

- Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 4, 9\}$, and $B = \{2, 4, 6, 7\}$. Find each of the following.
 - $A \cap B$
 - $A \cup B$
 - $P = \{x \in U : x > 4 \text{ or } x \leq 7\}$
 - $Q = \{x \in U : x > 4 \text{ and } x \leq 7\}$
- Suppose that S is the set of all squares and R is the set of all rectangles. Label each of the following statements as true or false.
 - $S \subseteq R$
 - $R \subseteq S$
 - $R \subseteq R$
 - $\emptyset \subseteq S$
 - $R \cup S = S$
 - $R \cup S = R$
 - $R \cap S = S$
 - $R \cap S = R$
- Suppose that F is the set of all integers divisible by four, S is the set of all integers divisible by six, and T is the set of all integers divisible by three. Label each of the following statements as true or false.
 - $S \subseteq T$
 - $F \subseteq S$
 - $F \cap T = S$
 - $F \cap T \subseteq S$
 - $F \subseteq S \cup T$
- List all three-digit numbers that can be formed using only the digits 2, 5, 7 and 9.
 - List all three-digit numbers that can be formed using only the digits 2, 5, 7 and 9 and repetition of digits is not allowed. (For example, 225 is not allowed.)
 - List all three-digit numbers that can be formed using only the digits 2, 5, 7 and 9, repetition of digits is not allowed, and are divisible by 3.
- Perform the division with remainder: $2017 \div 17$
- List all factors of 84.
- Consider the following numbers: 2011, 11060904, 321, 3106. Select all the numbers from the list that are divisible
 - by 2
 - by 3
 - by 6
- Which of the following numbers is a prime? 2007, 143, 151, 91
- Find the prime factorization of 720.
- Find the prime factorization for x if
 - $x = 12^{100}$
 - $x = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
 Note that there is a shorter notation for the product above: $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 10!$
- Label each of the following statements as true or false.
 - Every integer is a rational number.
 - Every positive integer can be written as a product of primes.
 - If a is divisible by 2 and b is divisible by 3, then the product ab is divisible by 6.
 - If the product ab is divisible by 6, then a is divisible by 2 and b is divisible by 3 (or vica versa, i.e. a is divisible by 3 and b is divisible by 2.)
 - If the product ab is divisible by 7, then a is divisible by 7 or b is divisible by 7.
 - The sum of two consecutive integers is always an odd number.
- Find the smallest positive integer that is divisible by 2, 3, 4, 5, and 6.
- Simplify each of the following.
 - -3^2
 - $-|-6|$
 - $12 \div 3 \cdot 2$
 - $15 - 3 + 2$
 - $12 - 2(7 - 4 \cdot 3)$
 - $|-8 + 5|$
 - $-\sqrt{49}$
 - $\sqrt{-49}$
 - -2^2
 - $(-2)^2$

14. Simplify each of the following.

a) $|-3^3 - 2|-5 - 2(-4)|$

f) $\sqrt{6^2 - 5\sqrt{16}}$

b) $\sqrt{-4^2 - (-1)^4 + 2 \cdot 3^2 \div 2 \cdot 6 - 1}$

g) $\frac{24 - (-2)^2 + 12 - 3 + 1}{-3^2 - 12 \div 3(-2)}$

c) $-3^2 - |-12 + 2 \cdot 5| - 2 + 1$

h) $|3 - 2 - |8 - 10||$

d) $-2^2 - 5(-2)$

j) $|3 - |2 - 8| - 10|$

e) $-3^2 - 2(4 - 5^2 + 3(10 - 7 + 2))$

k) $|3 - 2|-8 - 10||$

15. Add the algebraic expressions as indicated.

a) $(2a + 3b) + (-2a + 7b)$

b) $(3x - y + 2) + (-x + 6y - 2)$

c) $(3m - 4n) + (5m)$

16. Subtract the algebraic expressions as indicated.

a) $(2a + 3b) - (-2a + 7b)$

b) $(3x - y + 2) - (-x + 6y - 2)$

c) $(3m - 4n) - (5m)$

17. Multiply the algebraic expressions as indicated.

a) $(2a + 3b)(-2a + 7b)$

b) $(3x - y)(-x + 6y)$

c) $(3m - 4n)(5m)$

18. Simplify the algebraic expressions as indicated.

a) $-3(2a + 3b) + 5(-2a + 7b)$

d) $5x - 3(2x - y)$

b) $2(3x - y + 2) - 6(-x + 6y - 2)$

e) $-3m(3m - 4n) + 4n(5m)$

c) $2x(3x - y) - 5y(-x + 4y)$

f) $-y + 2(3x - y) - (2x - y) + 2(-2x + y)$

19. Simplify each of the following.

a) $x^3 \cdot x^8$

b) $(-x)^3(-x)^8$

c) $(-x^3)^8$

d) $(-x^8)^3$

e) $\frac{3x^2(-2y)^3}{4xy}$

20. Multiply the algebraic expressions as indicated.

a) $(x + 1)(3x - 5)$

d) $(3x^5 - 2)(3x^5 + 2)$

g) $(3a^3 - 1)^2$

b) $(3x - y)^2$

e) $(5x + 2)(5x - 2)$

h) $(3m - 2)(9m^2 + 6m + 4)$

c) $(3a - 4b)(5ab)$

f) $(5x - 2)^2$

i) $(3x - 1)^3$

21. Simplify each of the following.

a) $(2x - 1)^2 - 3x(x - 5)$

c) $(2x - 1)(x - 3) - 2(x - 4)^2$

b) $(-x + 2)^2 - (2x - 1)(x + 8)$

d) $x(2x - 5) - 3(2x + 1) - (x - 4)^2$

22. Evaluate the algebraic expression $\frac{-x + 2x^2 - 1}{x - 1}$ if

a) $x = 5$

b) $x = -5$

c) $x = 1$

d) $x = -1$

23. Consider the equation $2x^3 - 10(x^2 - 2) + 4x = -x^2 + 5$. For each of the following numbers given, determine whether it is a solution of the equation or not.

a) $x = -2$

b) $x = -1$

c) $x = 3$

24. Solve each of the following equations. Make sure to check your solutions.

a) $-3x + 5 = 20$

d) $\frac{x}{3} - 8 = -2$

g) $5x - 3(2x - 8) = 24$

b) $-3x + 5 = 7x + 5$

e) $5a - 3 = -3a + 21$

h) $x - (x - 2) - (-3x + 1) = x - 7$

c) $\frac{x - 8}{3} = -2$

f) $5(x - 1) - 3(2x - 7) = 2x + 1$

i) $3x - 4(2x - 8) = x - 6(x - 1)$

l) $5(y - 2) - 3(2y - 7) = 3y - 1 - 4(y - 3)$

j) $3(y - 1) - 5(3y + 2) = -13(y + 1)$

k) $4(x - 3) - 2(x - 1) = x - 2(4 - x)$

m) $(3x - 1) - (5 - 2x) = 5(x - 3) - 2$

25. What is the last digit of the number 2^{99} ?

26. Given that $A \cap B = \{1, 2, 4, 5\}$ and $A \cup B = \{1, 2, 3, 4, 5, 6\}$, how many different sets are possible for A ?

Answers

1. a) $\{4\}$ b) $\{1, 2, 4, 6, 7, 9\}$
 c) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ d) $\{5, 6, 7\}$
2. a) true b) false c) true d) true
 e) false f) true g) true h) false
3. a) true b) false c) false d) true e) false
4. a) There are 64 numbers as listed below.
- | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 222 | 252 | 272 | 292 | 522 | 552 | 572 | 592 |
| 225 | 255 | 275 | 295 | 525 | 555 | 575 | 595 |
| 227 | 257 | 277 | 297 | 527 | 557 | 577 | 597 |
| 229 | 259 | 279 | 299 | 529 | 559 | 579 | 599 |
- | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 722 | 752 | 772 | 792 | 922 | 952 | 972 | 992 |
| 725 | 755 | 775 | 795 | 925 | 955 | 975 | 995 |
| 727 | 757 | 777 | 797 | 927 | 957 | 977 | 997 |
| 729 | 759 | 779 | 799 | 929 | 959 | 979 | 999 |
- b) There are 24 such numbers.
- | | | | |
|-----|-----|-----|-----|
| 257 | 527 | 725 | 925 |
| 259 | 529 | 729 | 927 |
| 275 | 572 | 752 | 952 |
| 279 | 579 | 759 | 957 |
| 295 | 592 | 792 | 972 |
| 297 | 597 | 795 | 975 |
- c) There are 12 such numbers
- | | | | |
|-----|-----|-----|-----|
| 279 | 579 | 729 | 927 |
| 297 | 597 | 759 | 957 |
| | | 792 | 972 |
| | | 795 | 975 |
5. 118 R 11
6. 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
7. a) 11 060 904, 3106 b) 11 060 904, 321
 c) 11 060 904
8. 151
9. $720 = 2^4 \cdot 3^2 \cdot 5$
10. a) $2^{200} \cdot 3^{100}$ b) $2^8 \cdot 3^4 \cdot 5^2 \cdot 7$
11. a) true b) false (1 is the only exception)
 c) true d) false e) true f) true
12. 60
13. a) -9 b) -6 c) 8 d) 14 e) 22
 f) 3 g) -7 h) undefined i) -4 j) 4
14. a) 33 b) 6 c) -12 d) 6 e) 3
 f) 4 g) -30 h) 1 i) 13 j) 33
15. a) $10b$ b) $2x + 5y$ c) $8m - 4n$
16. a) $4a - 4b$ b) $4x - 7y + 4$ c) $-2m - 4n$
17. a) $-4a^2 + 8ab + 21b^2$ b) $-3x^2 + 19xy - 6y^2$
 c) $-20mn + 15m^2$
18. a) $-16a + 26b$ b) $12x - 38y + 16$ c) $6x^2 + 3xy - 20y^2$
 d) $-x + 3y$ e) $-9m^2 + 32mn$ f) 0
19. a) x^{11} b) $-x^{11}$ c) x^{24} d) $-x^{24}$ e) $-6xy^2$
20. a) $3x^2 - 2x - 5$ b) $9x^2 - 6xy + y^2$ c) $15a^2b - 20ab^2$
 d) $9x^{10} - 4$ e) $25x^2 - 4$ f) $25x^2 - 20x + 4$
 g) $9a^6 - 6a^3 + 1$ h) $27m^3 - 8$ i) $27x^3 - 27x^2 + 9x - 1$
21. a) $x^2 + 11x + 1$ b) $-x^2 - 19x + 12$ c) $9x - 29$
 d) $x^2 - 3x - 19$
22. a) 11 b) -9 c) undefined d) -1
23. a) no, $-44 \neq -2$ b) yes, $4 = 4$ c) yes, $-4 = -4$
24. a) -5 b) 0 c) 2 d) 18 e) 3
 f) 5 g) 0 h) -4 i) no solution
 j) 0 k) -2 l) all real numbers
 m) no solution
25. 8
26. 4

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