

1. Simplify each of the following expressions.

a) $(\sqrt{3})^4$ b) $(\sqrt[6]{5})^{12}$ c) $(-27)^0$ d) -27^0 e) -2^{-3} f) -3^{-2} g) $\frac{1}{7^{-2}}$

2. Perform the given set of operations on the intervals.

a) $(-1, 5) \cup [3, 8]$ c) $(-\infty, 14] \cup (-3, \infty)$ e) $(-\infty, -2) \cup [5, \infty)$
 b) $(-1, 5) \cap [3, 8]$ d) $(-\infty, 14] \cap (-3, \infty)$ f) $(-\infty, -2) \cap [5, \infty)$

3. Simplify each of the following expressions. Assume that all variables represent positive numbers.

a) $\frac{x^3 y^0 (-x^4 y^{-2} x^{-2})^{-3}}{(2x)^{-2} y x^0}$ c) $\sqrt{\frac{x^3}{x^2 (x-3)^5}}$ e) $\frac{x}{\sqrt{x}}$ g) $\frac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}}$
 b) $(x^{-3})^{-5} x^{-6}$ d) $\frac{1}{x^{-1}}$ f) $2^{-1} - 5^{-1}$ h) $\left(\frac{1 - x^{-3}}{3x^{-2}}\right)^0$

4. a) Solve the given equation over the real numbers by completing the square. $3x^2 = 4x + 1$

b) Use exact values to check your solution(s).

5. Simplify each of the following expressions.

a) $\frac{(x-5)^2}{x^2-25}$ b) $\frac{x^2-5x}{x^2-2x-15} \cdot \frac{x^2-9}{x^2-3x}$ c) $\frac{x^2-4x-12}{x^2-2x-8} \div \frac{x^2-6x}{x^2-16}$

6. Simplify each of the following. If the denominator is irrational, rationalize it.

a) $(7 - 3\sqrt{2})^2$ c) $\frac{2}{\sqrt{29} + 5}$ e) $3\sqrt{8} + 2\sqrt{18} - \sqrt{50}$ g) $\frac{\sqrt{5} - 1}{-\sqrt{5} + 1}$
 b) $(2 - \sqrt{5})^3 (2 + \sqrt{5})^3$ d) $\frac{4x - 9}{2\sqrt{x} - 3}$ f) $\frac{-6 + \sqrt{24}}{-10}$ h) $\frac{1}{1 + \sqrt{3}} - \frac{1}{1 - \sqrt{3}}$

7. Find the greatest or smallest value of each of the given expressions. State which (smallest or greatest) in each expression.

a) $3x^2 - 24x + 45$ b) $-2x^2 - 4x + 16$ c) $5x^2 - 30x + 55$ d) $-\frac{1}{2}x^2 + 4x - 8$

8. Completely factor each of the following binomials over the real numbers. If an expression cannot be factored, state so.

a) $x^2 + x$ b) $x^3 - x$ c) $x^4 + x^3$ d) $x^5 + x^3$ e) $x^6 - x^4$ f) $x^7 - x^3$

9. Completely factor each of the following trinomials over the real numbers. If an expression cannot be factored, state so.

a) $x^2 - 6x + 5$ c) $x^2 - 6x + 13$ e) $3x^2 - 14x - 5$
 b) $x^2 - 6x + 9$ d) $x^2 - 6x + 7$ f) $x^2 - x - 1$

10. Solve each of the following system of equations.

a)
$$\begin{cases} (x-2)^2 + (y-3)^2 = x^2 + (1-y)^2 \\ x = 3y - 1 \end{cases}$$

b)
$$\begin{cases} \frac{1}{2}(x+3) + \frac{1}{3}(y-2) = x-4 \\ \frac{1}{6}(x-1) - \frac{1}{2}(y+8) = y+3 \end{cases}$$

11. Solve each of the following compound inequalities.

a) $-\frac{1}{3}x + 1 < -8$ and $(x + 1)^2 - (x - 1)^2 \leq -16$ c) $(2x - 5)^2 > 4x^2 - 15x$ and $\frac{1}{3}x + 1 > \frac{5}{6}$
 b) $-\frac{1}{3}x + 1 < -8$ or $(x + 1)^2 - (x - 1)^2 \leq -16$ d) $(2x - 5)^2 > 4x^2 - 15x$ or $\frac{1}{3}x + 1 > \frac{5}{6}$

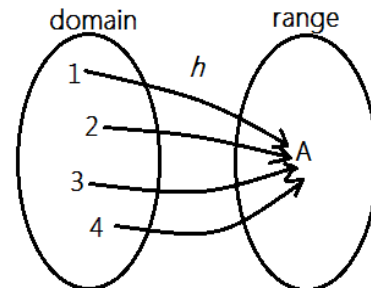
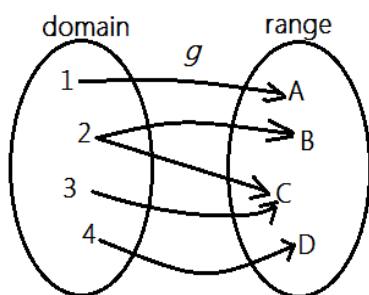
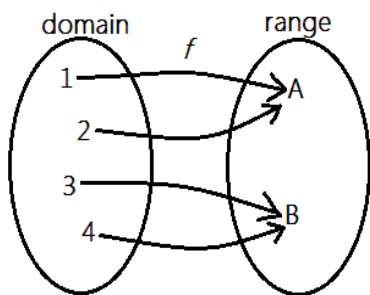
12. Solve each of the following equations over the real numbers. Make sure to check your solutions.

a) $\frac{5 - 4x}{3} - \frac{2x - 7}{5} = -2x + 2$ d) $2(x - 3) - \frac{x}{2} = \frac{3}{2}(x - 4)$ g) $x^2 + 134 = 22x$
 b) $x^3 - 2x^2 - 35x = 0$ e) $2x^2 - 32x = 0$ h) $2 = x - \sqrt{3x + 4}$
 c) $3|x + 3| - 5 = 10$ f) $4x + x^3 = 6x^2$ i) $5 - (2 - x)(x + 3) = (x - 2)^2$
 j) $1 + 4\sqrt{x - 2} = -x$

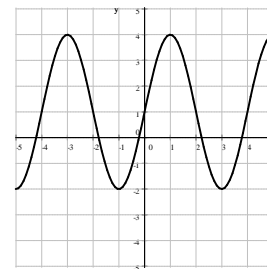
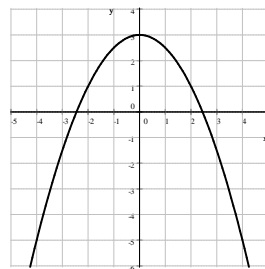
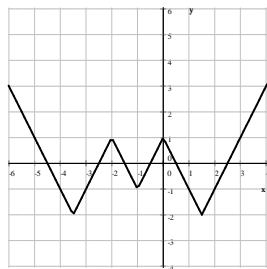
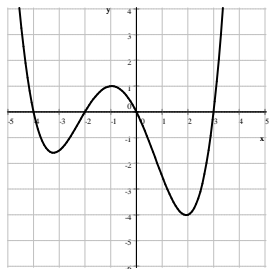
13. Solve each of the following absolute value equations.

a) $|2x + 1| = -7$ d) $|2x + 1| = x - 5$ f) $12 - |3x - 1| = 8$
 b) $|2x + 1| = 7$ e) $3\left|\frac{1}{2}x + 2\right| - 1 = 20$ g) $3\left|\frac{1}{2}x - 5\right| + 1 = -8$
 c) $|2x + 1| = |x - 5|$

14. Determine which of the given relations are functions.



15. The graph of each of the following functions are given, but not its equation. Use the graph to find an approximate value of each of the following.



a) $f(-1)$ c) $f(3)$ d) $g(-4)$ f) $g(4)$ g) $h(-4)$ i) $h(0)$ j) $j(-4)$ l) $j(3)$
 b) $f(2)$ e) $g(0)$ h) $h(-2)$ k) $j(1)$

16. In each case, graph the parabola given. State the coordinates of at least five points, including vertex and intercepts. a) $y = x^2 + 2x - 3$ b) $y = -2x^2 + 4x + 6$

17. Suppose that $f(x)$ is a function given by $f(x) = -x^2 + 10x - 1$. Find the value of each of the following.

- a) $f(0)$ c) $f(-3)$ d) $\frac{f(-3)}{f(3)}$ e) $f(-1)$ g) $f(5 - \sqrt{2})$ i) $f(f(10))$
 b) $f(3)$ f) $f(\sqrt{2})$ h) $f(f(1))$

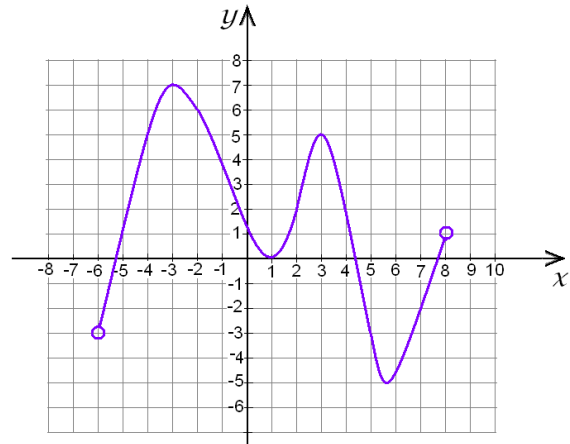
- j) Find all values of x with $f(x) = -1$ k) Find all values of x with $f(x) = 0$

18. Suppose that $f(x) = -2x + 5$ with domain \mathbb{R} . Compute each of the following.

- a) $f(2 + 3)$ c) $f(2 \cdot 3)$ e) $(f(3))^2$ g) $f(f(2))$
 b) $f(2) + f(3)$ d) $2f(3)$ f) $(f(3) + 3)^2$ h) $f(f(5))$

19. The graph of a function f is given, but not its equation.

- a) What is the domain of f ?
 b) What is the range of f ?
 c) Compute $f(-4) + f(4)$
 d) Compute $f(-3)f(-1)$
 e) Compute $f(-5)f(1)$



20. Re-write each of the following decimals as a fraction of two integers. You do NOT have to bring the fraction to lowest terms.

- a) $0.65\overline{2} = 0.65222222\dots$ b) $0.87\overline{9} = 0.87979797979\dots$ c) $0.99\overline{2016} = 0.99201620162016\dots$

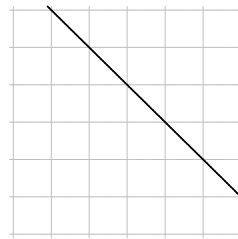
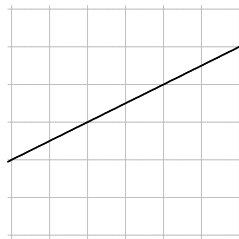
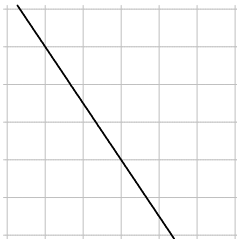
21. Compute the sum $91 + 103 + 115 + \dots + 715$

22. Find the exact value of the length of the main diagonal in a rectangular prism with edges 2 ft, 3 ft, and 5 ft long.

23. Express two consecutive 30% increases as a single change. What percentage of a change does this represent?

24. In each of the cases given, compute the slope of the line.

- a) a horizontal line c) the line passing through the points $A(3, -1)$ and $B(-5, 7)$
 b) a vertical line d) the line passing through the points $N(-3, 4)$ and $M(-3, 7)$
 e) f) g)



25. Find the value of k if the slope of the line connecting the points $A(3, -1)$ and $B(8, k)$ is 2.

26. The difference between two integers is 26. Their product is 1767. Find these numbers.

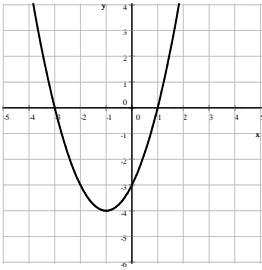
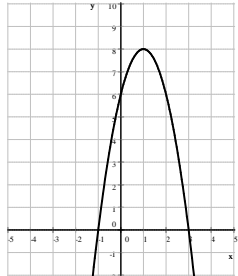
27. a) How many different relations are possible with domain $D = \{1, 2, 3\}$ and range being any non-empty subset of $S = \{a, b, c\}$?
- b) How many different functions are possible with domain $D = \{1, 2, 3\}$ and range being any non-empty subset of $S = \{a, b, c\}$?
- c*) How many different relations are possible with domain $D = \{1, 2, 3\}$ and range $R = \{a, b, c\}$?
- d) How many different functions are possible with domain $D = \{1, 2, 3\}$ and range $R = \{a, b, c\}$?

28. We are standing on the top of a 960 feet tall building and launch a small object upward. The object's vertical position, measured in feet, after t seconds is

$$h(t) = -16t^2 + 64t + 960$$

- a) How long until the object hits the ground?
- b) What is the highest point that the object will reach? When?
29. The population of a town has decreased from 75000 to 65250. What percent of a change does this represent?
30. The budget increased by 15%. If the new budget is 1811 250, how much was the old budget?
31. The sum of five times a number and -10 is 8 less than six times the sum of 7 and the opposite of the number. Find this number.
32. We have invested \$8000 into two bank accounts: one earns 6% interest, the other one earns 9% interest. How much money did we invest into each account if the combined interest was \$660?
33. The hypotenuse of a right triangle is 26 cm. The difference between the other two sides is 14 cm. Find the missing sides.
34. Lisa took 5 exams. The first 4 received scores of 72, 93, 86, and 82. How much did she score on the fifth exam if her average score is 74 points?
35. A number is exactly two greater than its own reciprocal. Find this number.
36. The base of a right triangle is 8 units longer than its height. Find the base if we know that the area of the triangle is 120 unit².
37. Today we had 120 students visiting the museum as part of a field trip. Due to a soccer game, the girls team needed to leave earlier. 25% of the attending female students left early, causing a 15% decrease in the total attendance. How many of the attending students were male?
38. Last week the shelter had lots of adoptable pets, all cats and dogs. On the great adoption day, three-fourth of the cats and one-sixth of the dogs were adopted. If these lucky pets represented half of the shelter's population, then what was the ratio of cats to dogs?

Answers

1. a) 9 b) 25 c) 1 d) -1 e) $-\frac{1}{8}$ f) $-\frac{1}{9}$ g) 49
2. a) $(-1, 8]$ b) $[3, 5)$ c) $(-\infty, \infty)$ d) $(-3, 14]$
e) $(-\infty, -2) \cup [5, \infty)$ f) \emptyset
3. a) $-\frac{4y^5}{x}$ b) x^9 c) x^8 d) x e) \sqrt{x} f) $\frac{3}{10}$
g) $\frac{xy}{x+y}$ h) 1
4. a) $\frac{2-\sqrt{7}}{3}$ and $\frac{2+\sqrt{7}}{3}$
5. b) If $x = \frac{2-\sqrt{7}}{3}$, then
- $$\begin{aligned} \text{LHS} &= 3 \left(\frac{2-\sqrt{7}}{3} \right)^2 = 3 \cdot \frac{(2-\sqrt{7})^2}{3^2} \\ &= \frac{3(11-4\sqrt{7})}{9} = \frac{11-4\sqrt{7}}{3} \\ \text{RHS} &= 4 \left(\frac{2-\sqrt{7}}{3} \right) + 1 = \frac{4(2-\sqrt{7})}{3} + \frac{3}{3} \\ &= \frac{8-4\sqrt{7}+3}{3} = \frac{11-4\sqrt{7}}{3} \end{aligned}$$
- Checking the other solution goes similarly.
6. a) $\frac{x-5}{x+5}$ b) 1 c) $\frac{x+4}{x}$
7. a) $67-42\sqrt{2}$ b) -1 c) $\frac{\sqrt{29}-5}{2}$ d) $2\sqrt{x}+3$
e) $7\sqrt{2}$ f) $\frac{3-\sqrt{6}}{5}$ g) -1 h) $\sqrt{3}$
8. a) -3 smallest value b) 18 greatest value
c) 10 smallest value d) 0 greatest value
9. a) $x(x+1)$ b) $x(x-1)(x+1)$ c) $x^3(x+1)$
d) $x^3(x^2+1)$ e) $x^4(x-1)(x+1)$
f) $x^3(x^2+1)(x-1)(x+1)$
10. a) $(x-1)(x-5)$ b) $(x-3)^2$
c) cannot be factored d) $(x-3-\sqrt{2})(x-3+\sqrt{2})$
e) $(3x+1)(x-5)$ f) $\left(x-\frac{1+\sqrt{5}}{2}\right)\left(x-\frac{1-\sqrt{5}}{2}\right)$
11. a) (2, 1) b) (7, -4)
12. a) no solution b) $(-\infty, -4] \cup (27, \infty)$ c) $\left(-\frac{1}{2}, 5\right)$
d) \mathbb{R}
13. a) -4 b) 7, 0, -5 c) -8, 2
d) identity, all numbers are solution e) 0, 16
f) 0, $3-\sqrt{5}$, $3+\sqrt{5}$ g) no real solution
- h) 7 (0 is extreme) i) 1
j) no solution (both 3 and 11 are extreme)
14. a) no solution b) 3, -4 c) $-6, \frac{4}{3}$
d) no solution e) 10, -18 f) $-1, \frac{5}{3}$
g) no real solution
15. f and h
16. a) (-6, 8) b) [-5, 7] c) 7 d) 28 e) 0
17. a) $y = x^2 + 2x - 3$
 $= (x+1)^2 - 4$
 $= (x+3)(x-1)$
vertex: (-1, -4)
x-intercepts: (-3, 0) and (1, 0)
y-intercept: (0, -3)
- b) $y = -2x^2 + 4x + 6$
 $= -2(x-1)^2 + 8$
 $= -2(x+1)(x-3)$
vertex: (1, 8)
x-intercepts: (-1, 0) and (3, 0)
y-intercept: (0, 6)
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18. a) -1 b) 20 c) -40 d) -2 e) -12
f) $10\sqrt{2}-3$ g) 22 h) 15 i) -12 j) 0, 10
k) $5+2\sqrt{6}$, $5-2\sqrt{6}$
19. a) 1 b) -4 c) 0 d) -1 e) 1 f) 3 g) -5
h) 1 i) 3 j) 1 k) 4 l) -2
20. a) -5 b) 0 c) -7 d) -2 e) 1 f) 4 g) 3 h) 15
21. a) $\frac{587}{900}$ b) $\frac{871}{990}$ c) $\frac{991\ 917}{999\ 900}$ 22. 21 359
23. $\sqrt{38}$ ft 24. 69% increase
25. a) 0 b) undefined c) -1 d) undefined
e) $-\frac{3}{2}$ f) $\frac{1}{2}$ g) -1 26. 9
27. -57, -31 and 31, 57 28. a) 343 b) 27 c) 265 d) 6
29. a) 10 seconds b) 1024 ft, after 2 seconds
30. 13% decrease 31. 1575 000 32. 4
33. \$2000 at 6% and \$6000 at 9% 33. 10 cm and 24 cm
34. 37 35. $1+\sqrt{2}$ and $1-\sqrt{2}$ 36. 20 unit
37. 48 38. cats to dogs is 4 to 3