numbers is never a prime number.

c) Suppose that x and y are positive integers. If we divide x by 5, the remainder is 2. If we divide y by 5, the remainder is 3. Then $x+y$ is divisible by 5.

d) There are five different one-digit prime numbers.

3. Simplify each of the following.

a) $\dfrac{2-3^{-1}}{3-2^{-1}}$

b) $\dfrac{\left(\frac{1}{2}\right)^{-2} - \left(\frac{2}{3}\right)^{-1}}{5^{-1} - 2^{-1}}$

c) $\left(\dfrac{2a^3b^{-2}}{-3a^{-1}b^6}\right)^0$

d) $(-2)^2(-3)^2 - \left((-3)^2 - (-2)\right)$

e) $\dfrac{-4a^2\left(-2a^{-1}\right)^3}{(2a)^{-1}}$

f) $(-1)^{2017} + (-1)^{2018} + (-1)^{2019}$

g) $\dfrac{3a^2b^{-3}a^{-5}}{-3a^{-2}b^3}$

h) $\left(\dfrac{1}{3} + \dfrac{1}{15}\right) \div \dfrac{11}{25}$

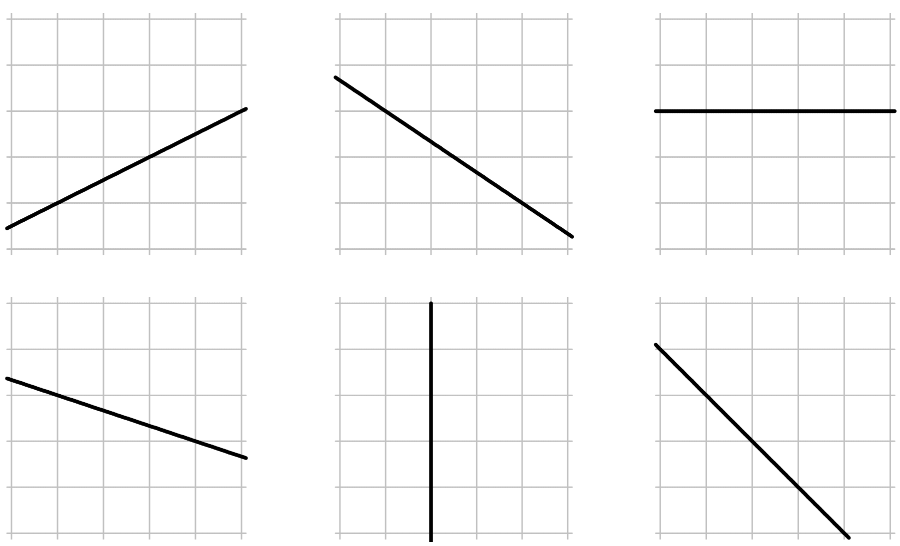
4. Compute the area of the parallelogram determined by the points $A(-3,4)$, $B(21,14)$, $C(21,30)$, and $D(-3,20)$.

5. Find the slope of each of the following lines.

a) the line connecting $A(3,-5)$ and $B(-2,10)$ b) the line $y = -\dfrac{2}{3}x + 6$ c) the line $3x - 4y = 12$

6. Find the slope of each of the graphed lines.

a) b) c)



d) e) f)

7. Find the value of k if we know that the line connecting $A(-1,3)$ and $B(5,k)$ has slope 2.

8. Factor each of the following over the integers.

   a) $x^2 - x$ d) $(a+b-c)^2 - (a-b+c)^2$ g) $x^2 - 6x - 16$ j) $12(x^2-4) + 3x(x^2-4)$

   b) $x^3 - x$ e) $12x^2 - 4x + 3x - 1$ h) $a^2 + 3a - 54$ k*) $x^2 - 6x + 9 - 25c^2$

   c) $9x^2 - 1$ f) $2x^2 + 18$ i) $m^2 - 8m + 15$

9. Solve each of the following equations.

   a) $(x-3)^2 - (2x-5)^2 = -x^2 - (x+1)^2$

   b) $\dfrac{3x+6}{4} - \dfrac{x-1}{3} = x - 4$

   c) $\dfrac{2}{3}(x-5) - \dfrac{3}{2}(x+2) = \dfrac{7}{6}$

   d) $\dfrac{2}{3}(x-1) - \dfrac{3}{4}(x+1) = -2$

   e) $(3 - 2(x - 4(x - 3(x-5)))) = 105$

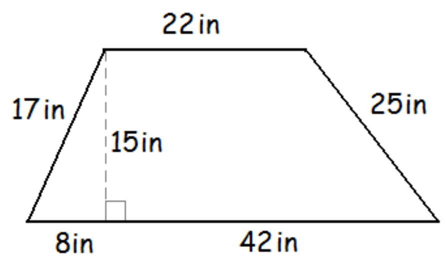
   f) $2x^4 = 8x^2$

   g) $2x^4 = 8x^3$

   h) $(3x-1)(x-3) - x = 3(x-2)^2$

   i) $(x-2)(x+4) = 40$

10. Compute the perimeter and area of the trapezoid shown on the picture.



11. Solve each of the following inequalities.

   a) $\dfrac{4x-1}{5} - \dfrac{x+1}{2} \geq x - 7$ b) $\dfrac{2}{5}x - 1 \leq \dfrac{1}{2}x - \dfrac{3}{4}$

   c) $(x+3)^2 - (x-3)^2 < 60$

12. Graph each of the given lines. a) $5x - 4y = -12$ b) $y = -\dfrac{2}{3}x + 1$

13. We invested a total of $\$4000$ in two bank accounts. One account earns an annual interest of 3%, the other account earns an annual interest of 5%. How much was invested into each account if the combined interest from the two accounts after one year was $\$183$?

14. We invested a total of $\$4000$ in two different stocks. After one year, one stock earned a 7% profit but the other stock sufffered a loss of 3%. How much was invested into each stock if the combined gain from the two stocks after one year was $\$50$?

15. We have 240 coins, all dimes and nickels. How many dimes and how many nickels do we have if the value of all coins is $\$18.80$?

16. The sum of four consecutive even numbers is 76. Find these numbers.

17. The square of a number is 60 greater than four times the number. Find this number.

18. In the fourth period, teachers took attendance in Springfield High School. Some of the female students had a field trip today. We know that if 15% of all girls left for a field trip, the total attendance school-wide drops by 10%. What percent of the students is female?

19. We eject a small object upward from the top of a $768\,\mathrm{ft}$ tall building. The vertical position (or height) h of the object, (measured in feet) t seconds after we threw it is $h = -16t^2 + 128t + 768$. How long does it take for the object to hit the ground?

20. Children's tickets cost $\$12$ each and adults' tickets cost $\$18$ each. We purchased 50 tickets for a total of $\$642$. How many of each tickets did we buy?

21. One side of a rectangle is one foot shorter than twice another side. Compute the area of the rectangle if its perimeter is $46\,\mathrm{ft}$.

22. Find all numbers for which the following is true: the opposite of the number is equal to the square of the number.

23. One number is four less than another. Find these numbers if their product is 96.

24. If we increase all sides of a square by 1 feet, the square's area will increase by $15\,\mathrm{ft}^2$. How long are the sides of the original square?

25. The farm raised chickens and cows. If all the animals together had 82 heads and 288 legs, how many chickens and how many cows are there on the farm?

26. One side of a rectangle is $24\,\mathrm{cm}$ shorter than four times another side. Find the length of the sides if the area of the rectangle is $160\,\mathrm{cm}^2$.

27. Ann and Betsy are friends. One day Betsy tells Ann: "*If you gave me five dollars, we would end up with the same amount of money.*" Ann responds: "*Yes, but if you gave me five dollars instead, then I would have twice as much money as you.*" How much money did they have?

28. Which number has more divisors, 2^{100} or 3^{100}? (Hint: if you do not know the answer with 100 for exponents, try 1, 2, 3, 4, etc and the idea will come!)

29. Simplify each of the following.

a) $\dfrac{3\cdot 2^{2020}-2^{2019}}{2^{2018}}$ b) $2^{2018}+2^{2018}$

Answers

1. a) $(4,9)$ b) $(-2,1)$ c) $(0,-4)$ d) $(7,-5)$ 2. a) true b) false c) true d) false

3. a) $\dfrac{2}{3}$ b) $-\dfrac{25}{3}$ c) 1 d) 25 e) 64 f) -1 g) $-\dfrac{1}{ab^6}$ h) $\dfrac{10}{11}$

4. $A=384$ unit^2 (Hint: Use horizontal lines to split up the parallelogram to a rectangle and two right triangles.)

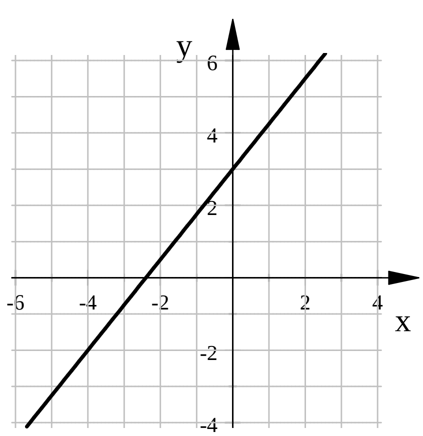
5. a) -3 b) $-\dfrac{2}{3}$ c) $\dfrac{3}{4}$ 6. a) $\dfrac{1}{2}$ b) $-\dfrac{2}{3}$ c) 0 d) $-\dfrac{1}{3}$ e) undefined f) -1 7. 15

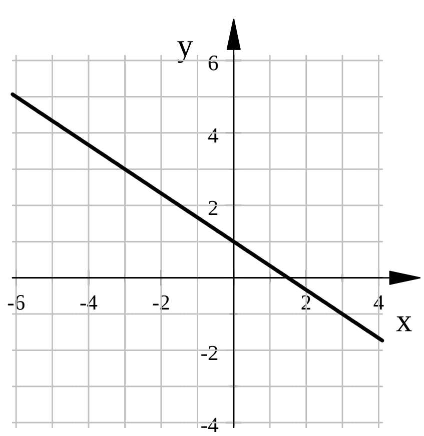
8. a) $x\,(x-1)$ b) $x\,(x-1)\,(x+1)$ c) $(3x-1)\,(3x+1)$ d) $4a\,(b-c)$ e) $(3x-1)\,(4x+1)$ f) $2\left(x^2+9\right)$ g) $(x+2)\,(x-8)$ h) $(a+9)\,(a-6)$ i) $(m-3)\,(m-5)$ j) $3\,(x+4)\,(x-2)\,(x+2)$ k*) $(x+5c-3)\,(x-5c-3)$

9. a) $1,15$ b) 10 c) -9 d) 7 e) 1 f) $2,0,-2$ g) $4,0$ h) 9 i) $6,-8$ 10. $P=114\,\mathrm{in}$, $A=540\,\mathrm{in}^2$

11. a) $(-\infty,9]$ b) $\left[-\dfrac{5}{2},\infty\right)$ c) $(-\infty,5)$

12. a) $5x-4y=-12$ b) $y=-\dfrac{2}{3}x+1$





13. \$850 at 3% and \$3150 at 5% 14. \$1700 gained 7% and \$2300 lost 3% 15. 104 nickels and 136 dimes

16. $16, 18, 20, 22$ 17. -6 and 10 18. $66.\overline{6}\%$ 19. 12 seconds 20. 43 children's tickets, 7 adult tickets

21. $120\,\text{ft}^2$ 22. $0, -1$ 23. -12 with -8 and 8 with 12 24. $7\,\text{ft}$ 25. 20 chickens and 62 cows

26. $10\,\text{cm}$ by $16\,\text{cm}$ 27. Ann has \$35, Betsy has \$25 28. Both have 101 divisors. 29. a) 10 b) 2^{2019}

Solutions

18. Let g denote the number of female students. Let S denote the total number of students. 15% of the girls is the same as 10% of all students.

$$\begin{aligned} 0.15g &= 0.1S \qquad \text{multiply by 100} \\ 15g &= 10S \\ g &= \frac{10}{15}S = \frac{2}{3}S \end{aligned}$$

Thus two thirds of the students are girls, which is $\frac{2}{3} = 0.\overline{6} = \frac{0.\overline{6}}{1} = \frac{66.\overline{6}}{200} = 66.\overline{6}\%$.

23. If we label the smaller number by x, the greater number is $x + 4$ and sothe equation we solve is $x(x+4) = 60$.

26. If one side is x, the other side is $4x - 24$ and so the equation is $x(4x - 24) = 160$.

27. Let us denote the money that Ann had by a and the money that Betsy had by b. Betsy tells Ann: "*If you gave me five dollars, we would end up with the same amount of money.*" If that happened, Ann would have $a - 5$ and Betsy $b + 5$. So we have an equation, $a - 5 = b + 5$. Then Ann says: "*If you gave me five dollars instead, then I would have twice as much money as you.*" If that happened, Ann would have $a + 5$ and Betsy $b - 5$. Then Ann'smoney would be twice Betsy's money, giving us the equation $a + 5 = 2(b - 5)$. We solve this system of equation.

$$\begin{aligned} a - 5 &= b + 5 \\ a + 5 &= 2(b - 5) \end{aligned}$$

We simplify each equations and get $a = 35$ and $b = 25$.

29. a) Solution 1: We simply divide.

$$\begin{aligned} \frac{3 \cdot 2^{2020} - 2^{2019}}{2^{2018}} &= \left(3 \cdot 2^{2020} - 2^{2019}\right) \cdot \frac{1}{2^{2018}} = 3 \cdot 2^{2020} \cdot \frac{1}{2^{2018}} - 2^{2019} \cdot \frac{1}{2^{2018}} = \frac{3 \cdot 2^{2020}}{2^{2018}} - \frac{2^{2019}}{2^{2018}} \\ &= 3 \cdot 2^{2020-2018} - 2^{2019-2018} = 3 \cdot 2^2 - 2^1 = 12 - 2 = 10 \end{aligned}$$

Solution 2: Factor out 2018 from the numerator and then cancel.

$$\frac{3 \cdot 2^{2020} - 2^{2019}}{2^{2018}} = \frac{2^{2018} \cdot \left(3 \cdot 2^2 - 2^1\right)}{2^{2018}} = 12 - 2 = 10$$

b) $2^{2018} + 2^{2018} = 2 \cdot 2^{2018} = 2^{1+2018} = 2^{2019}$

Last revised: November 22, 2018