

1. Simplify each of the following.

$$\begin{array}{llll} \text{a) } \frac{\frac{1}{2} + \frac{1}{5} \cdot 3}{\frac{1}{2} - \frac{1}{5} \cdot 3} & \text{b) } -\frac{2}{5} + \frac{1}{4} \left(-\frac{3}{2}\right)^2 & \text{c) } 1 - \frac{2}{3 - \frac{4}{5 - \frac{1}{6}}} & \text{d) } \frac{1}{3} - \left(\left(-\frac{2}{3}\right)^2 - \frac{5}{6}\right) \end{array}$$

2. Simplify each of the following. Use exact values in each step.

$$\begin{array}{llll} \text{a) } 3\sqrt{5} - \sqrt{5} & \text{c) } \frac{\sqrt{45}}{\sqrt{5}} & \text{e) } (2\sqrt{3} - 1)(\sqrt{3} + 2) & \text{h) } (\sqrt{3} - \sqrt{2})^2 \\ \text{b) } \sqrt{18} - \sqrt{50} & \text{d) } \sqrt{2}(3\sqrt{2} - 1) & \text{f) } (\sqrt{10} - 3)(\sqrt{10} + 3) & \text{i) } (2 - \sqrt{3})^3 \\ & & \text{g) } (2\sqrt{5} - 1)^2 & \text{j) } (\sqrt{5} + 1)^2 - (\sqrt{5} - 1)^2 \end{array}$$

3. Compute each of the following sums.

$$\text{a) } 20 + 23 + 26 + \dots + 152 \quad \text{b) } 137 + 142 + 147 + \dots + 632 \quad \text{c) } 387 + 398 + 409 + \dots + 2587$$

4. Prove that each of the given decimal represents a rational number by converting each of them to a quotient of two integers. You do not need to reduce the fractions.

$$\text{a) } 3.407 \quad \text{b) } 0.3\overline{20} \quad \text{c) } 0.7\overline{819} \quad \text{d) } 0.33\overline{4}$$

5. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find each of the given sets.

$$\begin{array}{lll} \text{a) } \{x \in U : x \geq 8 \text{ and } x \text{ is even}\} & \text{c) } \{x \in U : x < 3 \text{ and } x > 7\} & \text{e) } \{x \in U : x \geq 4 \text{ and } x > 6\} \\ \text{b) } \{x \in U : x \geq 8 \text{ or } x \text{ is even}\} & \text{d) } \{x \in U : x < 3 \text{ or } x > 7\} & \text{f) } \{x \in U : x \geq 4 \text{ or } x > 6\} \end{array}$$

6. Compute the prime factorization of 2500.

7. Use the prime factorization to find the greatest common factor of 72 and 960.

8. Label each of the following statements as true or false.

- Every positive integer has an even number of factors.
- The product of two consecutive integers is always even.
- A degree three equation can have up to three different solutions.
- If we square an odd number and then subtract one, the result is always divisible by 4.
- There is no prime number that is divisible by 5.
- If n is a perfect square, then all exponents in the prime-factorization of n are even.
- If A and B are any sets such that $A \cup B = A$, then $A = B$.
- If A and B are any sets such that $A \cup B = A$, then $B \subseteq A$.

9. Simplify each of the following.

$$\begin{array}{llll} \text{a) } x^6 \cdot x^7 & \text{e) } (2xy^3)^3 & \text{h) } \sqrt{x^8} & \text{k) } \frac{(2x^5y)^2 (-xy^2x^3)^3}{(-x^3y)^4} \\ \text{b) } (-x)^6 \cdot (-x)^7 & \text{f) } \frac{(x^3)^8}{x^3 \cdot x^8} & \text{i) } (-5a^2b)^2 & \text{l) } \frac{2a^3 (-2ab)^3 (-ab^5)^2}{4b^5 (-2a^3b)^2} \\ \text{c) } (-x^6) \cdot (-x^7) & \text{g) } (x^3)^2 & \text{j) } (-5a^2b)^3 & \end{array}$$

10. Simplify each of the following. Assume that x is positive.

$$\text{a) } (\sqrt{2})^6 \quad \text{b) } (\sqrt{3})^4 \quad \text{c) } (\sqrt{x})^{10} \quad \text{d) } \sqrt{\sqrt{x}}$$

11. Simplify each of the following.

a) $(2x + 3) + (5x - 1)$

d) $(2x + 3)(5x - 1)$

g) $(2(x + 3) - 5)(x - 1)$

b) $(2x + 3) - (5x - 1)$

e) $2x + 3(5x - 1)$

c) $-2(2x + 3) - 8(5x - 1)$

f) $(2x + 3)5x - 1$

h) $(2x + 3)^2 - (5x - 1)^2$

12. Factor out the greatest common factor (GCF) from each of the following.

a) $15a^4b - 6a^3b^2 + 3a^2b$

b) $2a^2xy^2 + 6b^2xy^2$

c) $12a^3bc + 18a^2b^2c^2 - 6a^2c$

13. Completely factor each of the following over the integers.

a) $3x^2 - 48$

e) $x^2 + 6x$

i) $(3x^5 - x + 5)^2 - (3x^5 + x - 5)^2$

b) $A^{10} - 9$

f) $x^2 - 1$

j) $18a^2bx^2 - 2a^2by^2$

c) $-2x^2 - 50$

g) $x^4 - 1$

d) $100 - x^2$

h) $(5a + 3b)^2 - (3a - b)^2$

14. Completely factor each of the following over the real numbers by completing the square.

a) $-2x^2 - 8x + 24$

b) $10x^3 - 60x^2 + 100x$

c) $-x^2 + 12x - 36$

15. Solve each of the given equations. Make sure to check your solutions.

a) $3x - 8 = 19$

c) $\frac{x - 5}{3} = -2$

e) $3(x - 1) - 2(3x + 5) = -x + 7$

b) $\frac{2}{3}x - \frac{5}{8} = -\frac{23}{24}$

d) $\frac{x - \frac{2}{5}}{3\frac{1}{3}} = \frac{1}{200}$

f) $\frac{2}{3}\left(x - \frac{1}{2}\right) - \frac{3}{4}\left(\frac{2}{3}x - \frac{3}{4}\right) = -\frac{1}{6}x + \frac{1}{4}$

16. Solve each of the following equations. Make sure to check your solutions.

a) $2x - 5(x - 3) = (x + 1)^2 - (x - 2)^2$

g) $\frac{1}{2}(x - 3) + \frac{1}{2}(x + 1) = 3x - 1$

b) $\frac{2}{3}x + \frac{3}{4} = \frac{4}{5}$

h) $\frac{2}{3}(x - 1) - \frac{1}{2}(x + 5) = \frac{1}{6}(x - 2)$

c) $6x + x^2 = 7$

i) $\frac{3}{4}x - \frac{2}{5} - \left(\frac{1}{2} - \frac{x}{4}\right) = x - \frac{9}{10}$

d) $x(x + 1)(3x - 7)(x + 5)^2 = 0$

j) $x^2 = 36$

e) $(x - 3)^2 - (2x + 1)^2 = 8 - 3x^2$

k) $x^5 = 36x^4$

f) $-x(x - 2) + 3(x - 1)^2 = 3$

l) $x^5 = 36x^3$

17. Expand each of the following.

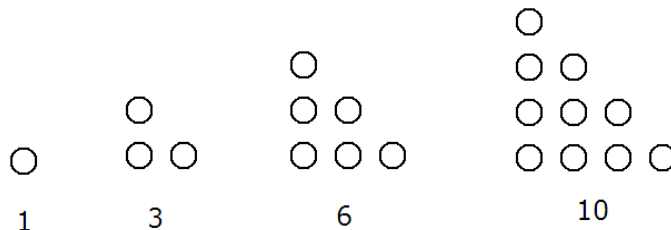
a) $(a - b)(a + b)$

b) $(a - b)(a^2 + ab + b^2)$

c) $(a - b)(a^3 + a^2b + ab^2 + b^3)$

18. In a class of 30, students discuss their summer activities. 20 students report that they traveled to other cities or countries in the summer. 18 students report that they visited local beaches or waterparks. If 7 students claimed to do none of the above, how many students did both traveling and waterparks? (Hint: draw a Venn diagram!)

19. $A(-3, -1)$, $B(-5, 4)$, and $C(5, 8)$ are three vertices of a rectangle. Find the fourth vertex of the rectangle.
20. The picture shows the first few triangular numbers. Find the hundredth triangular number.



21. Apply the difference of squares theorem to evaluate the given expressions mentally.

a) $99^2 - 98^2$ b) $60^2 - 40^2$ c) $35^2 - 15^2$ d) $21^2 - 19^2$

Answers

1. a) -11 b) $\frac{13}{80}$ c) $\frac{5}{63}$ d) $\frac{13}{18}$
2. a) $2\sqrt{5}$ b) $-2\sqrt{2}$ c) 3 d) $6 - \sqrt{2}$
 e) $4 + 3\sqrt{3}$ f) 1 g) $21 - 4\sqrt{5}$ h) $5 - 2\sqrt{6}$
 i) $26 - 15\sqrt{3}$ j) $4\sqrt{5}$
3. a) 3870 b) 38450 c) 298887
4. a) $\frac{3407}{1000}$ b) $\frac{317}{990}$ c) $\frac{7812}{9990}$ d) $\frac{301}{900}$
5. a) $\{8, 10\}$ b) $\{2, 4, 6, 8, 9, 10\}$ c) \emptyset
 d) $\{1, 2, 8, 9, 10\}$ e) $\{7, 8, 9, 10\}$
 f) $\{4, 5, 6, 7, 8, 9, 10\}$
6. $2500 = 2^2 \cdot 5^4$
7. 24
8. a) false b) true c) true d) true
 e) false f) true g) false h) true
9. a) x^{13} b) $-x^{13}$ c) x^{13} d) x^{42} e) $8x^3y^9$
 f) x^{13} g) x^6 h) x^4 i) $25a^4b^2$
 j) $-125a^6b^3$ k) $-4x^{10}y^4$ l) $-a^2b^6$
10. a) 8 b) 9 c) x^5 d) $\sqrt[4]{x}$
11. a) $7x + 2$ b) $-3x + 4$ c) $-44x + 2$
 d) $10x^2 + 13x - 3$ e) $17x - 3$ f) $10x^2 + 15x - 1$
 g) $2x^2 - x - 1$ h) $-21x^2 + 22x + 8$
12. a) $3a^2b(5a^2 - 2ab + 1)$ b) $2xy^2(a^2 + 3b^2)$
 c) $6a^2c(2ab + 3b^2c - 1)$
13. a) $3(x + 4)(x - 4)$ b) $(A^5 + 3)(A^5 - 3)$
 c) $-2(x^2 + 25)$ d) $-(x + 10)(x - 10)$
 e) $x(x + 6)$ f) $(x + 1)(x - 1)$
 g) $(x^2 + 1)(x + 1)(x - 1)$ h) $4(a + 2b)(4a + b)$
 i) $-12x^5(x - 5)$ j) $2a^2b(3x + y)(3x - y)$
14. a) $-2(x + 6)(x - 2)$ b) $10x(x^2 - 6x + 10)$
 c) $-(x - 6)^2$
15. a) 9 b) $-\frac{1}{2}$ c) -1 d) $\frac{5}{12}$ e) -10 f) $\frac{1}{16}$
16. a) 2 b) $\frac{3}{40}$ c) $-7, 1$ d) $0, -1, -5, \frac{7}{3}$
 e) 0 f) $0, 2$ g) 0 h) There is no solution.
 i) All numbers are solution. j) $-6, 6$ k) $0, 36$
 l) $-6, 0, 6$
17. a) $a^2 - b^2$ b) $a^3 - b^3$ c) $a^4 - b^4$
18. 15
19. $(7, 3)$
20. 5050
21. a) 197 b) 2000 c) 1000 d) 80