

1. Which number has more divisors, 24 or 36?

2. Find the prime factorization for each of the following.

a) 1200 b) 10^{100} c) $75 \cdot 80$

3. Simplify each of the given expressions.

a) $\frac{\frac{3}{8} \cdot \left(-\frac{1}{3}\right)^2}{\frac{1}{4} - \frac{1}{3}}$ b) $\left(-\frac{2}{3}\right)^2$ c) $-\left(-\frac{3}{5}\right)^2$ d) $\frac{(2x^2y)^5}{(2xy)^3}$ e) $\frac{(-2xy^2)^3(-2x^2y)^4}{(-4x^3y)^2}$

4. Compute each of the following sums.

a) $42 + 50 + 58 + \dots + 482$ b) $-35 + (-25) + \dots + 755$ c) $31 + 40 + 49 + \dots + 1102$

5. Perform the indicated operations and simplify.

a) $(2\sqrt{7} - 1)(\sqrt{7} + 3)$ d) $(3\sqrt{7} - 8)(3\sqrt{7} + 8)$ g) $\frac{12 - \sqrt{20}}{2}$ i) $(-\sqrt{2})^8$
 b) $(3\sqrt{2} - 1)^2$ e) $\sqrt{50} - \sqrt{18} + 2\sqrt{72}$ h) $\frac{\sqrt{80}}{\sqrt{45}}$ j) $(-\sqrt{2})^9$
 c) $(\sqrt{2} - 1)^4$ f) $(2\sqrt{5} - 4)^3(2\sqrt{5} + 4)^3$

6. Simplify each of the following.

a) $(1, 4) \cup (2, 7)$ c) $(-\infty, 3) \cup (-\infty, 5]$ e) $(-\infty, 10] \cup (7, \infty)$ g) $(1, 5) \cup [12, \infty)$
 b) $(1, 4) \cap (2, 7)$ d) $(-\infty, 3) \cap (-\infty, 5]$ f) $(-\infty, 10] \cap (7, \infty)$ h) $(1, 5) \cap [12, \infty)$

7. Evaluate each of the given expressions with the values given for the variable(s).

a) $2x^2 - x - 3$ if $x = 2\sqrt{3} - 5$ b) $-a^2 + 5a - 1$ if $a = 4 - \sqrt{3}$ c) $x^2 - 8x + 6 = 0$ if $x = 4 - \sqrt{10}$

8. Graph each of the following.

a) $y = -\frac{2}{3}x + 1$ b) $x + y = 3$ c) $2x - 3y = -1$

9. Find the perimeter and area of the quadrilateral determined by the points $A(-3, 5)$, $B(-3, -2)$, $C(2, -2)$, and $D(2, 5)$.

10. Perform the following operations. Show all steps.

a) $18 - 2(-5) - 2(11 - 2(-5))$ d) $\left|-\frac{1}{2} - 2\right| - \left|\frac{3}{4} - 3\right|$ g) $\left|-\frac{1}{2} - \left|2 - \frac{3}{4} - 3\right|\right|$
 b) $\frac{-3^2 + (-1)^3}{7 - 3(-1)^3}$ e) $\left|-\frac{1}{2} - 2 - \left|\frac{3}{4} - 3\right|\right|$ h) $\left|-\frac{1}{2}\right| - 2 - \frac{3}{4} - 3$
 c) $-2^2(24 - 2(-3) - 5(-2)^2) - 12$ f) $\left|-\frac{1}{2} - 2\left|\frac{3}{4} - 3\right|\right|$ i) $\sqrt{3\sqrt{49} - \sqrt{25}}$

11. Completely factor each of the following over the integers.

a) $(3a + 1)^2 - 49$ d) $x^{10} - 49$ g) $(2x^2 - x + 2)^2 - (2x^2 + x - 4)^2$
 b) $200x + 2x^3$ e) $405 - 5x^4$
 c) $5x^6 - 80x^2$ f) $12x^2 - 110x + 2x^3$

12. If A is the set of all integers divisible by 5 and B is the set of all integers divisible by 3, what is $A \cap B$?

13. Evaluate $\frac{3xy + 2x^2 - 2y^2}{x + 2y}$ if

a) $x = \frac{1}{2}$ and $y = -3$ b) $x = -1$ and $y = -2$ c) $x = -6$ and $y = 3$

14. Simplify each of the following algebraic expressions.

a) $(3x^2 - 5x + 8) + (-3x^2 - 5x + 12)$

f) $(2x - 3)(2x + 3)$

b) $(3x^2 - 5x + 8) - (-3x^2 - 5x + 12)$

g) $2(m - 3)^2 - (2m + 5)(m - 3)$

c) $3(x - 5) - 4(x - 2)$

h) $(a - 4)(2a + 1) - (a - 3)^2$

d) $5(2a + 1) - 2(2 - a) - 7(a + 8)$

i) $(3x - 1)^3$

e) $(x - 5)(x - 2)$

15. Solve each of the following equations.

a) $-\frac{2}{5}x + \frac{1}{2} = \frac{1}{5}$

f) $20x^5 = 5x^3$

b) $\frac{3x + 1}{5} - \frac{2x - 4}{3} = 2x - 15$

g) $(2x - 5)^2 - (2x + 6)^2 = 4x - 11$

c) $x(3x - 1) - 2x - 3 = 3(x - 1)^2$

h) $60x - 20x^2 = 5x^3$

d) $\frac{2}{3}\left(x - \frac{1}{2}\right) - \frac{1}{2}\left(x + \frac{2}{3}\right) = \frac{1}{6}(x + 2)$

i) $x^4(x^2 + 1)(x^4 + 9) = 0$

e) $20x^5 = 5x^4$

j) $4(x - 3) - 3(x + 1) = 2x - 10 - (x + 5)$

k) $(2x - 1)^2 - (x - 2)^2 = 2x^2 + 1$

16. Solve each of the following inequalities.

h) $3(x - 2) - 5(6x - 1) \geq 3x - 1$

j) $\frac{2x - 3}{7} - \frac{3x - 1}{2} \leq x - 11$

i) $\frac{3x - 8}{4} < \frac{5x - 17}{3}$

k) $(3x - 1)^2 + (x - 1)^2 \geq 10x^2 + 6$

17. Solve each of the given compound inequalities. Present your answer in interval notation.

a) $-3x + 1 < 40$ and $\frac{2}{3}x - 1 \geq 5$

d) $\frac{2x - 5}{3} > -1$ or $\frac{1}{2}(x - 1) \geq \frac{2}{3}(x + 1)$

b) $-3x + 1 < 40$ or $\frac{2}{3}x - 1 \geq 5$

e) $(x - 5)^2 \geq (x + 1)^2$ and $x^2 + 3 > x(x - 3)$

c) $\frac{2x - 5}{3} > -1$ and $\frac{1}{2}(x - 1) \geq \frac{2}{3}(x + 1)$

f) $(x - 5)^2 \geq (x + 1)^2$ or $x^2 + 3 > x(x - 3)$

18. Prove that each of the following numbers is rational by re-writing them as a fraction of two integers.

a) $0.\overline{304} = 0.304040404\dots$ b) $7.\overline{2673} = 7.2673673673673$ c) $0.\overline{9} = 0.99999\dots$

19. The opposite of a number is thirty-five more than the sum of -5 and the number. Find this number.

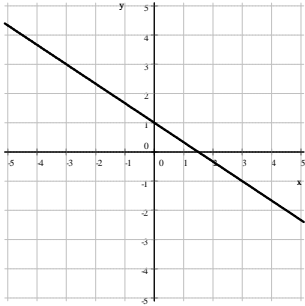
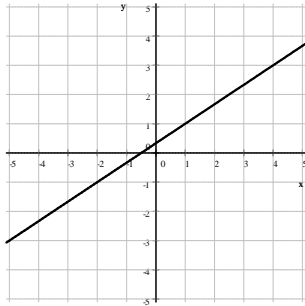
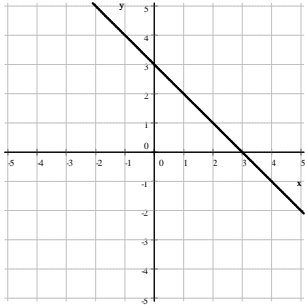
20. The sum of three consecutive numbers is 63. Find these numbers.

21. Find the sides of the square if we know the following. If we increased the length of each side by 1 feet, the area of the square would increase by 17 ft^2 .

22. We have some \$5 bills and some \$10 bills. The number of \$5 bills is ten less than six times the number of \$10. How many \$5 bills do we have if the value of all bills is \$430?

23. A number is 42 less than its own square. Find this number.
24. Find all numbers with the following property: if we multiply the number by seven, the result is the original number.
25. The first row in a theater has 15 seats in it. The second row has three more seats than the first row. The third row has three more seats than the second row. And so on, each row has three more seats than the row before. If the last row has 81 seats in it, how many seats are there in the entire theater?
26. Find all numbers with the following property: if we square the number, the result is the original number.
27. We have a jar full of coins, all dimes and quarters. How many of each coins do we have if the total value of all coins is \$24.70 and if we also know that there are 30 more dimes than quarters?
28. Find all numbers with the following property: if we raise the number, to the third power, the result is the original number.
29. One number is twelve less than another. Find these numbers if their product is 160.
30. One number is twelve less than twice another. Find these numbers if their product is 320.
- 31*. Suppose that $A = 2^{99}$. Express each of the following in terms of A .
- a) 2^{100} b) $2^{99} + 2^{100} - 3 \cdot 2^{101}$ c) 8^{99}

Answers

1. 36 (24 has 8 divisors, 36 has 9)
2. a) $1200 = 2^4 \cdot 3 \cdot 5^2$ b) $10^{100} = 2^{100} \cdot 5^{100}$
c) $75 \cdot 80 = 2^4 \cdot 3 \cdot 5^3$
3. a) $-\frac{1}{2}$ b) $\frac{4}{9}$ c) $-\frac{9}{25}$ d) $4x^7y^2$ e) $-8x^5y^8$
4. a) 14672 b) 28800 c) 67980
5. a) $11 + 5\sqrt{7}$ b) $19 - 6\sqrt{2}$ c) $17 - 12\sqrt{2}$
d) -1 e) $14\sqrt{2}$ f) 64 g) $6 - \sqrt{5}$
h) $\frac{4}{3}$ i) 16 j) $-16\sqrt{2}$
6. a) (1, 7) b) (2, 4) c) $(-\infty, 5]$ d) $(-\infty, 3)$
e) \mathbb{R} same as $(-\infty, \infty)$ f) (7, 10]
g) can not be simplified h) \emptyset
7. a) $76 - 42\sqrt{3}$ b) $3\sqrt{3}$ c) 0
8. a) $y = -\frac{2}{3}x + 1$ c) $2x - 3y = -1$
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- 
- b) $x + y = 3$
- 
8. $P = 24$ unit, $A = 35$ unit²
9. a) -14 b) -1 c) -40 d) $\frac{1}{4}$ e) $\frac{19}{4}$
f) 5 g) $\frac{9}{4}$ h) $\frac{23}{8}$ i) 4
10. a) $3(3a + 8)(a - 2)$ b) $2x(x^2 + 100)$
c) $5x^2(x^2 + 4)(x + 2)(x - 2)$ d) $(x^5 + 7)(x^5 - 7)$
e) $-5(x^2 + 9)(x + 3)(x - 3)$ f) $2x(x + 11)(x - 5)$
g) $-4(2x^2 - 1)(x - 3)$
11. the set of all integers divisible by 15
12. a) 4 b) 0 c) undefined
13. a) $-10x + 20$ b) $6x^2 - 4$ c) $-x - 7$
d) $5a - 55$ e) $x^2 - 7x + 10$ f) $4x^2 - 9$
g) $-11m + 33$ h) $a^2 - a - 13$
i) $27x^3 - 27x^2 + 9x - 1$
14. a) $\frac{3}{4}$ b) 8 c) 2 d) There is no solution.
e) $0, \frac{1}{4}$ f) $-\frac{1}{2}, 0, \frac{1}{2}$ g) 0 h) 2, 0, -6
i) 0 j) \mathbb{R} (all real numbers are solution)
k) 2, -2
15. a) $(-\infty, 0]$ b) $(4, \infty)$ c) $[5, \infty)$ d) $(-\infty, -\frac{1}{2}]$
16. a) $[9, \infty)$ b) $(-13, \infty)$ c) \emptyset (empty set)
d) $(-\infty, -7] \cup (1, \infty)$ e) $(-1, 2]$ f) $(-\infty, \infty)$
17. a) $\frac{301}{990}$ b) $\frac{72601}{9990}$ c) 1
18. -15 20. 20, 21, 22 21.8 ft
22. 12 ten-dollar bills and 62 five-dollar bills
23. -6 and 7 24. 0 25. 1104 26. 0, 1
27. 62 quarters and 92 dimes 28. 0, 1, -1
29. -8 with -20 or 8 with 20
30. -10 with -32 and 16 with 20
- 31*. a) $2A$ b) $-9A$ c) A^3