2.2.2 Simplifying Fractions

Learning Objective(s)
1 Find an equivalent fraction with a given denominator.
2 Simplify a fraction to lowest terms.

Introduction

Fractions are used to represent a part of a whole. Fractions that represent the same part of a whole are called **equivalent fractions**. Factoring, multiplication, and division are all helpful tools for working with equivalent fractions.

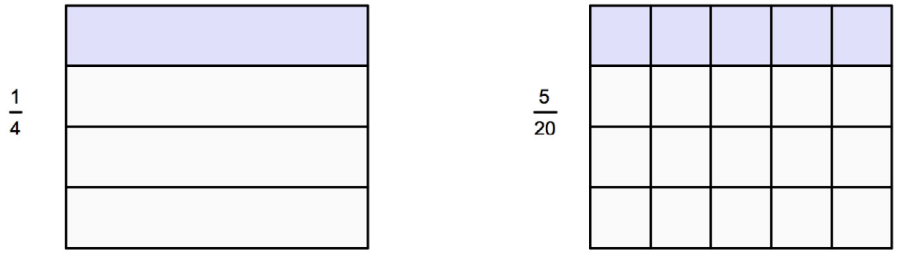
Equivalent Fractions

Objective 1

We use equivalent fractions every day. Fifty cents can be 2 quarters, and we have $\frac{2}{4}$ of a dollar, because there are 4 quarters in a dollar. Fifty cents is also 50 pennies out of 100 pennies, or $\frac{50}{100}$ of a dollar. Both of these fractions are the same amount of money, but written with a different numerator and denominator.

Think about a box of crackers that contains 3 packets of crackers. Two of these packets are $\frac{2}{3}$ of the box. Suppose each packet has 30 crackers in it. Two packets are also 60 (30 • 2) crackers out of 90 (30 • 3) crackers. This is $\frac{60}{90}$ of the box. The fractions $\frac{2}{3}$ and $\frac{60}{90}$ both represent two packets of crackers, so they are equivalent fractions.

Equivalent fractions represent the same part of a whole, even if the numerator and denominator are different. For example, $\frac{1}{4} = \frac{5}{20}$. In these diagrams, both fractions represent one of four rows in the rectangle.

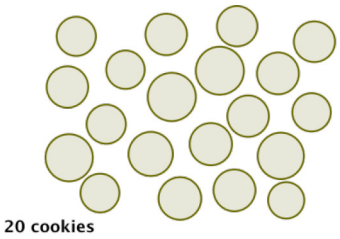


Since $\frac{1}{4}$ and $\frac{5}{20}$ are naming the same part of a whole, they are equivalent.
There are many ways to name the same part of a whole using equivalent fractions.

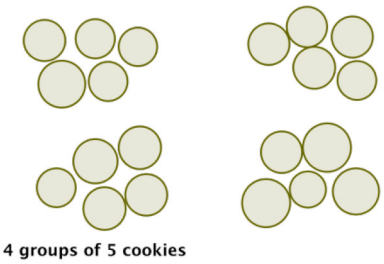
Let's look at an example where you need to find an equivalent fraction.

Example

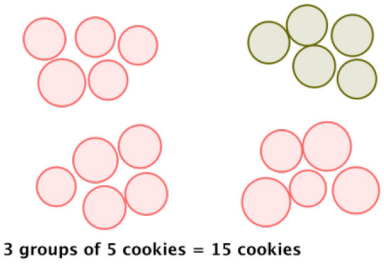
Problem **John is making cookies for a bake sale. He made 20 large cookies, but he wants to give away only 3/4 of them for the bake sale. What fraction of the cookies does he give away, using 20 as the denominator?**



Start with 20 cookies.



Because the denominator of $\frac{3}{4}$ is 4, make 4 groups of cookies, 5 in each group.



$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5}$ because there are 5 cookies in each group.

$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$

Answer He gives away $\frac{15}{20}$ of the cookies.

When you regroup and reconsider the parts and whole, you are multiplying the numerator and denominator by the same number. In the above example, you multiply 4 by 5 to get the needed denominator of 20, so you also need to multiply the numerator 3 by 5, giving the new numerator of 15.

Finding Equivalent Fractions

To find equivalent fractions, multiply or divide *both* the numerator and the denominator by the same number.

Examples:

$$\frac{20}{25}=\frac{20\div 5}{25\div 5}=\frac{4}{5}$$

$$\frac{2}{7}=\frac{2\cdot 6}{7\cdot 6}=\frac{12}{42}$$

Self Check A

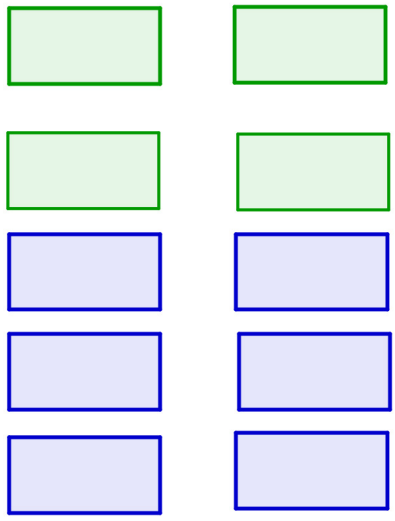
Write an equivalent fraction to $\frac{2}{3}$ that has a denominator of 27.

Simplifying Fractions

Objective 2

A fraction is in its **simplest form**, or **lowest terms**, when it has the least numerator and the least denominator possible for naming this part of a whole. The numerator and denominator have no common factor other than 1.

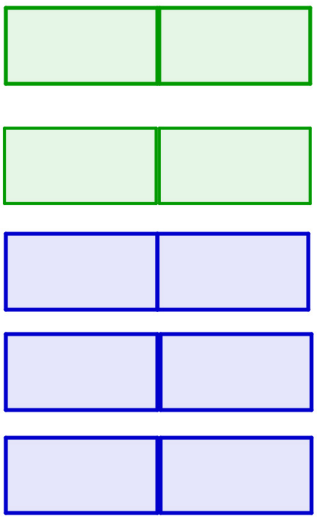
Here are 10 blocks, 4 of which are green. So, the fraction that is green is $\frac{4}{10}$. To simplify, you find a common factor and then regroup the blocks by that factor.



$$\frac{\text{green blocks}}{\text{blocks}}=\frac{4}{10}$$

Example

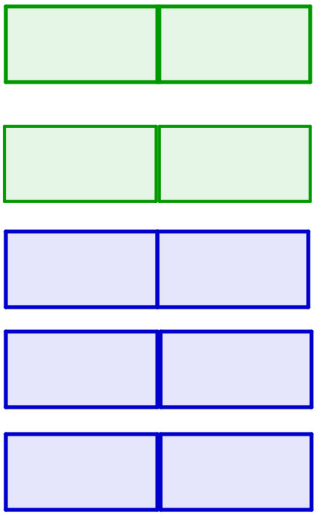
Problem **Simplify $\frac{4}{10}$.**



$$\frac{\text{green blocks}}{\text{blocks}} = \frac{2(2)}{5(2)}$$

We start with 4 green blocks out of 10 total blocks.

Group the blocks in twos, since 2 is a common factor. You have 2 groups of green blocks and a total of 5 groups, each group containing 2 blocks.



$$\frac{\text{green blocks}}{\text{blocks}} = \frac{2(2)}{5(2)} = \frac{2}{5}$$

Now, consider the groups as the part and you have 2 green groups out of 5 total groups.

Answer $\frac{4}{10} = \frac{2}{5}$ The simplified fraction is $\frac{2}{5}$.

Once you have determined a common factor, you can divide the blocks into the groups by dividing both the numerator and denominator to determine the number of groups that you have.

For example, to simplify $\frac{6}{9}$ you find a common factor of 3, which will divide evenly into both 6 and 9. So, you divide 6 and 9 into groups of 3 to determine how many groups of 3 they contain. This gives $\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$, which means 2 out of 3 groups, and $\frac{2}{3}$ is equivalent to $\frac{6}{9}$.

It may be necessary to group more than one time. Each time, determine a common factor for the numerator and denominator using the tests of divisibility, when possible. If both numbers are even numbers, start with 2. For example:

Example		
Problem	**Simplify** $\frac{32}{48}$.	
	$\frac{32}{48} = \frac{32 \div 2}{48 \div 2} = \frac{16}{24}$	32 and 48 have a common factor of 2. Divide each by 2.
	$\frac{16}{24} = \frac{16 \div 2}{24 \div 2} = \frac{8}{12}$	16 and 24 have a common factor of 2. Divide each by 2.
	$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$	8 and 12 have a common factor of 4. Divide each by 4.
Answer	$\frac{32}{48} = \frac{2}{3}$	$\frac{2}{3}$ is the simplified fraction equivalent to $\frac{32}{48}$.

In the example above, 16 is a factor of both 32 and 48, so you could have shortened the solution.

$$\frac{32}{48} =$$

$$\frac{2 \cdot 16}{3 \cdot 16} =$$

$$\frac{2}{3}$$

You can also use **prime factorization** to help regroup the numerator and denominator.

Example		
Problem	**Simplify** $\frac{54}{72}$.	
	$\frac{54}{72} = \frac{2 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$	The prime factorization of 54 is $2 \cdot 3 \cdot 3 \cdot 3$. The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.
	$\frac{3 \cdot (2 \cdot 3 \cdot 3)}{2 \cdot 2 \cdot (2 \cdot 3 \cdot 3)}$	Rewrite, finding common factors.
	$\frac{3}{2 \cdot 2} \cdot 1$	$\frac{2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 3} = 1$
	$\frac{3}{4}$	Multiply: $2 \cdot 2$.
Answer	$\frac{54}{72} = \frac{3}{4}$	$\frac{3}{4}$ is the simplified fraction equivalent to $\frac{54}{72}$.

Notice that when you *simplify* a fraction, you *divide* the numerator and denominator by the same number, in the same way you *multiply* by the same number to find an *equivalent* fraction with a greater denominator. In the example above, you could have divided the numerator and denominator by 9, a common factor of 54 and 72.

$$\frac{54 \div 9}{72 \div 9} = \frac{6}{8}$$

Since the numerator (6) and the denominator (8) still have a common factor, the fraction is not yet in lowest terms. So, again divide by the common factor 2.

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Repeat this process of dividing by a common factor until the only common factor is 1.

Simplifying Fractions to Lowest Terms

To simplify a fraction to lowest terms, divide both the numerator and the denominator by their common factors. Repeat as needed until the only common factor is 1.

Self Check B

Simplify $\frac{36}{72}$.

Summary

Multiplication of binomials and polynomials requires use of the distributive property and integer operations. Whether the polynomials are monomials, binomials, or trinomials, carefully multiply each term in one polynomial by each term in the other polynomial. Be careful to watch the addition and subtraction signs and negative coefficients. A product is written in simplified if all of its like terms have been combined.

2.2.2 Self Check Solutions

Self Check A

Write an equivalent fraction to $\frac{2}{3}$ that has a denominator of 27.

$\frac{18}{27}$

The multiplying factor is 9, so the denominator is 3 • 9 = 27 and the numerator is 2 • 9 = 18.

Self Check B

Simplify $\frac{36}{72}$.

$\frac{36}{72} = \frac{36 \div 36}{72 \div 36} = \frac{1}{2}$. This is in lowest terms since 1 is the only common factor of 1 and 2.

2.2.3 Comparing Fractions

Learning Objective(s)
1 Determine whether two fractions are equivalent.
2 Use > or < to compare fractions.

Introduction

You often need to know when one fraction is greater or less than another fraction. Since a fraction is a part of a whole, to find the greater fraction you need to find the fraction that contains more of the whole. If the two fractions simplify to fractions with a **common denominator**, you can then compare numerators. If the denominators are different, you can find a common denominator first and then compare the numerators.

Determining Equivalent Fractions

Objective 1

Two fractions are **equivalent fractions** when they represent the same part of a whole. Since equivalent fractions do not always have the same numerator and denominator, one way to determine if two fractions are equivalent is to find a common denominator and rewrite each fraction with that denominator. Once the two fractions have the same denominator, you can check to see if the numerators are equal. If they are equal, then the two fractions are equal as well.

One way to find a common denominator is to check to see if one denominator is a factor of the other denominator. If so, the greater denominator can be used as the common denominator.

Example

Problem **Are $\frac{2}{6}$ and $\frac{8}{18}$ equivalent fractions?**

Does $\frac{2}{6} = \frac{8}{18}$?

To solve this problem, find a common denominator for the two fractions. This will help you compare the two fractions. Since 6 is a factor of 18, you can write both fractions with 18 as the denominator.

$\frac{2 \cdot 3}{6 \cdot 3} = \frac{6}{18}$

Start with the fraction $\frac{2}{6}$.

Multiply the denominator, 6, by 3 to get a new denominator of 18. Since you multiply the denominator by 3, you must also multiply the numerator by 3.

	$\frac{8}{18}$	The fraction $\frac{8}{18}$ already has a denominator of 18, so you can leave it as is.
	$\frac{6}{18}$ does not equal $\frac{8}{18}$	Compare the fractions. Now that both fractions have the same denominator, 18, you can compare numerators.
Answer	$\frac{2}{6}$ and $\frac{8}{18}$ are not equivalent fractions.	

When one denominator is not a factor of the other denominator, you can find a common denominator by multiplying the denominators together.

Example

Problem — **Determine whether $\frac{3}{6}$ and $\frac{5}{10}$ are equivalent fractions.**

	6 • 10 = 60	Use 60 as a common denominator.
	$\frac{3}{6} = \frac{3 \cdot 10}{6 \cdot 10} = \frac{30}{60}$	Multiply the numerator and denominator of $\frac{3}{6}$ by 10 to get 60 in the denominator.
	$\frac{5}{10} = \frac{5 \cdot 6}{10 \cdot 6} = \frac{30}{60}$	Multiply numerator and denominator of $\frac{5}{10}$ by 6.
	$\frac{30}{60} = \frac{30}{60}$	Now that the denominators are the same, compare the numerators.
Answer	Yes, $\frac{3}{6}$ and $\frac{5}{10}$ are equivalent fractions.	Since 30 is the value of the numerator for both fractions, the two fractions are equal.

Notice in the above example you can use 30 as the least common denominator since both 6 and 10 are factors of 30. Any common denominator will work.

In some cases you can simplify one or both of the fractions, which can result in a common denominator.

Example

Problem **Determine whether $\frac{2}{3}$ and $\frac{40}{60}$ are equivalent fractions.**

$$\frac{40}{60} = \frac{40 \div 10}{60 \div 10} = \frac{4}{6}$$

Simplify $\frac{40}{60}$. Divide the numerator and denominator by the common factor 10.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

$\frac{4}{6}$ is still not in lowest terms, so divide the numerator and the denominator again, this time by the common factor 2.

$$\frac{2}{3} = \frac{2}{3}$$

Compare the fractions. The numerators and denominators are the same.

Answer Yes, $\frac{2}{3}$ and $\frac{40}{60}$ are equivalent fractions.

Note: In the example above you could have used the common factor of 20 to simplify $\frac{40}{60}$ directly to $\frac{2}{3}$.

Determining Equivalent Fractions

To determine whether or not two fractions are equivalent:

Step 1: Rewrite one or both of the fractions so that they have common denominators.
Step 2: Compare the numerators to see if they have the same value. If so, then the fractions are equivalent.

Self Check A

Which of the following fraction pairs are equivalent?

A) $\frac{5}{7}$ *and* $\frac{7}{5}$

B) $\frac{12}{30}$ *and* $\frac{6}{10}$

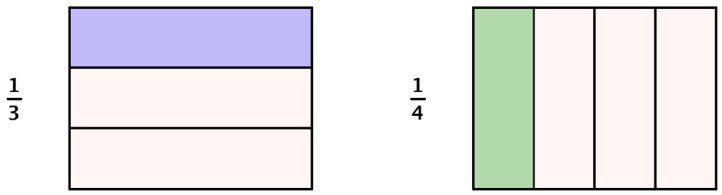
C) $\frac{4}{20}$ *and* $\frac{1}{5}$

D) $\frac{8}{11}$ *and* $\frac{8}{22}$

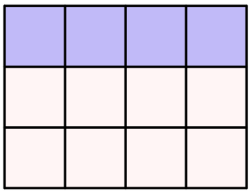
Comparing Fractions Using < and >

Objective 2

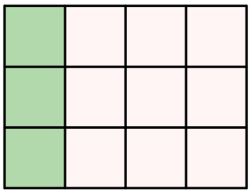
When given two or more fractions, it is often useful to know which fraction is greater than or less than the other. For example, if the discount in one store is $\frac{1}{3}$ off the original price and the discount in another store is $\frac{1}{4}$ off the original price, which store is offering a better deal? To answer this question, and others like it, you can compare fractions.



To determine which fraction is greater, you need to find a common denominator. You can then compare the fractions directly. Since 3 and 4 are both factors of 12, you will divide the whole into 12 parts, create equivalent fractions for $\frac{1}{3}$ and $\frac{1}{4}$, and then compare.

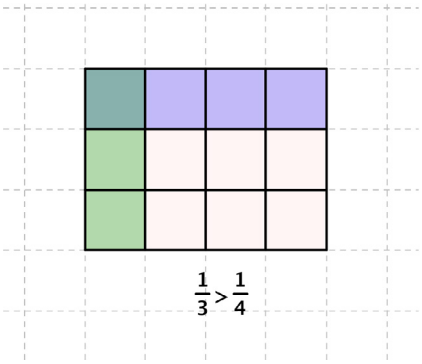


$\frac{1}{3}=\frac{4}{12}$



$\frac{1}{4}=\frac{3}{12}$

Now you see that $\frac{1}{3}$ contains 4 parts of 12, and $\frac{1}{4}$ contains 3 parts of 12. So, $\frac{1}{3}$ is greater than $\frac{1}{4}$.



$\frac{1}{3}>\frac{1}{4}$

As long as the denominators are the same, the fraction with the greater numerator is the greater fraction, as it contains more parts of the whole. The fraction with the lesser numerator is the lesser fraction as it contains fewer parts of the whole.

Recall that the symbol < means “less than”, and the symbol > means “greater than”. These symbols are inequality symbols. So, the true statement 3 < 8 is read as “3 is less than 8” and the statement 5 > 3 is read as “5 is greater than 3”. One way to help you remember the distinction between the two symbols is to think that the smaller end of the symbol points to the lesser number.

As with comparing whole numbers, the inequality symbols are used to show when one fraction is “greater than” or “less than” another fraction.

Comparing Fractions

To compare two fractions:

Step 1: Compare denominators. If they are different, rewrite one or both fractions with a common denominator.
Step 2: Check the numerators. If the denominators are the same, then the fraction with the greater numerator is the greater fraction. The fraction with the lesser numerator is the lesser fraction. And, as noted above, if the numerators are equal, the fractions are equivalent.

Example

Problem **Use < or > to compare the two fractions $\frac{4}{5}$ and $\frac{14}{20}$.**

Is $\frac{4}{5} > \frac{14}{20}$, or is $\frac{4}{5} < \frac{14}{20}$?	You cannot compare the fractions directly because they have different denominators. You need to find a common denominator for the two fractions.
$\frac{4}{5} = \frac{?}{20}$	Since 5 is a factor of 20, you can use 20 as the common denominator.
$\frac{4 \cdot 4}{5 \cdot 4} = \frac{16}{20}$	Multiply the numerator and denominator by 4 to create an equivalent fraction with a denominator of 20.
$\frac{16}{20} > \frac{14}{20}$	Compare the two fractions. $\frac{16}{20}$ is greater than $\frac{14}{20}$.
Answer $\frac{4}{5} > \frac{14}{20}$	If $\frac{16}{20} > \frac{14}{20}$, then $\frac{4}{5} > \frac{14}{20}$, since $\frac{4}{5} = \frac{16}{20}$.

Self Check B

Which of the following is a true statement?

A) $\frac{5}{6} < \frac{24}{30}$

B) $\frac{25}{100} > \frac{9}{12}$

C) $\frac{4}{16} > \frac{1}{3}$

D) $\frac{3}{8} < \frac{20}{40}$

Summary

You can compare two fractions with like denominators by comparing their numerators. The fraction with the greater numerator is the greater fraction, as it contains more parts of the whole. The fraction with the lesser numerator is the lesser fraction as it contains fewer parts of the whole. If two fractions have the same denominator, then equal numerators indicate equivalent fractions.

2.2.3 Self Check Solutions

Self Check A

Which of the following fraction pairs are equivalent?

A) $\frac{5}{7}$ *and* $\frac{7}{5}$ B) $\frac{12}{30}$ *and* $\frac{6}{10}$

C) $\frac{4}{20}$ *and* $\frac{1}{5}$ D) $\frac{8}{11}$ *and* $\frac{8}{22}$

$\frac{4}{20}$ *and* $\frac{1}{5}$

Take the fraction $\frac{1}{5}$ and multiply both the numerator and denominator by 4. You are left with the fraction $\frac{4}{20}$. This means that the two fractions are equivalent.

Self Check B

Which of the following is a true statement?

A) $\frac{5}{6} < \frac{24}{30}$ B) $\frac{25}{100} > \frac{9}{12}$ C) $\frac{4}{16} > \frac{1}{3}$ D) $\frac{3}{8} < \frac{20}{40}$

$\frac{3}{8} < \frac{20}{40}$

Simplifying $\frac{20}{40}$, you get the equivalent fraction $\frac{1}{2}$. Since you still don't have a common denominator, write $\frac{1}{2}$ as an equivalent fraction with a denominator of 8: $\frac{1}{2} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$.

You find that $\frac{3}{8} < \frac{4}{8}$, so $\frac{3}{8} < \frac{20}{40}$ as well.