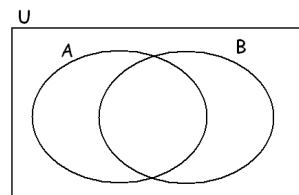


1. Suppose that  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{1, 2, 3, 4, 5\}$ , and  $B = \{1, 4, 9\}$ . Find each of the following.

- a)  $A \cap B$    b)  $A \cup B$    c)  $A \cup \bar{B}$    d)  $\overline{A \cup B}$    e)  $\bar{A} \cup B$

2. Consider the Venn-diagram shown on the picture, depicting sets  $A$  and  $B$  in a universal set  $U$ . Shade the region corresponding to each of the following sets.



- a)  $A \cap B$    c)  $A \cup \bar{B}$    e)  $\bar{A} \cup B$   
 b)  $A \cup B$    d)  $\overline{A \cup B}$

3. Compute each of the following sets.

- a)  $(1, 5) \cup [2, 8]$    c)  $(-\infty, 5) \cup (-2, \infty)$    e)  $(3, 4) \cup [1, 7]$    g)  $(-\infty, -3) \cup (-1, 12)$   
 b)  $(1, 5) \cap [2, 8]$    d)  $(-\infty, 5) \cap (-2, \infty)$    f)  $(3, 4) \cap [1, 7]$    h)  $(-\infty, -3) \cap (-1, 12)$

4. Label each of the following statements as true or false.

- a) If a natural number  $n$  is divisible by 6, then its square is divisible by 9.  
 b) For all sets  $A$ ,  $A \cap \bar{A} = U$ .  
 c) For all sets  $A$  and  $B$ , if  $A \cup B = A$ , then  $A$  is a subset of  $B$ .  
 d) For all sets  $A$  and  $B$ , if  $A \cup B = A$ , then  $B$  is a subset of  $A$ .  
 e) Suppose that  $x$  and  $y$  are positive integers. If the product  $xy$  is divisible by 3, then  $x$  is divisible by 3 or  $y$  is divisible by 3.  
 f) Suppose that  $x$  and  $y$  are positive integers. If the product  $xy$  is divisible by 6, then  $x$  is divisible by 6 or  $y$  is divisible by 6.  
 g) There is no right triangle with all three sides odd integers.

5. Simplify each of the following expressions.

- a)  $\frac{x^2 - 10x + 21}{9 - x^2}$    e)  $\frac{12x - 8}{-4}$    j)  $\frac{2^{-2}3^{-1}}{2^03^{-2}5^{-1}}$   
 b)  $\frac{5x - 30}{x^2 - 36} \cdot \frac{3x + 18}{5}$    f)  $\sqrt{125} - 3\sqrt{80} + \sqrt{45}$    k)  $\frac{2^{-2} - 3^{-1}}{2^0 - 3^{-2}}$   
 c)  $\frac{a^3 - 4a}{a^2 + 2a} \div \frac{a^2 - 8a + 12}{a^2 - 6a}$    h)  $(\sqrt{3} - 1)^3 (\sqrt{3} + 1)^3$    l)  $\left(\frac{-2a^0ba^{-5}}{3a^{-2}b^2a^{-3}}\right)^{-3}$   
 d)  $\frac{x - 2}{4 - 2x}$    i)  $\frac{\sqrt{40}}{6}$    m)  $\frac{-2a^7b^0(2a^5b^{-3}a^{-1})^{-1}}{b^5(4a^{-3}b^2)^{-3}(-a)^{-4}}$

6. Expand each of the following.

- a)  $(3x - 1)^2$    b)  $(3x - 1)^3$    c)  $(3x^2 - 2)^2$    d)  $(x - y)(x^2 + xy + y^2)$

7. Rationalize the denominator in each of the following expressions.

- a)  $\frac{3}{\sqrt{5}}$    b)  $\frac{1}{\sqrt{10} - 3}$    c)  $\frac{2}{\sqrt{7} + 1}$    d)  $\frac{3 + \sqrt{3}}{3 - \sqrt{3}}$    e)  $\frac{2\sqrt{x}}{\sqrt{x} - 3}$

8. Convert each of the following decimals to a fraction of integers.

- a) 1.037   b)  $0.78\overline{70}$    c)  $0.61\overline{8}$

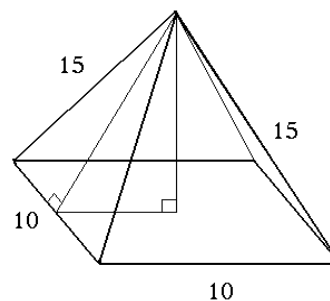
9. Compute each of the following sums.

- a)  $8 + 16 + 24 + \dots + 2016$    b)  $20 + 23 + 26 + \dots + 98$    c)  $320 + 335 + 350 + \dots + 2120$

10. Factor  $3x^2 - 7x - 6$  over the real numbers **by completing the square**.
11. A conference had 50 attendees, 35 men and 15 women.
- If every attendee shook hands with everyone else, how many hand shakes took place?
  - If women greet *only* women with a hand shake, and men greet *only* men with a hand shake, how many hand shakes took place?
12. Completely factor each of the following over the real numbers.
- $x^2 - y^2 + 2y - 1$
  - $24x - 9x^2 - 16$
  - $3a^2 - 5a - 2$
  - $30x - 3x^2 - 78$
  - $13x + 2x^2 - 24$
  - $x^2 - 6x + 8$
  - $4b^2 - b - 5$
  - $5a^5 - 80a$
13. Solve each of the following equations over the real numbers. Make sure to check your solution(s).
- $2x^3 = 20x^2 + 1750x$
  - $8x^3 = 50x^2$
  - $5 - 2\sqrt{x} = -7$
  - $\frac{3x + 17}{2} = x - 1 + \frac{x + 19}{2}$
  - $8p^3 = 50p$
  - $\sqrt{x - 1} = x - 7$
  - $\frac{3}{8}x - \frac{1}{5} = -\frac{7}{40}$
  - $x^2 = 4x - 5$
  - $x - 1 = 2\sqrt{3x - 8}$
  - $\frac{2}{3}(x - 7) = \frac{4}{5}(x + 1)$
  - $x - 2\sqrt{x - 3} = 3$
  - $x - \sqrt{4x - 3} = 0$
  - $8a + 2a^2 = 42$
  - $\sqrt{2x - 3} = -11$
  - $x(x - 5) + 1 = 3(x - 2)$
  - $\sqrt{2x + 1} + 8 = 3$
- r)  $7x^2 + (x + 3)(2x - 1) = (3x + 1)^2$       s)  $2 - (3 - x)(2x + 5) = (x - 1)(2x - 1)$
14. Find the smallest value of the given expressions.
- $x^2 - 20x + 38$
  - $x^4 + 2x^2 - 2$
  - $x^4 - 2x^2 - 2$
15. a) Graph the line  $3x - 4y = -5$ . Make sure to find two lattice points.  
 b) Graph the straight lines  $3x + 5y = 5$  and  $y = -x - 1$  in the same coordinate system. Use your graph to find the coordinates of the point where the lines intersect.
16. Graph the parabola  $y = x^2 - 6x + 5$ . Clearly state the coordinates of five points on the parabola, including vertex and intercepts.
17. Solve each of the following inequalities. Present your answer in interval notation.
- $\frac{2x - 1}{3} - \frac{3x + 1}{4} \leq x - 6$
  - $\frac{2}{3}x - \frac{5}{6} \geq -\frac{1}{3}$
  - $(x + 6)^2 < (x + 4)^2$
  - $2(x - 3) - x - 16 > 3x + 2 - 4(1 - 2x)$
18. Solve each of the following compound inequalities. Present your answer in interval notation.
- $\frac{2x - 1}{-3} > 5$  and  $(x - 5)^2 < (x + 1)^2$
  - $\frac{2}{3}x - 1 \leq 5$  and  $\frac{3}{4}x + 1 < \frac{2}{3}x - 1$
  - $\frac{2x - 1}{-3} > 5$  or  $(x - 5)^2 < (x + 1)^2$
  - $\frac{2}{3}x - 1 \leq 5$  or  $\frac{3}{4}x + 1 < \frac{2}{3}x - 1$
19. a) One number is four less than three times another number. Find these numbers if their sum is 64.  
 b) One number is four less than three times another number. Find these numbers if their product is 160.
20. Find the distance between the points  $A(-3, -5)$  and  $B(3, 3)$ .

21. a) Find the exact value of the area of a triangle with sides 8 m, 6 m, and 6 m long.  
 b) Find the exact value of the area of a regular triangle with sides 5 meters long.
22. An arch is in the shape of a semicircle. At a point along the base 1 foot from an end of the arch, the height of the arch is 5 feet. Find the maximum height of the arch. Present exact value of the answer.

23. The picture shows a straight pyramid with a square base. The sides of the base are 10 in long. The other sides are 15 in long.
- a) Find the exact value of the height of a triangular face. This is called the slant height.  
 b) Use part a) to find the height of the pyramid.

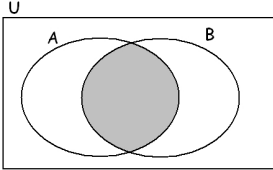
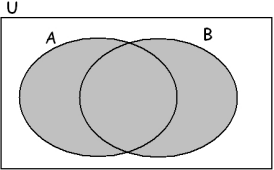
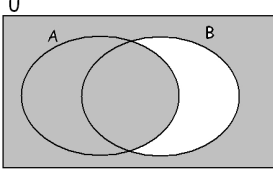
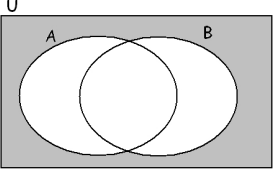
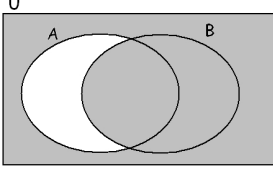


24. Find the sides in a rectangle if we know that one side is 12 cm and the diagonal of the rectangle is 2 cm longer than the other side.
25. Compute the perimeter and area of the parallelogram determined by the points  $A(1, 3)$ ,  $B(5, 6)$ ,  $C(5, 15)$ , and  $D(1, 12)$ .
26. Solve each of the following word problems.
- a) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft.
- b) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is  $84 \text{ ft}^2$ .
- c) One side of a rectangle is 4 in shorter than 3 times the other side. Find the sides of the rectangle if its area is  $319 \text{ in}^2$ .
- d) A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is \$610. How much of each denomination of bill does he have?
- e) The first row in a theater has 18 seats in it. The second row has four more seats than the first row. The third row has four more seats than the second row. And so on, each row has four more seats than the row before. If the last row has 114 seats in it, how many seats are there in the entire theater?
- f) During a volatile day in the markets, our stock has first gained 12% of its value. In a few hours, the stock lost 10% of its value. Express the two changes together as a single change. What percentage of a change is this?
- g) Eight times a number is nine less than the square of the number. Find this number.
- h) A TV set went on a 12% off sale. If the sale price is \$1848, what was the original price?
- i) The hypotenuse of right triangle is 50 meters long. The difference between the other two sides is 34 meters. Find the missing sides. Use exact values.
- j) During a volatile day in the markets, our stock has first gained 20% of its value. In a few hours, the stock lost 20% of its value. Express the two changes together as a single change. What percentage of a change is this?
- k) Find the sides of the square if we know the following: if we increase each side by 2 cm, the square's area will increase by  $48 \text{ cm}^2$ .
- l) We throw an object upward from the top of a 1904 ft tall building. The vertical position  $h$  of the object, (measured in feet)  $t$  seconds after we threw it is

$$h = -16t^2 + 160t + 1904$$

How long does it take for the object to hit the ground?

## Answers

1. a)  $\{1, 4\}$  b)  $\{1, 2, 3, 4, 5, 9\}$   
 c)  $\{1, 2, 3, 4, 5, 6, 7, 8, 10\}$  d)  $\{6, 7, 8, 10\}$   
 e)  $\{1, 4, 6, 7, 8, 9, 10\}$
2. a)  b)   
 c)  d)   
 e) 
3. a)  $(1, 8]$  b)  $[2, 5)$  c)  $(-\infty, \infty)$  d)  $(-2, 5)$   
 e)  $[1, 7]$  f)  $(3, 4)$   
 g)  $(-\infty, -3) \cup (-1, 12)$  cannot be simplified  
 h)  $\emptyset$
4. a) true b) true c) false d) true  
 e) true f) false g) true
5. a)  $\frac{-x+7}{x+3}$  b) 3 c)  $a$  d)  $-\frac{1}{2}$  e)  $-3x+2$   
 f)  $-4\sqrt{5}$  g)  $11-4\sqrt{7}$  h) 8 i)  $\frac{\sqrt{10}}{3}$   
 j)  $\frac{15}{4}$  k)  $-\frac{3}{32}$  l)  $-\frac{27b^3}{8}$  m)  $-\frac{64b^4}{a^2}$
6. a)  $9x^2 - 6x + 1$  b)  $27x^3 - 27x^2 + 9x - 1$   
 c)  $9x^4 - 12x^2 + 4$  d)  $x^3 - y^3$
7. a)  $\frac{3}{\sqrt{5}}$  b)  $\sqrt{10} + 3$  c)  $\frac{\sqrt{7}-1}{3}$  d)  $2 + \sqrt{3}$   
 e)  $\frac{2x + 6\sqrt{x}}{x-9}$
8. a)  $\frac{1037}{1000}$  b)  $\frac{7792}{9900}$  c)  $\frac{557}{900}$
9. a) 255 024 b) 1593 c) 147 620

10.  $3\left(x + \frac{2}{3}\right)(x-3) = (3x+2)(x-3)$

11. a) 1225 b) 700

12. a)  $(x-y+1)(x+y-1)$

b)  $2(x+8)\left(x-\frac{3}{2}\right) = (x+8)(2x-3)$

c)  $-9\left(x-\frac{4}{3}\right)^2$  d)  $(x-2)(x-4)$

e)  $(a-2)(3a+1)$  f)  $(4b-5)(b+1)$

g)  $-3(x^2-10x+26)$

h)  $5a(a^2+4)(a+2)(a-2)$

13. a)  $-35, 0, 25$  b) identity, all real numbers are

solution c)  $\frac{1}{15}$  d)  $-41$  e)  $-7, 3$

f)  $\frac{25}{4}, 0$  g)  $-\frac{5}{2}, 0, \frac{5}{2}$  h) no real solution

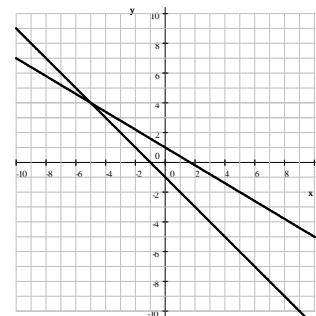
i) 3, 7 j) 5 (0 does not work) k) no real solution

l) 36 m) 10 (5 does not work) n) 3, 11 o) 1, 3

p) 1, 7 q) no real solution r)  $-4$  s) 7

14. a)  $-62$  b)  $-2$  c)  $-3$

15. a)  b)  $(-5, 4)$



16.  $y$ -intercept:  $(0, 5)$

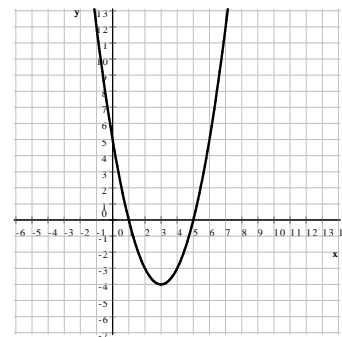
vertex:  $(3, -4)$

$x$ -intercepts:

$(1, 0)$  and  $(5, 0)$

additional points:

$(2, -3), (4, -3), (6, 5)$



17. a)  $[5, \infty)$  b)  $(-\infty, -5)$  c)  $\left[\frac{3}{4}, \infty\right)$  d)  $(-\infty, -2)$

18. a)  $\emptyset$  b)  $(-\infty, -7) \cup (2, \infty)$  c)  $(-\infty, -24)$   
d)  $(-\infty, 9]$
19. a) 17 and 47 b) 8 with 20 and  $-\frac{20}{3}$  with  $-24$
20. 10 units 21. a)  $8\sqrt{5} \text{ m}^2$  b)  $\frac{25\sqrt{3}}{4} \text{ m}^2$
22. 13 ft
23. a)  $\sqrt{200} \text{ in} = 10\sqrt{2} \text{ in}$  b)  $\sqrt{175} \text{ in} = 5\sqrt{7} \text{ in}$
24. 12 cm by 35 cm 25.  $P = 28$  unit,  $A = 36 \text{ unit}^2$
26. a) 9 ft and 23 ft b) 6 ft and 14 ft  
c) 11 in by 29 in  
d) 33 ten-dollar bills and 56 five-dollar bills  
e) 1650 f) 0.8% increase g)  $-1, 9$  h) \$2100  
i) 14 m, 48 m, 50 m j) 4% decrease k) 11 cm  
l) 17 seconds

Last revised: November 9, 2017