

2.3 Multiplying Fractions and Mixed Numbers

Learning Objective(s)

- 1 Multiply two or more fractions.
- 2 Multiply a fraction by a whole number.
- 3 Multiply two or more mixed numbers.
- 4 Solve application problems that require multiplication of fractions or mixed numbers.
- 5 Find the area of triangles.

Introduction

Just as you add, subtract, multiply, and divide when working with whole numbers, you also use these operations when working with fractions. There are many times when it is necessary to multiply fractions and **mixed numbers**. For example, this recipe will make 4 crumb piecrusts:

$$\begin{array}{ll} 5 \text{ cups graham crackers} & 8 \text{ T. sugar} \\ 1\frac{1}{2} \text{ cups melted butter} & \frac{1}{4} \text{ tsp. vanilla} \end{array}$$

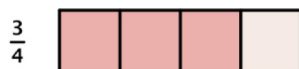
Suppose you only want to make 2 crumb piecrusts. You can multiply all the ingredients by $\frac{1}{2}$, since only half of the number of piecrusts are needed. After learning how to multiply a fraction by another fraction, a whole number or a mixed number, you should be able to calculate the ingredients needed for 2 piecrusts.

Multiplying Fractions

Objective 1

When you multiply a fraction by a fraction, you are finding a “fraction of a fraction.”

Suppose you have $\frac{3}{4}$ of a candy bar and you want to find $\frac{1}{2}$ of the $\frac{3}{4}$:



By dividing each fourth in half, you can divide the candy bar into eighths.



Then, choose half of those to get $\frac{3}{8}$.



In both of the above cases, to find the answer, you can multiply the numerators together and the denominators together.

Multiplying Two Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{\text{product of the numerators}}{\text{product of the denominators}}$$

Example:

$$\frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$$

Multiplying More Than Two Fractions

$$\frac{a}{b} \cdot \frac{c}{d} \cdot \frac{e}{f} = \frac{a \cdot c \cdot e}{b \cdot d \cdot f}$$

Example:

$$\frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5} = \frac{1 \cdot 2 \cdot 3}{3 \cdot 4 \cdot 5} = \frac{6}{60}$$

Example		
Problem	$\frac{2}{3} \cdot \frac{4}{5}$	Multiply.
	$\frac{2 \cdot 4}{3 \cdot 5}$	Multiply the numerators and multiply the denominators.
	$\frac{8}{15}$	Simplify, if possible. This fraction is already in lowest terms.
Answer	$\frac{8}{15}$	

If the resulting **product** needs to be simplified to lowest terms, divide the numerator and denominator by common factors.

Example		
Problem	$\frac{2}{3} \cdot \frac{1}{4}$	Multiply. Simplify the answer.
	$\frac{2 \cdot 1}{3 \cdot 4}$	Multiply the numerators and multiply the denominators.
	$\frac{2}{12}$	Simplify, if possible.
	$\frac{2 \div 2}{12 \div 2}$	Simplify by dividing the numerator and denominator by the common factor 2.
<i>Answer</i>	$\frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$	

You can also simplify the problem before multiplying, by dividing common factors.

Example		
Problem	$\frac{2}{3} \cdot \frac{1}{4}$	Multiply. Simplify the answer.
	$\frac{2 \cdot 1}{3 \cdot 4} = \frac{1 \cdot 2}{3 \cdot 4}$	Reorder the numerators so that you can see a fraction that has a common factor.
	$\frac{1 \cdot 1}{3 \cdot 2}$	Simplify. $\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$
<i>Answer</i>	$\frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$	

You do not have to use the “simplify first” shortcut, but it could make your work easier because it keeps the numbers in the numerator and denominator smaller while you are working with them.

Self Check A

$\frac{3}{4} \cdot \frac{1}{3}$ Multiply. Simplify the answer.

Multiplying a Fraction by a Whole Number

Objective 2

When working with both fractions and whole numbers, it is useful to write the whole number as an **improper fraction** (a fraction where the numerator is greater than or equal to the denominator). All whole numbers can be written with a “1” in the denominator. For example: $2 = \frac{2}{1}$, $5 = \frac{5}{1}$, and $100 = \frac{100}{1}$. Remember that the denominator tells how many parts there are in one whole, and the numerator tells how many parts you have.

Multiplying a Fraction and a Whole Number

$$a \cdot \frac{b}{c} = \frac{a}{1} \cdot \frac{b}{c}$$

Example:

$$4 \cdot \frac{2}{3} = \frac{4}{1} \cdot \frac{2}{3} = \frac{8}{3}$$

Often when multiplying a whole number and a fraction the resulting product will be an improper fraction. It is often desirable to write improper fractions as a mixed number for the final answer. You can simplify the fraction before or after rewriting as a mixed number. See the examples below.

Example		
Problem	$7 \cdot \frac{3}{5}$	Multiply. Simplify the answer and write as a mixed number.
	$\frac{7}{1} \cdot \frac{3}{5}$	Rewrite 7 as the improper fraction $\frac{7}{1}$.
	$\frac{7 \cdot 3}{1 \cdot 5} = \frac{21}{5}$	Multiply the numerators and multiply the denominators.
	$4 \frac{1}{5}$	Rewrite as a mixed number. $21 \div 5 = 4$ with a remainder of 1.
Answer	$7 \cdot \frac{3}{5} = 4 \frac{1}{5}$	

Example		
Problem	$4 \cdot \frac{3}{4}$	Multiply. Simplify the answer and write as a mixed number.
	$\frac{4}{1} \cdot \frac{3}{4}$	Rewrite 4 as the improper fraction $\frac{4}{1}$.
	$\frac{4 \cdot 3}{1 \cdot 4}$	Multiply the numerators and multiply the denominators.
	$\frac{12}{4} = 3$	Simplify.
<i>Answer</i>	$4 \cdot \frac{3}{4} = 3$	

Self Check B

$3 \cdot \frac{5}{6}$ Multiply. Simplify the answer and write it as a mixed number.

Multiplying Mixed Numbers

Objective 3

If you want to multiply two mixed numbers, or a fraction and a mixed number, you can again rewrite any mixed number as an improper fraction.

So, to multiply two mixed numbers, rewrite each as an improper fraction and then multiply as usual. Multiply numerators and multiply denominators and simplify. And, as before, when simplifying, if the answer comes out as an improper fraction, then convert the answer to a mixed number.

Example		
Problem	$2\frac{1}{5} \cdot 4\frac{1}{2}$	Multiply. Simplify the answer and write as a mixed number.
	$2\frac{1}{5} = \frac{11}{5}$	Change $2\frac{1}{5}$ to an improper fraction. $5 \cdot 2 + 1 = 11$, and the denominator is 5.
	$4\frac{1}{2} = \frac{9}{2}$	Change $4\frac{1}{2}$ to an improper fraction. $2 \cdot 4 + 1 = 9$, and the denominator is 2.

	$\frac{11}{5} \cdot \frac{9}{2}$	Rewrite the multiplication problem, using the improper fractions.
	$\frac{11 \cdot 9}{5 \cdot 2} = \frac{99}{10}$	Multiply numerators and multiply denominators.
	$\frac{99}{10} = 9 \frac{9}{10}$	Write as a mixed number. $99 \div 10 = 9$ with a remainder of 9.
<i>Answer</i>	$2 \frac{1}{5} \cdot 4 \frac{1}{2} = 9 \frac{9}{10}$	

Example		
Problem	$\frac{1}{2} \cdot 3 \frac{1}{3}$	Multiply. Simplify the answer and write as a mixed number.
	$3 \frac{1}{3} = \frac{10}{3}$	Change $3 \frac{1}{3}$ to an improper fraction. $3 \cdot 3 + 1 = 10$, and the denominator is 3.
	$\frac{1}{2} \cdot \frac{10}{3}$	Rewrite the multiplication problem, using the improper fraction in place of the mixed number.
	$\frac{1 \cdot 10}{2 \cdot 3} = \frac{10}{6}$	Multiply numerators and multiply denominators.
	$\frac{10}{6} = 1 \frac{4}{6}$	Rewrite as a mixed number. $10 \div 6 = 1$ with a remainder of 4.
	$1 \frac{2}{3}$	Simplify the fractional part to lowest terms by dividing the numerator and denominator by the common factor 2.
<i>Answer</i>	$\frac{1}{2} \cdot 3 \frac{1}{3} = 1 \frac{2}{3}$	

As you saw earlier, sometimes it's helpful to look for common factors in the numerator and denominator before you simplify the products.

Example

Problem

$1\frac{3}{5} \cdot 2\frac{1}{4}$

Multiply. Simplify the answer and write as a mixed number.

$1\frac{3}{5} = \frac{8}{5}$

Change $1\frac{3}{5}$ to an improper fraction. $5 \cdot 1 + 3 = 8$, and the denominator is 5.

$2\frac{1}{4} = \frac{9}{4}$

Change $2\frac{1}{4}$ to an improper fraction. $4 \cdot 2 + 1 = 9$, and the denominator is 4.

$\frac{8}{5} \cdot \frac{9}{4}$

Rewrite the multiplication problem using the improper fractions.

$\frac{8 \cdot 9}{5 \cdot 4} = \frac{9 \cdot 8}{5 \cdot 4}$

Reorder the numerators so that you can see a fraction that has a common factor.

$\frac{9 \cdot 8}{5 \cdot 4} = \frac{9 \cdot 2}{5 \cdot 1}$

Simplify. $\frac{8}{4} = \frac{8 \div 4}{4 \div 4} = \frac{2}{1}$

$\frac{18}{5}$

Multiply.

$\frac{18}{5} = 3\frac{3}{5}$

Write as a mixed fraction.

Answer

$1\frac{3}{5} \cdot 2\frac{1}{4} = 3\frac{3}{5}$

In the last example, the same answer would be found if you multiplied numerators and multiplied denominators without removing the common factor. However, you would get $\frac{72}{20}$, and then you would need to simplify more to get your final answer.

Self Check C

$1\frac{3}{5} \cdot 3\frac{1}{3}$ Multiply. Simplify the answer and write as a mixed number.

Now that you know how to multiply a fraction by another fraction, by a whole number, or by a mixed number, you can use this knowledge to solve problems that involve multiplication and fractional amounts. For example, you can now calculate the ingredients needed for the 2 crumb piecrusts.

Example			
Problem	5 cups graham crackers $1\frac{1}{2}$ cups melted butter	8 T. sugar $\frac{1}{4}$ tsp. vanilla	The recipe at left makes 4 piecrusts. Find the ingredients needed to make only 2 piecrusts.
	$5 \cdot \frac{1}{2} = \frac{5}{1} \cdot \frac{1}{2} = \frac{5}{2}$ <p>$2\frac{1}{2}$ cups of graham crackers are needed.</p>		<p>Since the recipe is for 4 piecrusts, you can multiply each of the ingredients by $\frac{1}{2}$ to find the measurements for just 2 piecrusts.</p> <p>5 cups graham crackers: Since the result is an improper fraction, rewrite $\frac{5}{2}$ as the improper fraction $2\frac{1}{2}$.</p>
	$8 \cdot \frac{1}{2} = \frac{8}{1} \cdot \frac{1}{2} = \frac{8}{2} = 4$ <p>4 T. sugar is needed.</p>		<p>8 T. sugar: This is another example of a whole number multiplied by a fraction.</p>
	$\frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$ <p>$\frac{3}{4}$ cup melted butter is needed.</p>		<p>$1\frac{1}{2}$ cups melted butter: You need to multiply a mixed number by a fraction. So, first rewrite $1\frac{1}{2}$ as the improper fraction $\frac{3}{2}$: $2 \cdot 1 + 1$, and the denominator is 2. Then, rewrite the multiplication problem, using the improper fraction in place of the mixed number. Multiply.</p>

$$\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$\frac{1}{4}$ tsp. vanilla: Here, you multiply a fraction by a fraction.

$\frac{1}{8}$ tsp. vanilla is needed.

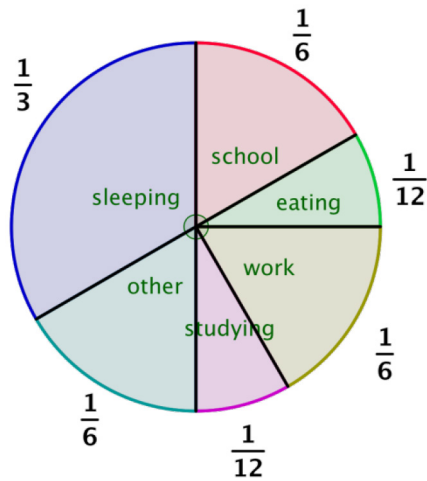
Answer The ingredients needed for 2 pie crusts are: $2\frac{1}{2}$ cups graham crackers
4 T. sugar
 $\frac{3}{4}$ cup melted butter
 $\frac{1}{8}$ tsp. vanilla

Often, a problem indicates that multiplication by a fraction is needed by using phrases like “half of,” “a third of,” or “ $\frac{3}{4}$ of.”

Example	
Problem	The cost of a vacation is \$4,500 and you are required to pay $\frac{1}{5}$ of that amount when you reserve the trip. How much will you have to pay when you reserve the trip?
$4,500 \cdot \frac{1}{5}$	You need to find $\frac{1}{5}$ of 4,500. “Of” tells you to multiply.
$\frac{4,500}{1} \cdot \frac{1}{5}$	Change 4,500 to an improper fraction by rewriting it with 1 as the denominator.
$\frac{4,500}{5}$	Divide.
900	Simplify.
Answer	You will need to pay \$900 when you reserve the trip.

Example

Problem



The pie chart at left represents the fractional part of daily activities. Given a 24-hour day, how many hours are spent sleeping? Attending school? Eating? Use the pie chart to determine your answers.

$$\frac{1}{3} \cdot 24 = \text{number of hours sleeping}$$

$$\frac{1}{3} \cdot \frac{24}{1}$$

$$\frac{24}{3} = 8$$

8 hours sleeping

$$\frac{1}{6} \cdot 24 = \text{number of hours spent at school}$$

$$\frac{1}{6} \cdot \frac{24}{1}$$

$$\frac{24}{6} = 4$$

4 hours attending school

Sleeping is $\frac{1}{3}$ of the pie, so the number of hours spent sleeping is $\frac{1}{3}$ of 24.

Rewrite 24 as an improper fraction with a denominator of 1.

Multiply numerators and multiply denominators. Simplify $\frac{24}{3}$ to 8.

Attending school is $\frac{1}{6}$ of the pie, so the number of hours spent attending school is $\frac{1}{6}$ of 24.

Rewrite 24 as an improper fraction with a denominator of 1.

Multiply numerators and multiply denominators. Simplify $\frac{24}{6}$ to 4.

$$\frac{1}{12} \cdot 24 = \text{number of hours spent eating}$$

$$\frac{1}{12} \cdot \frac{24}{1}$$

$$\frac{24}{12} = 2$$

2 hours spent eating

Answer Hours spent:
sleeping: 8 hours
attending school: 4 hours
eating: 2 hours

Eating is $\frac{1}{12}$ of the pie, so the number of hours spent eating is $\frac{1}{12}$ of 24.

Rewrite 24 as an improper fraction with a denominator of 1.

Multiply numerators and multiply denominators. Simplify $\frac{24}{12}$ to 2.

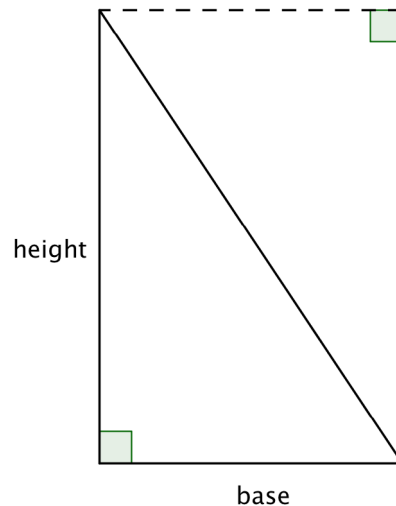
Self Check D

Neil bought a dozen (12) eggs. He used $\frac{1}{3}$ of the eggs for breakfast. How many eggs are left?

Area of Triangles

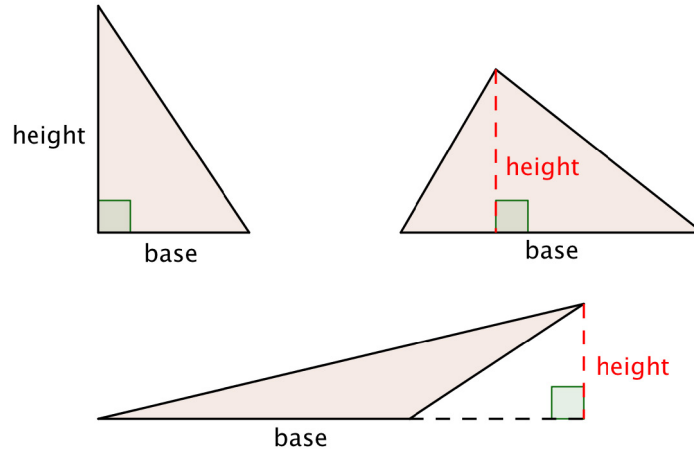
Objective 5

The formula for the area of a triangle can be explained by looking at a right triangle. Look at the image below—a rectangle with the same height and base as the original triangle. The area of the triangle is one half of the rectangle!



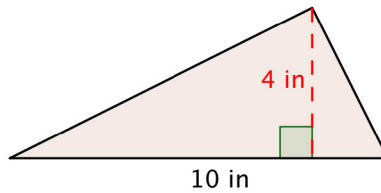
Since the area of two congruent triangles is the same as the area of a rectangle, you can come up with the formula $Area = \frac{1}{2}b \cdot h$ to find the area of a triangle.

When you use the formula for a triangle to find its area, it is important to identify a base and its corresponding height, which is perpendicular to the base.



Example

Problem A triangle has a height of 4 inches and a base of 10 inches. Find the area.



$A = \frac{1}{2}bh$ Start with the formula for the area of a triangle.

$A = \frac{1}{2} \cdot 10 \cdot 4$ Substitute 10 for the base and 4 for the height.

$A = \frac{1}{2} \cdot 40$ Multiply.

$A = 20$

Answer $A = 20 \text{ in}^2$

Summary

You multiply two fractions by multiplying the numerators and multiplying the denominators. Often the resulting product will not be in lowest terms, so you must also simplify. If one or both fractions are whole numbers or mixed numbers, first rewrite each as an improper fraction. Then multiply as usual, and simplify.

2.3 Self Check Solutions

Self Check A

$\frac{3}{4} \cdot \frac{1}{3}$ Multiply. Simplify the answer.

$$\frac{3 \cdot 1}{4 \cdot 3} = \frac{3}{12}, \text{ then simplify: } \frac{3 \div 3}{12 \div 3} = \frac{1}{4}.$$

Self Check B

$3 \cdot \frac{5}{6}$ Multiply. Simplify the answer and write it as a mixed number.

$2\frac{1}{2}$. Multiplying the two numbers gives $\frac{15}{6}$, and since $15 \div 6 = 2R3$, the mixed number is $2\frac{3}{6}$. The fractional part simplifies to $\frac{1}{2}$.

Self Check C

$1\frac{3}{5} \cdot 3\frac{1}{3}$ Multiply. Simplify the answer and write as a mixed number.

$5\frac{1}{3}$. First, rewrite each mixed number as an improper fraction: $1\frac{3}{5} = \frac{8}{5}$ and $3\frac{1}{3} = \frac{10}{3}$.

Next, multiply numerators and multiply denominators: $\frac{8}{5} \cdot \frac{10}{3} = \frac{80}{15}$. Then write as a

mixed fraction $\frac{80}{15} = 5\frac{5}{15}$. Finally, simplify the fractional part by dividing both numerator and denominator by the common factor 5.

Self Check D

Neil bought a dozen (12) eggs. He used $\frac{1}{3}$ of the eggs for breakfast. How many eggs are left?

$\frac{1}{3}$ of 12 is 4 ($\frac{1}{3} \cdot \frac{12}{1} = \frac{12}{3} = 4$), so he used 4 of the eggs. Because $12 - 4 = 8$, there are 8 eggs left.