2.5 Adding and Subtracting Fractions and Mixed Numbers with Like Denominators

Learning Objective(s)
1 Add fractions with like denominators.
2 Subtract fractions with like denominators.
3 Add mixed numbers with like denominators.
4 Subtract mixed numbers with like denominators.
5 Solve application problems that require the addition of fractions or mixed numbers.

Introduction

Fractions are used in many areas of everyday life: recipes, woodworking, rainfall, timecards, and measurements, to name just a few. Sometimes you have parts of wholes that you need to combine. Just as you can add whole numbers, you can add fractions and mixed numbers. Consider, for example, how to determine the monthly rainfall if you know the daily rainfall in inches. You have to add fractions. Also, consider several painters who are working to paint a house together with multiple cans of paint. They might add the fractions of what remains in each can to determine if there is enough paint to finish the job or if they need to buy more.

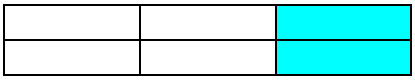
Adding Fractions with Like Denominators

Objective 1

When the pieces are the same size, they can easily be added. Consider the pictures below showing the fractions $\frac{3}{6}$ and $\frac{2}{6}$.

This picture represents $\frac{3}{6}$ shaded because 3 out of 6 blocks are shaded.

This picture represents $\frac{2}{6}$ shaded because 2 out of 6 blocks are shaded.



If you add these shaded blocks together, you are adding $\frac{3}{6} + \frac{2}{6}$.

You can create a new picture showing 5 shaded blocks in a rectangle containing 6 blocks.

So, $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

Without drawing rectangles and shading boxes, you can get this answer simply by adding the numerators, 3 + 2, and keeping the denominator, 6, the same. This procedure works for adding any fractions that have the same denominator, called **like denominators**.

Example		
Problem	$\frac{3}{5}+\frac{1}{5}$	**Add.**
	$\frac{3+1}{5}$ $\frac{4}{5}$	Since the denominator of each fraction is 5, these fractions have like denominators. So, add the numerators and write the sum over the denominator, 5.
Answer	$\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$	

Example		
Problem	$\frac{3}{8}+\frac{5}{8}$	**Add. Simplify the answer.**
	$\frac{3}{8}+\frac{5}{8}=\frac{3+5}{8}$	The denominators are alike, so add the numerators.
	$=\frac{8}{8}$	
	$\frac{8}{8}=1$	Simplify the fraction.
Answer	$\frac{3}{8}+\frac{5}{8}=1$	

Example		
Problem	$\frac{11}{12}+\frac{5}{12}$	**Add. Simplify the answer and write as a mixed number.**
	$\frac{11}{12}+\frac{5}{12}=\frac{11+5}{12}$ $=\frac{16}{12}$	The denominators are alike, so add the numerators.
	$\frac{16}{12}=\frac{16\div4}{12\div4}=\frac{4}{3}$	Simplify the fraction. 16 and 12 have a common factor of 4.
	$\frac{4}{3}=1\frac{1}{3}$	Write the improper fraction as a mixed number, by dividing: 4 ÷ 3 = 1 with a remainder of 1.
Answer	$\frac{11}{12}+\frac{5}{12}=1\frac{1}{3}$	

In the previous example, the fraction was simplified and then converted to a mixed number. You could just as easily have first converted the improper fraction to a mixed number and then simplified the fraction in the mixed number. Notice that the same answer is reached with both methods.

$$\frac{16}{12}=1\frac{4}{12}$$

The fraction $\frac{4}{12}$ can be simplified. $\frac{4}{12}=\frac{4\div4}{12\div4}=\frac{1}{3}$

But, don't forget about the 1 that is part of the mixed number! The final answer is $1\frac{1}{3}$.

Adding Fractions with Like Denominators

1. Add the numerators (the number in the top of each fraction).
2. Keep the denominator (the bottom number) the same.
3. Simplify to lowest terms.

Self Check A

$\frac{7}{10}+\frac{8}{10}$ Add. Simplify the answer and write as a mixed number.

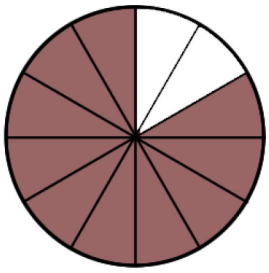
Sometimes subtraction, rather than addition, is required to solve problems that involve fractions. Suppose you are making pancakes and need $4\frac{1}{2}$ cups of flour but you only have $2\frac{3}{4}$ cups. How many additional cups will you have to get to make the pancakes? You can solve this problem by subtracting the mixed numbers.

Subtracting Fractions with Like Denominators

Objective **2**

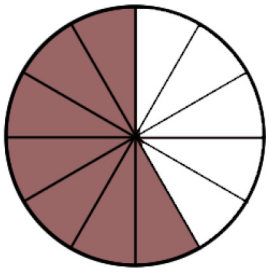
The most simple fraction subtraction problems are those that have two proper fractions with a **common denominator**. That is, each denominator is the same. The process is just as it is for addition of fractions with **like denominators**, except you subtract! You subtract the second numerator from the first and keep the denominator the same.

Imagine that you have a cake with equal-sized pieces. Some of the cake has already been eaten, so you have a fraction of the cake remaining. You could represent the cake pieces with the picture below.



The cake is cut into 12 equal pieces to start. Two are eaten, so the remaining cake can be represented with the fraction $\frac{10}{12}$. If three more pieces of cake are eaten, what fraction of the cake is left? You can represent that problem with the expression $\frac{10}{12}-\frac{3}{12}$.

If you subtract 3 pieces, you can see below that $\frac{7}{12}$ of the cake remains.



You can solve this problem without the picture by subtracting the numerators and keeping the denominator the same:

$$\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$$

Subtracting Fractions with Like Denominators

If the denominators (bottoms) of the fractions are the same, subtract the numerators (tops) and keep the denominator the same. *Remember to simplify the resulting fraction, if possible.*

Example		
Problem	$\frac{6}{7}-\frac{1}{7}$	**Subtract.**
	$\frac{6-1}{7}=\frac{5}{7}$	Both fractions have a denominator of 7, so subtract the numerators and keep the same denominator.
Answer	$\frac{6}{7}-\frac{1}{7}=\frac{5}{7}$	

Example		
Problem	$\frac{5}{9}-\frac{2}{9}$	**Subtract. Simplify the answer.**
	$\frac{5}{9}-\frac{2}{9}=\frac{3}{9}$	The fractions have a **like denominator**, also known as a common denominator, so subtract the numerators.
	$\frac{3\div3}{9\div3}=\frac{1}{3}$	Simplify the fraction.
Answer	$\frac{5}{9}-\frac{2}{9}=\frac{1}{3}$	

Self Check B

$\frac{11}{16}-\frac{7}{16}$ Subtract and simplify the answer.

Adding Mixed Numbers

Objective 3

Just as you can add whole numbers and proper fractions, you can also add mixed numbers. To add mixed numbers, add the whole numbers together and the fraction parts of the mixed numbers together and then recombine to express the value as a mixed number. The steps for adding two mixed numbers are shown in the examples below.

You can keep the whole numbers and the fractions together using a vertical method for adding mixed numbers as shown below.

Example		
Problem	$2\frac{1}{8}+3\frac{3}{8}$	**Add. Simplify the answer and write as a mixed number.**
	$2\frac{1}{8}$ $+\ 3\frac{3}{8}$	Arrange the mixed numbers vertically so the whole numbers align and the fractions align.
	$2\frac{1}{8}$ $+\ 3\frac{3}{8}$ $5\frac{4}{8}$	Add whole numbers. Add fractions.
	$5\frac{4}{8}=5\frac{1}{2}$	Simplify the fraction.
Answer	$2\frac{1}{8}+3\frac{3}{8}=5\frac{1}{2}$	

When adding mixed numbers you may need to regroup if the fractional parts add to more than one whole.

Example		
Problem	$6\frac{5}{7}+8\frac{4}{7}$	*Add. Simplify the answer and* **write as a mixed number.**
	$6\frac{5}{7}$ $+\ 8\frac{4}{7}$	Arrange the mixed numbers vertically so the whole numbers align and the fractions align.
	$6\frac{5}{7}$ $+\ 8\frac{4}{7}$ $14\frac{9}{7}$	Add whole numbers. Add fractions.

	$\frac{9}{7} = 1\frac{2}{7}$	Write the improper fraction as a mixed number.
	$14 + 1\frac{2}{7} = 15\frac{2}{7}$	Combine whole numbers and fraction to write a mixed number.
Answer	$6\frac{5}{7} + 8\frac{4}{7} = 15\frac{2}{7}$	

Self Check C

$3\frac{7}{9} + 1\frac{4}{9}$ Add. Simplify the answer and write as a mixed number.

Subtracting Mixed Numbers

Objective **4**

Subtracting mixed numbers works much the same way as adding mixed numbers. To subtract mixed numbers, subtract the whole number parts of the mixed numbers and then subtract the fraction parts in the mixed numbers. Finally, combine the whole number answer and the fraction answer to express the answer as a mixed number.

Example		
Problem	$6\frac{4}{5} - 3\frac{1}{5}$	**Subtract. Simplify the answer and write as a mixed number.**
	$6 - 3 = 3$	Subtract the whole numbers and subtract the fractions.
	$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$	
	$3\frac{3}{5}$	Combine the fraction and the whole number. Make sure the fraction in the mixed number is simplified.
Answer	$6\frac{4}{5} - 3\frac{1}{5} = 3\frac{3}{5}$	

Sometimes it might be easier to express the mixed number as an improper fraction first and then solve. Consider the example below.

Example		
Problem	$8\frac{1}{3}-4\frac{2}{3}$	**Subtract. Simplify the answer and write as a mixed number.**
	$8\frac{1}{3}=\frac{8\cdot3+1}{3}=\frac{24+1}{3}=\frac{25}{3}$ $4\frac{2}{3}=\frac{4\cdot3+2}{3}=\frac{12+2}{3}=\frac{14}{3}$	Write each mixed number as an improper fraction.
	$\frac{25}{3}-\frac{14}{3}=\frac{11}{3}$	Since the fractions have a like denominator, subtract the numerators.
	$\frac{11}{3}=3\frac{2}{3}$	Write the answer as a mixed number. Divide 11 by 3 to get 3 with a remainder of 2.
Answer	$8\frac{1}{3}-4\frac{2}{3}=3\frac{2}{3}$	

Since addition is the inverse operation of subtraction, you can check your answer to a subtraction problem with addition. In the example above, if you add $4\frac{2}{3}$ to your answer of $3\frac{2}{3}$, you should get $8\frac{1}{3}$.

$$4\frac{2}{3}+3\frac{2}{3}$$
$$4+3+\frac{2}{3}+\frac{2}{3}$$
$$7+\frac{4}{3}$$
$$7+1\frac{1}{3}$$
$$8\frac{1}{3}$$

Subtracting Mixed Numbers with Regrouping

Objective 4

Sometimes when subtracting mixed numbers, the fraction part of the second mixed number is larger than the fraction part of the first number. Consider the problem:

$7\frac{1}{6}-3\frac{5}{6}$. The standard procedure would be to subtract the fractions, but $\frac{1}{6}-\frac{5}{6}$ would result in a negative number. You don't want that!
You can regroup one of the whole numbers from the first number, writing the first mixed number in a different way:

$$7\frac{1}{6}=7+\frac{1}{6}=6+1+\frac{1}{6}$$

$$6+\frac{6}{6}+\frac{1}{6}=6+\frac{7}{6}=6\frac{7}{6}$$

Now, you can write an equivalent problem to the original:

$$6\frac{7}{6}-3\frac{5}{6}$$

Then, you just subtract like you normally subtract mixed numbers:

$$6-3=3$$
$$\frac{7}{6}-\frac{5}{6}=\frac{2}{6}=\frac{1}{3}$$

So, the answer is $3\frac{1}{3}$.

Example		
Problem	$7\frac{1}{4}-3\frac{3}{4}$	**Subtract. Simplify the answer and write as a mixed number.**
	$7\frac{1}{4}=6+1+\frac{1}{4}$ $6+\frac{4}{4}+\frac{1}{4}$ $6+\frac{5}{4}$ $6\frac{5}{4}$	Since the second fraction part, $\frac{3}{4}$, is larger than the first fraction part, $\frac{1}{4}$, regroup one of the whole numbers and write it as $\frac{4}{4}$.

	$7\frac{1}{4}-3\frac{3}{4}$ $6\frac{5}{4}-3\frac{3}{4}$	Rewrite the subtraction expression using the equivalent fractions.
	$6-3=3$ $\frac{5}{4}-\frac{3}{4}=\frac{2}{4}$	Subtract the whole numbers, subtract the fractions.
	$\frac{2}{4}=\frac{1}{2}$	Simplify the fraction
	$3\frac{1}{2}$	Combine the whole number and the fraction.
Answer	$7\frac{1}{4}-3\frac{3}{4}=3\frac{1}{2}$	

Sometimes a mixed number is subtracted from a whole number. In this case, you can also rewrite the whole number as a mixed number in order to perform the subtraction. You use an equivalent mixed number that has the same denominator as the fraction in the other mixed number.

Example		
Problem	$8-4\frac{2}{5}$	**Subtract. Simplify the answer and write as a mixed number.**
	$8=7+1$ $7+\frac{5}{5}$ or $7\frac{5}{5}$	Regroup one from the whole number and write it as $\frac{5}{5}$.
	$7\frac{5}{5}-4\frac{2}{5}$	Rewrite the subtraction expression using the equivalent fractions.
	$7-4=3$ $\frac{5}{5}-\frac{2}{5}=\frac{3}{5}$	Subtract the whole numbers, subtract the fractions.
	$3\frac{3}{5}$	Combine the whole number and the fraction.
Answer	$8-4\frac{2}{5}=3\frac{3}{5}$	

Subtracting Mixed Numbers

If the fractional part of the mixed number being subtracted is larger than the fractional part of the mixed number from which it is being subtracted, or if a mixed number is being subtracted from a whole number, follow these steps:

1. Subtract 1 from the whole number part of the mixed number being subtracted.
2. Add that 1 to the fraction part to make an improper fraction. For example, $7\frac{2}{3} = 6 + \frac{3}{3} + \frac{2}{3} = 6\frac{5}{3}$.
3. Then, subtract as with any other mixed numbers.

Alternatively, you can change both numbers to improper fractions and then subtract.

Self Check D

$15 - 13\frac{1}{4}$ Subtract. Simplify the answer and write as a mixed number.

Adding and Subtracting Fractions to Solve Problems

Objective 5

Knowing how to add fractions is useful in a variety of situations. When reading problems, look for phrases that help you know you want to add the fractions.

Example

Problem **A stack of pamphlets is placed on top of a book. If the stack of pamphlets is $3\frac{1}{4}$ inches thick and the book is $5\frac{3}{4}$ inches thick, how high is the pile?**

$3\frac{1}{4} + 5\frac{3}{4}$ Find the total height of the pile by adding the thicknesses of the stack of pamphlets and the book.

$3 + 5 + \frac{1}{4} + \frac{3}{4}$ Group the whole numbers and fractions to make adding easier.

$8 + \frac{1}{4} + \frac{3}{4}$ Add whole numbers.

$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$ Add fractions.

$8 + 1 = 9$ Combine whole number and fraction.

Answer The pile is 9 inches high.

Knowing how to subtract fractions and mixed numbers is useful in a variety of situations. When reading problems, look for key words that indicate that the problem can be solved using subtraction.

Example	
Problem	**Sherry loves to quilt, and she frequently buys fabric she likes when she sees it. She purchased 5 yards of blue print fabric and decided to use $2\frac{3}{8}$ yards of it in a quilt. How much of the blue print fabric will she have left over after making the quilt?**
	$5-2\frac{3}{8}$ Write an expression using subtraction to describe the situation.
	$4\frac{8}{8}-2\frac{3}{8}$ Rewrite the whole number as a mixed number.
	$4\frac{8}{8}-2\frac{3}{8}=2\frac{5}{8}$ Subtract. Check that the mixed number is simplified.
Answer	Sherry has $2\frac{5}{8}$ yards of blue print fabric left over.

Summary

Adding and subtracting fractions with like denominators involves adding or subtracting the numerators and keeping the denominator the same. Always simplify the answer. Adding mixed numbers involves adding the fractional parts, adding the whole numbers, and then recombining them as a mixed number.

When subtracting mixed numbers, if the fraction in the second mixed number is larger than the fraction in the first mixed number, rewrite the first mixed number by regrouping one whole as a fraction. Alternatively, rewrite all fractions as improper fractions and then subtract. This process is also used when subtracting a mixed number from a whole number.

2.5 Self Check Solutions

Self Check A

$\frac{7}{10}+\frac{8}{10}$ Add. Simplify the answer and write as a mixed number.

$\frac{7+8}{10}=\frac{15}{10}=1\frac{5}{10}=1\frac{1}{2}$.

Self Check B

$\frac{11}{16}-\frac{7}{16}$ Subtract and simplify the answer.

$\frac{11}{16}-\frac{7}{16}=\frac{11-7}{16}=\frac{4}{16}=\frac{1}{4}$

Self Check C

$3\frac{7}{9}+1\frac{4}{9}$ Add. Simplify the answer and write as a mixed number.

Adding the fractions: $\frac{7}{9}+\frac{4}{9}=\frac{11}{9}=1\frac{2}{9}$. Adding the whole numbers, 3+1 = 4. Combining these, $4+1\frac{2}{9}=5\frac{2}{9}$.

Self Check D

$15-13\frac{1}{4}$ Subtract. Simplify the answer and write as a mixed number.

Regrouping, $15=14+1=14+\frac{4}{4}=14\frac{4}{4}$

$15-13\frac{1}{4}=14\frac{4}{4}-13\frac{1}{4}$

Subtracting the whole numbers, 14-13 = 1. Subtracting fractions, $\frac{4}{4}-\frac{1}{4}=\frac{3}{4}$

$15-13\frac{1}{4}=1\frac{3}{4}$

2.6 Adding and Subtracting Fractions and Mixed Numbers with Unlike Denominators

Learning Objective(s)
1 Find the least common multiple (LCM) of two or more numbers.
2 Find the Least Common Denominator
3 Add fractions with unlike denominators.
4 Add mixed numbers
5 Subtract fractions with unlike denominators..
6 Subtract mixed numbers without regrouping.
7 Subtract mixed numbers with regrouping.
8 Solve application problems that require the subtraction of fractions or mixed numbers.

Finding Least Common Multiples

Objective 1

Sometimes fractions do not have the same denominator. They have **unlike denominators**. Think about the example of the house painters. If one painter has $\frac{2}{3}$ can of paint and his painting partner has $\frac{1}{2}$ can of paint, how much do they have in total? How can you add these fractions when they do not have like denominators?

The answer is that you can rewrite one or both of the fractions so that they have the same denominator. This is called finding a **common denominator**. While any common denominator will do, it is helpful to find the **least common multiple** of the two numbers in the denominator because this will save having to simplify at the end. The least common multiple is the least number that is a multiple of two or more numbers. Least common multiple is sometimes abbreviated LCM.

There are several ways to find common multiples, some of which you used when comparing fractions. To find the least common multiple (LCM), you can list the multiples of each number and determine which multiples they have in common. The least of these numbers will be the least common multiple. Consider the numbers 4 and 6. Some of their multiples are shown below. You can see that they have several common multiples, and the *least* of these is 12.

4	8	**12**	16	20	**24**	28	32	**36**	40	44	**48**	52	56	**60**	64
6	**12**	18	**24**	30	**36**	42	**48**	54	**60**	66	68				

Example		
Problem	**Find the least common multiple of 30 and 50.**	
	30, 60, 90, 120, **150**, 180, 210, 240	List some multiples of 30.
	50, 100, **150**, 200, 250	List some multiples of 50.
	150 is found on both lists of multiples.	Look for the least number found on both lists.
Answer	The least common multiple of 30 and 50 is 150.	

The other method for finding the least common multiple is to use **prime factorization**. This is the method you need for working with rational expressions. The following shows how the factor method works with the numeric example, 4 and 6.

Start by finding the prime factorization of each denominator:

$$4 = 2 \cdot 2$$
$$6 = 3 \cdot 2$$

Identify the greatest number of times any factor appears in either factorization and multiply those factors to get the least common multiple. For 4 and 6, it would be:

$$3 \cdot 2 \cdot 2 = 12$$

Notice that 2 is included twice, because it appears twice in the prime factorization of 4. 12 is the least common multiple of 4 and 6.

The next example also shows how to use prime factorization.

Example		
Problem	**Find the least common multiple of 28 and 40.**	
	$28 = 2 \cdot 2 \cdot 7$	Write the prime factorization of 28.
	$40 = 2 \cdot 2 \cdot 2 \cdot 5$	Write the prime factorization of 40.
	$2 \cdot 2 \cdot 2 \cdot 5 \cdot 7 = 280$	Write the factors the greatest number of times they appear in either factorization and multiply.
Answer	The least common multiple of 28 and 40 is 280.	

Self Check A
Find the least common multiple of 12 and 80.

Finding Least Common Denominators

Objective 2

You can use the least common multiple of two denominators as the **least common denominator** for those fractions. Then you rewrite each fraction using the same denominator.

The example below shows how to use the least common multiple as the least common denominator.

Example	
Problem	**Rewrite the fractions $\frac{2}{3}$ and $\frac{1}{2}$ as fractions with a least common denominator.**
	Multiples of 3: 3, **6**, 9, 12 Multiples of 2: 2, 4, **6** 6 is the least common denominator. — Find the least common multiple of the denominators. This is the least common denominator.
	$\frac{2}{3}\cdot\frac{2}{2}=\frac{4}{6}$ — Rewrite $\frac{2}{3}$ with a denominator of 6.
	$\frac{1}{2}\cdot\frac{3}{3}=\frac{3}{6}$ — Rewrite $\frac{1}{2}$ with a denominator of 6.
Answer	The fraction $\frac{2}{3}$ can be rewritten as $\frac{4}{6}$. The fraction $\frac{1}{2}$ can be rewritten as $\frac{3}{6}$.

Self Check B
Find the least common denominator of $\frac{3}{4}$ and $\frac{1}{6}$. Then express each fraction using the least common denominator.

Adding Fractions with Unlike Denominators

Objective 3

To add fractions with unlike denominators, first rewrite them with like denominators. Then, you know what to do! The steps are shown below.

Adding Fractions with Unlike Denominators

1. Find a common denominator.
2. Rewrite each fraction using the common denominator.
3. Now that the fractions have a common denominator, you can add the numerators.
4. Simplify to lowest terms, expressing improper fractions as mixed numbers.

You can always find a common denominator by multiplying the two denominators together. See the example below.

Example		
Problem	$\frac{2}{3}+\frac{1}{5}$	**Add. Simplify the answer.**
	$3 \cdot 5 = 15$	Since the denominators are not alike, find a common denominator by multiplying the denominators.
	$\frac{2}{3}\cdot\frac{5}{5}=\frac{10}{15}$	Rewrite each fraction with a denominator of 15.
	$\frac{1}{5}\cdot\frac{3}{3}=\frac{3}{15}$	
	$\frac{10}{15}+\frac{3}{15}=\frac{13}{15}$	Add the fractions by adding the numerators and keeping the denominator the same. Make sure the fraction cannot be simplified.
Answer	$\frac{2}{3}+\frac{1}{5}=\frac{13}{15}$	

You can find a common denominator by finding the common multiples of the denominators. The least common multiple is the easiest to use.

Example		
Problem	$\frac{3}{7}+\frac{2}{21}$	**Add. Simplify the answer.**
	Multiples of 7: 7, 14, **21** Multiples of 21: **21**	Since the denominators are not alike, find the least common denominator by finding the least common multiple (LCM) of 7 and 21.
	$\frac{3}{7}\cdot\frac{3}{3}=\frac{9}{21}$	Rewrite each fraction with a denominator of 21.
	$\frac{2}{21}$	
	$\frac{9}{21}+\frac{2}{21}=\frac{11}{21}$	Add the fractions by adding the numerators and keeping the denominator the same. Make sure the fraction cannot be simplified.
Answer	$\frac{3}{7}+\frac{2}{21}=\frac{11}{21}$	

You can also add more than two fractions as long as you first find a common denominator for all of them. An example of a sum of three fractions is shown below. In this example, you will use the prime factorization method to find the LCM.

Example		
Problem	$\frac{3}{4}+\frac{1}{6}+\frac{5}{8}$	**Add. Simplify the answer and write as a mixed number.**
	4 = 2 • 2 6 = 3 • 2 8 = 2 • 2 • 2 LCM: 2 • 2 • 2 • 3 = 24	Since the denominators are not alike, find the least common denominator by finding the least common multiple (LCM) of 4, 6 and 8.
	$\frac{3}{4}\cdot\frac{6}{6}=\frac{18}{24}$ $\frac{1}{6}\cdot\frac{4}{4}=\frac{4}{24}$ $\frac{5}{8}\cdot\frac{3}{3}=\frac{15}{24}$	Rewrite each fraction with a denominator of 24.

	$\frac{18}{24}+\frac{4}{24}+\frac{15}{24}=\frac{37}{24}$	Add the fractions by adding the numerators and keeping the denominator the same.
	$\frac{37}{24}=1\frac{13}{24}$	Write the improper fraction as a mixed number and simplify the fraction.
Answer	$\frac{3}{4}+\frac{1}{6}+\frac{5}{8}=1\frac{13}{24}$	

Self Check C

$\frac{2}{3}+\frac{4}{5}+\frac{1}{12}$ Add. Simplify the answer and write as a mixed number.

Adding Mixed Numbers

Objective 4

When adding mixed numbers you may also need to find a common denominator first. Consider the example below.

Example

Problem	$8\frac{5}{6}+7\frac{4}{9}$	*Add. Simplify the answer and* **write as a mixed number.**
	Multiples of 6: 6, 12, **18** Multiple of 9: 9, **18**	Find a least common denominator for the fractions.
	$\frac{5}{6}\cdot\frac{3}{3}=\frac{15}{18}$ $\frac{4}{9}\cdot\frac{2}{2}=\frac{8}{18}$	Express each fraction with a denominator of 18.
	$8\frac{15}{18}$ $+\,7\frac{8}{18}$	Arrange the mixed numbers vertically so the whole numbers align and the fractions align.

	$8\frac{15}{18}$ $+\ 7\frac{8}{18}$ $15\frac{23}{18}$	Add whole numbers. Add fractions.
	$\frac{23}{18}=1\frac{5}{18}$	Write the improper fraction as a mixed number.
	$15+1+\frac{5}{18}$ $16\frac{5}{18}$	Combine whole numbers and fraction to write a mixed number.
Answer	$8\frac{5}{6}+7\frac{4}{9}=16\frac{5}{18}$	

Self Check D

$3\frac{3}{5}+1\frac{4}{9}$ Add. Simplify the answer and write as a mixed number.

Subtracting Fractions with Unlike Denominators

Objective 5

If the denominators are not the same (they have **unlike denominators**), you must first rewrite the fractions with a common denominator. The **least common denominator**, which is the least common multiple of the denominators, is the most efficient choice, but any common denominator will do. Be sure to check your answer to be sure that it is in simplest form. You can use prime factorization to find the **least common multiple** (LCM), which will be the least common denominator (LCD). See the example below.

Example		
Problem	$\frac{1}{5}-\frac{1}{6}$	**Subtract. Simplify the answer.**
	$5 \cdot 6 = 30$	The fractions have unlike denominators, so you need to find a common denominator. Recall that a common denominator can be found by multiplying the two denominators together.
	$\frac{1}{5}\cdot\frac{6}{6}=\frac{6}{30}$	Rewrite each fraction as an equivalent fraction with a denominator of 30.
	$\frac{1}{6}\cdot\frac{5}{5}=\frac{5}{30}$	
	$\frac{6}{30}-\frac{5}{30}=\frac{1}{30}$	Subtract the numerators. Simplify the answer if needed.
Answer	$\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$	

The example below shows using multiples to find the least common multiple, which will be the least common denominator.

Example		
Problem	$\frac{5}{6}-\frac{1}{4}$	**Subtract. Simplify the answer.**
	Multiples of 6: 6, **12,** 18, 24 Multiples of 4: 4, 8 **12,** 16, 20 12 is the least common multiple of 6 and 4.	Find the least common multiple of the denominators – this is the least common denominator.
	$\frac{5}{6}\cdot\frac{2}{2}=\frac{10}{12}$	Rewrite each fraction with a denominator of 12.
	$\frac{1}{4}\cdot\frac{3}{3}=\frac{3}{12}$	
	$\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$	Subtract the fractions. Simplify the answer if needed.
Answer	$\frac{5}{6}-\frac{1}{4}=\frac{7}{12}$	

Self Check E

$\frac{2}{3}-\frac{1}{6}$ Subtract and simplify the answer.

Subtracting Mixed Numbers

Objective 6

Sometimes you have to find a common denominator in order to solve a mixed number subtraction problem.

Example

Problem $7\frac{1}{2}-2\frac{1}{3}$ **Subtract. Simplify the answer and write as a mixed number.**

$2 \cdot 3 = 6$	Recall that a common denominator can easily be found by multiplying the denominators together.
$\frac{1}{2}\cdot\frac{3}{3}=\frac{3}{6}$ $\frac{1}{3}\cdot\frac{2}{2}=\frac{2}{6}$	Rewrite each fraction using the common denominator 6.
$\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$	Subtract the fractions.
$7-2=5$	Subtract the whole numbers.
$5\frac{1}{6}$	Combine the whole number and the fraction.

Answer $7\frac{1}{2}-2\frac{1}{3}=5\frac{1}{6}$

Self Check F

$9\frac{4}{5}-4\frac{2}{3}$ Subtract. Simplify the answer and write it as a mixed number.

Subtracting Mixed Numbers with Regrouping

Objective 7

The regrouping approach shown in the last section will also work with unlike denominators.

Example

Problem $7\frac{1}{5}-3\frac{1}{4}$ **Subtract. Simplify the answer and write as a mixed number.**

Multiples of 5: 5, 10, 15, **20,** 25
Multiples of 4: 4, 8, 12, 16, **20,** 24
Find a least common denominator. 20 is the least common multiple, so use it for the least common denominator.

$\frac{1}{5}\cdot\frac{4}{4}=\frac{4}{20}$ and $\frac{1}{4}\cdot\frac{5}{5}=\frac{5}{20}$
Rewrite each fraction using the common denominator.

$7\frac{4}{20}-3\frac{5}{20}$
Write the expression using the mixed numbers with the like denominator.

$7\frac{4}{20}=6+1+\frac{4}{20}$
$6+\frac{20}{20}+\frac{4}{20}$
$6+\frac{24}{20}$
$6\frac{24}{20}$
Since the second fraction part, $\frac{5}{20}$, is larger than the first fraction part, $\frac{4}{20}$, regroup one of the whole numbers and write it as $\frac{20}{20}$.

$7\frac{4}{20}-3\frac{5}{20}$
$6\frac{24}{20}-3\frac{5}{20}$
Rewrite the subtraction expression using the equivalent fractions.

$6-3=3$
$\frac{24}{20}-\frac{5}{20}=\frac{19}{20}$
Subtract the whole numbers, subtract the fractions.

$3\frac{19}{20}$
Combine the whole number and the fraction.

Answer $7\frac{1}{5}-3\frac{1}{4}=3\frac{19}{20}$

Self Check G

$6\frac{1}{2}-2\frac{5}{6}$ Subtract. Simplify the answer and write as a mixed number.

Adding and Subtracting Fractions to Solve Problems

Objective 8

Example

Problem

A cake recipe requires $2\frac{1}{4}$ cups of milk and $1\frac{1}{2}$ cups of melted butter. If these are the only liquids, how much liquid is in the recipe?

$2\frac{1}{4}+1\frac{1}{2}$ Find the total amount of liquid by adding the quantities.

$2+1+\frac{1}{4}+\frac{1}{2}$ Group the whole numbers and fractions to make adding easier.

$3+\frac{1}{4}+\frac{1}{2}$ Add whole numbers.

$3+\frac{1}{4}+\frac{2}{4}=3+\frac{3}{4}$ Add fractions. Recall that $\frac{1}{2}=\frac{2}{4}$.

$3\frac{3}{4}$ Combine whole number and fraction.

Answer

There are $3\frac{3}{4}$ cups of liquid in the recipe.

Self Check H

What is the total rainfall in a three-day period if it rains $3\frac{1}{4}$ inches the first day, $\frac{3}{8}$ inch the second day, and $2\frac{1}{2}$ inches on the third day?

Example	
Problem	**Pilar and Farouk are training for a marathon. On a recent Sunday, they both completed a run. Farouk ran $12\frac{7}{8}$ miles and Pilar ran $14\frac{3}{4}$ miles. How many more miles did Pilar run than Farouk?**
	$14\frac{3}{4} - 12\frac{7}{8}$ Write an expression using subtraction to describe the situation.
	$14\frac{6}{8} - 12\frac{7}{8}$ Rewrite the mixed numbers using the least common denominator.
	$14\frac{6}{8}$ $13 + 1 + \frac{6}{8}$ $13 + \frac{8}{8} + \frac{6}{8}$ $13\frac{14}{8}$ Since the fraction part of the second mixed number is larger than the fraction part of the first mixed number, regroup one as a fraction and rewrite the first mixed number.
	$13\frac{14}{8} - 12\frac{7}{8}$ Write the subtraction expression in its new form.
	$1\frac{7}{8}$ Subtract.
Answer	Pilar ran $1\frac{7}{8}$ miles more than Farouk.

Example

Problem

Mike and Jose are painting a room. Jose used $\frac{2}{3}$ of a can of paint and Mike used $\frac{1}{2}$ of a can of paint. How much more paint did Jose use? Write the answer as a fraction of a can.

$\frac{2}{3}-\frac{1}{2}$ Write an expression using subtraction to describe the situation.

$\frac{2\cdot 2}{3\cdot 2}=\frac{4}{6}$ Rewrite the fractions using a common denominator.

$\frac{1\cdot 3}{2\cdot 3}=\frac{3}{6}$

$\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$ Subtract. Check that the fraction is simplified.

Answer

Jose used $\frac{1}{6}$ of a can more paint than Mike.

Self Check I

Mariah's sunflower plant grew $18\frac{2}{3}$ inches in one week. Her tulip plant grew $3\frac{3}{4}$ inches in one week. How many more inches did the sunflower grow in a week than the tulip?

Summary

To adding or subtracting fractions with unlike denominators, first find a common denominator. The least common denominator is easiest to use. The least common multiple can be used as the least common denominator.

2.6 Self Check Solutions

Self Check A

Find the least common multiple of 12 and 80.

240

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 240$.

Self Check B

Find the least common denominator of $\frac{3}{4}$ and $\frac{1}{6}$. Then express each fraction using the least common denominator.

LCD: 12

$\frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12}, \quad \frac{1}{6} \cdot \frac{2}{2} = \frac{2}{12}$

Self Check C

$\frac{2}{3} + \frac{4}{5} + \frac{1}{12}$ Add. Simplify the answer and write as a mixed number.

$\frac{40}{60} + \frac{48}{60} + \frac{5}{60} = \frac{93}{60} = 1\frac{33}{60} = 1\frac{11}{20}$.

Self Check D

$3\frac{3}{5} + 1\frac{4}{9}$ Add. Simplify the answer and write as a mixed number.

$3 + 1 + \frac{3}{5} + \frac{4}{9} \quad 4 + \frac{27}{45} + \frac{20}{45} = 4 + \frac{47}{45} = 4 + 1\frac{2}{45} = 5\frac{2}{45}$.

Self Check E

$\frac{2}{3} - \frac{1}{6}$ Subtract and simplify the answer.

$\frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

Self Check F

$9\frac{4}{5}-4\frac{2}{3}$ Subtract. Simplify the answer and write it as a mixed number.

9 – 4 = 5; $\frac{4}{5}-\frac{2}{3}=\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$. Combining them gives $5\frac{2}{15}$.

Self Check G

$6\frac{1}{2}-2\frac{5}{6}$ Subtract. Simplify the answer and write it as a mixed number.

$6\frac{1}{2}-2\frac{5}{6}=6\frac{3}{6}-2\frac{5}{6}$. Regrouping, $5\frac{9}{6}-2\frac{5}{6}=3\frac{4}{6}=3\frac{2}{3}$

Self Check H

What is the total rainfall in a three-day period if it rains $3\frac{1}{4}$ inches the first day, $\frac{3}{8}$ inch the second day, and $2\frac{1}{2}$ inches on the third day?

$6\frac{1}{8}$ inches

$3\frac{2}{8}+\frac{3}{8}+2\frac{4}{8}=5\frac{9}{8}=6\frac{1}{8}$

Self Check I

Mariah's sunflower plant grew $18\frac{2}{3}$ inches in one week. Her tulip plant grew $3\frac{3}{4}$ inches in one week. How many more inches did the sunflower grow in a week than the tulip?

$18\frac{2}{3}-3\frac{3}{4}=18\frac{8}{12}-3\frac{9}{12}=17\frac{20}{12}-3\frac{9}{12}=14\frac{11}{12}$ inches.