1. Solve each of the following system of equations.

   a) $\begin{cases} 2x - y = -1 \\ 5x - 2y = 2 \end{cases}$ b) $\begin{cases} 2x - 5y = -9 \\ x - y = -3 \end{cases}$ c) $\begin{cases} 3x + 5y = -20 \\ \frac{1}{3}x - \frac{1}{2}y = 2 \end{cases}$

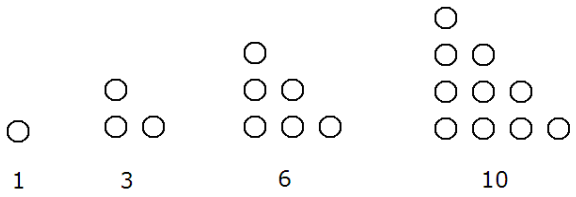
2. Compute each of the following sums.

   a) $150 + 155 + 160 + ... + 400$ b) $290 + 280 + 270 + ... + (-350)$

3. a) Find a formula for $1 + 2 + 3 + 4 + ... + n$ in terms of the positive integer n.

   b) Recall the triangular numbers, 1, 3, 6, 10, and so on. The first four triangular numbers are shown on the picture. Find the 100th triangular number.

   c) Prove that the sum of two consecutive triangular numbers is always a square number.



4. Label each of the following as true or false.

   a) The product of two prime numbers is never a prime number.
   b) The sum of two prime numbers is never a prime number.
   c) Suppose that x and y are positive integers. If we divide x by 5, the remainder is 2. If we divide y by 5, the remainder is 3. Then $x + y$ is divisible by 5.
   d) There are five different one-digit prime numbers.

5. Simplify each of the following.

   a) $\dfrac{2 - 3^{-1}}{3 - 2^{-1}}$

   b) $\dfrac{x^2 - x - 2}{x^2 - 1}$

   c) $\dfrac{\left(\frac{1}{2}\right)^{-2} - \left(\frac{2}{3}\right)^{-1}}{5^{-1} - 2^{-1}}$

   d) $\left(\dfrac{2a^3b^{-2}}{-3a^{-1}b^6}\right)^0$

   e) $\left(\sqrt{5}\right)^4$

   f) $\left(2\sqrt{5} - 4\right)^3 \left(2\sqrt{5} + 4\right)^3$

   g) $\sqrt{x^{15}}$

   h) $\dfrac{x^2 - 10x + 21}{x^2 - 2x - 35} \div \dfrac{x^2 - 9}{2x^2 + 10x}$

   i) $\dfrac{\frac{3}{x} + 1}{\frac{9}{x^2} - 1}$

   j) $\dfrac{1}{\frac{1}{x} - \frac{1}{y}}$

   k) $\dfrac{3a^2b^{-3}a^{-5}}{-3a^{-2}b^3}$

   l) $\dfrac{3a^2 + b^{-3}}{-3 + a^{-2}}$

   m) $\dfrac{1}{2}\left(\sqrt{a-2} + \sqrt{a+2}\right)^2$

   n) $\dfrac{2}{\sqrt{3}+5} - \dfrac{2}{\sqrt{3}-5}$

6. Let $\Phi = \dfrac{1 + \sqrt{5}}{2}$. (This number is called the golden mean or golden ratio.)

   a) Compute $\Phi^2 - \dfrac{1}{\Phi}$. b) Prove that $\dfrac{1}{\Phi} = \Phi - 1$.

7. Rationalize the denominator in $\dfrac{\sqrt{8} - \sqrt{6}}{\sqrt{8} + \sqrt{6}}$.

8. Prove that the decimal $0.28\overline{719} = 0.28719719719.....$ represents a rational number by re-writing it as a fraction of two integers. You do NOT have to bring the fraction to lowest terms.

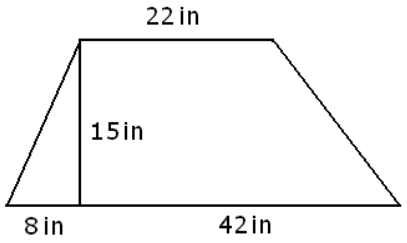
9. Compute the perimeter and area of the parallelogram determined by the points $A(-3, 4)$, $B(21, 14)$, $C(21, 30)$, and $D(-3, 20)$.

10. Compute the exact value of the distance between the points given.
    a) $A(2,-9)$ and $B(5,-5)$ b) $P(-4,6)$ and $Q(3,5)$
    c) Find the value(s) of k for which the distance between the points $A(-3,1)$ and $B(3,k)$ is 10 units.

11. Factor each of the following over the integers.
    a) $x^2 - x$ c) $9x^2 - 1$ e) $12x^2 - x - 1$ g) $9x^2 - 6x - 1$
    b) $x^3 - x$ d) $(a+b-c)^2 - (a-b+c)^2$ f) $x^2 - 5$ h*) $x^2 - 6x + 9 - 25c^2$

12. Solve each of the following equations.
    a) $(x-3)^2 - (2x-5)^2 = -x^2 - (x+1)^2$
    b) $\dfrac{3x+6}{4} - \dfrac{x-1}{3} = x - 4$
    c) $\dfrac{2}{3}(x-5) - \dfrac{3}{2}(x+2) = \dfrac{7}{6}$
    d) $\sqrt{5x+1} + 1 = x$
    e) $(3 - 2(x - 4(x - 3(x - 5)))) = 105$
    f) $2x^2 - 8x = -26$
    g) $(3x-1)(x-3) - x = 3(x-2)^2$
    h) $2x^2 = x + 3$

13. Compute the perimeter and area of the trapezoid shown on the picture.



14. Solve each of the following inequalities.
    a) $\dfrac{4x-1}{5} - \dfrac{x+1}{2} \geq x - 7$
    b) $\dfrac{2}{5}x - 1 \leq \dfrac{1}{2}x - \dfrac{3}{4}$
    c) $(x+3)^2 - (x-3)^2 < 60$

15. Solve each of the following compound inequalities.
    a) $x(x-3) - 3x > (x-2)^2$ and $\dfrac{-3x+5}{2} < -8$
    b) $x(x-3) - 3x > (x-2)^2$ or $\dfrac{-3x+5}{2} < -8$
    c) $(x+2)^2 - (x-1)^2 < 3(x-10)$ and $\dfrac{1}{2}x - \dfrac{1}{3} < \dfrac{1}{3} - \dfrac{1}{6}x$
    d) $(x+2)^2 - (x-1)^2 < 3(x-10)$ or $\dfrac{1}{2}x - \dfrac{1}{3} < \dfrac{1}{3} - \dfrac{1}{6}x$

16. a) Graph the line $5x - 4y = -7$
    b) Graph the parabola $y = x^2 - 8x + 7$. State the coordinates of at least five points, including vertex and intercepts.

17. Compute the exact value of the perimeter and area of a right triangle if its shorter sides are
    a) 10 cm and 24 cm long b) 5 ft and 10 ft long

18. We invested a total of \$4000 in two bank accounts. One account earns an annual interest of 3%, the other account earns an annual interest of 5%. How much was invested into each account if the combined interest from the two accounts after one year was \$183?

19. We invested a total of \$4000 in two different stocks. After one year, one stock earned a 7% profit but the other stock sufffered a loss of 3%. How much was invested into each stock if the combined gain from the two stocks after one year was \$50?

20. If we increase all sides of a square by 1 feet, the square's area will increase by 105 ft^2. How long are the sides of the original square?

21. The hypotenuse of a right triangle is 74 m long. The difference between the other two sides is 46 m. Find the sides of this triangle.

22. We have 240 coins, all dimes and nickels. How many dimes and how many nickels do we have if the value of all coins is \$18.80?

23. The sum of four consecutive even numbers is 76. Find these numbers.

24. The square of a number is 60 greater than four times the number. Find this number.

25. In the fourth period, teachers took attendance in Springfield High School. Some of the female students had a field trip today. We know that if 15% of all girls left for a field trip, the total attendance school-wide drops by 10%. What percent of the students is female?

26. Children's tickets cost \$12 each and adults' tickets cost \$18 each. We purchased 50 tickets for a total of \$642. How many of each tickets did we buy?

27. The longer side of a rectangle is 15 ft long. The diagonal is 17 ft long. Compute the area of the rectangle.

28. The hypotenuse of a right triangle is 8 m shorter than twice the shortest side. The second longest side is 8 m longer than the shortest side. Find the sides of this triangle.

29. How many people are in the room if we know that everyone shook hands with everyone and there were 595 handshakes?

30. Find all numbers for which the following is true: the opposite of the number is equal to the square of the number.

31. One number is four less than twice another. Find these numbers if their product is 70.

32. If we increase all sides of a square by 1 feet, the square's area will increase by $15\,\text{ft}^2$. How long are the sides of the original square?

33. The square of a number is 8 greater than twice the opposite of the number. Find this number.

34. The farm raised chickens and cows. If all the animals together had 82 heads and 288 legs, how many chickens and how many cows are there on the farm?

35. One side of a rectangle is 24 cm shorter than four times the other side. Find the length of the sides if the area of the rectangle is $160\,\text{cm}^2$.

36. Children's tickets cost \$7 each and adults' tickets cost \$12 each. We purchased 60 tickets for a total of \$460. How many of each tickets did we buy?

37*. Ann and Betsy are friends. One day Betsy tells Ann: "*If you gave me five dollars, we would end up with the same amount of money.*" Ann responds: "*Yes, but if you gave me five dollars instead, then I would have twice as much money as you.*" How much money did they have?

38*. Consider the expression $\left|n^2 - 14n + 40\right|$. Find all integer values of n for which the value of this expression is a prime number.

39*. Find the area of the triangle determined by the points $A\,(-7, 4)$, $B\,(-3, -2)$, and $C\,(5, 1)$.

40*. The conference of medical staff was very well attended. Nurses and doctors have attended. All together, there were 45 people present. Every doctor shook hands with all other doctors and every nurse shook hands with all other nurses, but there wasn't a single handshake between a doctor and a nurse. How many doctors and nurses were there if the total number of handshakes was 486?

Answers

1. a) $x = 4, y = 9$ b) $x = -2, y = 1$
   c) $x = 0, y = -4$

2. a) $14\,025$ b) -1950

3. a) $\dfrac{n(n+1)}{2}$ b) 5050
   c) see solutions at the end

4. a) true b) false c) true d) false

5. a) $\dfrac{2}{3}$ b) $\dfrac{x-2}{x-1}$ c) $-\dfrac{25}{3}$ d) 1 e) 25 f) 64
   g) $x^7\sqrt{x}$ h) $\dfrac{2x}{x+3}$ i) $-\dfrac{x}{x-3}$ j) $\dfrac{xy}{y-x}$
   k) $-\dfrac{1}{ab^6}$ l) $\dfrac{3a^4b^3+a^2}{-3a^2b^3+b^3}$ m) $a+\sqrt{a^2-4}$ n) $\dfrac{10}{11}$

6. a) 2 b) see solutions at the end 7. $7-2\sqrt{12}$

8. $\dfrac{28\,691}{99\,900}$ 9. $P = 84$ unit, $A = 384$ unit^2

10. a) 5 unit b) $\sqrt{50} = 5\sqrt{2}$ unit c) $-7, 9$

11. a) $x(x-1)$ b) $x(x-1)(x+1)$
    c) $(3x-1)(3x+1)$ d) $4a(b-c)$
    e) $(3x-1)(4x+1)$ f) x^2-5
    g) $9x^2-6x-1$ h) $(x+5c-3)(x-5c-3)$

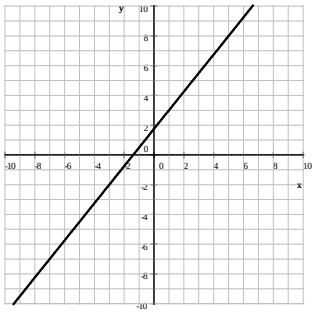
12. a) $1, 15$ b) 10 c) -9 d) 7 (0 is extreneous)
    e) 1 f) no real solution g) 9 h) $-1, \dfrac{3}{2}$

13. $P = 114\,\text{in}$, $A = 540\,\text{in}^2$

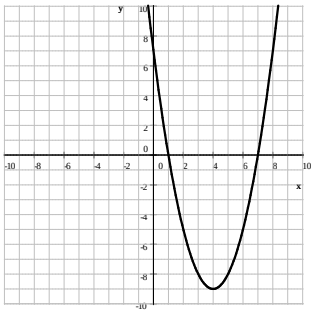
14. a) $(-\infty, 9]$ b) $\left[-\dfrac{5}{2}, \infty\right)$ c) $(-\infty, 5)$

15. a) $\varnothing$ b) $(-\infty, -2) \cup (7, \infty)$ c) $(-\infty, -11)$
    d) $(-\infty, 1)$

16. a) $5x - 4y = -7$



b) $y = x^2 - 8x + 7$
$y-$intercept: $(0, 7)$ vertex: $(4, -9)$
$x-$intercepts: $(1, 0)$ and $(7, 0)$
additional points:
$(3, -8)$ and $(5, -8)$



17. a) $P = 60\,\text{cm}$ $A = 120\,\text{cm}^2$
    b) $P = 15 + 5\sqrt{5}\,\text{ft}$ $A = 25\,\text{ft}$

18. \$850 at 3% and \$3150 at 5%

19. \$1700 gained 7% and \$2300 lost 3%

20. $52\,\text{ft}$ 21. $24\,\text{m}$, $70\,\text{m}$, and $74\,\text{m}$

22. 104 nickels and 136 dimes 23. $16, 18, 20, 22$

24. -6 and 10 25. $66.\overline{6}\%$

26. 43 children's tickets and 7 adult tickets 27. $120\,\text{ft}^2$

28. $24\,\text{m}$, $32\,\text{m}$, and $40\,\text{m}$ 29. 35 30. $0, -1$

31. -5 with -14 and 7 with 10 32. $7\,\text{ft}$

33. $-4, 2$ 34. 20 chickens and 62 cows

35. $10\,\text{cm}$ by $16\,\text{cm}$ 36. 52 children and 8 adult tickets

37. Ann has \$35 and Betsy has \$25

38. $3, 5, 9, 11$ 39. 30 unit^2

40. 21 doctors and 24 nurses or 24 doctors and 21 nurses

Solutions

3. c) The nth triangular number is $\dfrac{n(n+1)}{2}$. The next one is obtained by substituting $n+1$ into n:

$$\frac{n(n+1)}{2} \quad \text{becomes} \quad \frac{(n+1)((n+1)+1)}{2} = \frac{(n+1)(n+2)}{2}$$

We add the two:

$$\begin{aligned} S &= \frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} && \text{we have a common denominator} \\ &= \frac{n(n+1)+(n+1)(n+2)}{2} && \text{factor out } n+1 \\ &= \frac{(n+1)[n+(n+2)]}{2} = \frac{(n+1)(2n+2)}{2} = \frac{(n+1)(2)(n+1)}{2} = (n+1)^2 \end{aligned}$$

6. Let $\Phi = \dfrac{1+\sqrt{5}}{2}$. (This number is called the golden mean or golden ratio.)

a) Compute $\Phi^2 - \dfrac{1}{\Phi}$.

$$\begin{aligned} \Phi^2 - \frac{1}{\Phi} &= \left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{1}{\frac{1+\sqrt{5}}{2}} = \frac{6+2\sqrt{5}}{4} - \frac{2}{\sqrt{5}+1} = \frac{3+\sqrt{5}}{2} - \frac{2}{\sqrt{5}+1}\cdot\frac{\sqrt{5}-1}{\sqrt{5}-1} \\ &= \frac{3+\sqrt{5}}{2} - \frac{2(\sqrt{5}-1)}{5-1} = \frac{3+\sqrt{5}}{2} - \frac{2(\sqrt{5}-1)}{4} = \frac{3+\sqrt{5}}{2} - \frac{\sqrt{5}-1}{2} = \frac{3+\sqrt{5}-(\sqrt{5}-1)}{2} = \frac{4}{2} = 2 \end{aligned}$$

b) Prove that $\dfrac{1}{\Phi} = \Phi - 1$.

Solution: $\dfrac{1}{\Phi} = \dfrac{2}{1+\sqrt{5}}$ we rationalize the denominator

$$\begin{aligned} \frac{1}{\Phi} &= \frac{2}{1+\sqrt{5}} = \frac{2}{\sqrt{5}+1}\cdot\frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{2(\sqrt{5}-1)}{5-1} = \frac{2(\sqrt{5}-1)}{4} = \frac{\sqrt{5}-1}{2} \\ \Phi - 1 &= \frac{1+\sqrt{5}}{2} - 1 = \frac{1+\sqrt{5}}{2} - \frac{2}{2} = \frac{\sqrt{5}+1-2}{2} = \frac{\sqrt{5}-1}{2} \qquad \text{Thus } \frac{1}{\Phi} = \Phi - 1 \end{aligned}$$

25. Let g denote the number of female students. Let S denote the total number of students. 15% of the girls is the same as 10% of all students.

$$\begin{aligned} 0.15g &= 0.1S && \text{multiply by 100} \\ 15g &= 10S \\ g &= \frac{10}{15}S = \frac{2}{3}S \end{aligned}$$

Thus two thirds of the students are girls, which is $66.\overline{6}\%$.

37. Consider the expression $\left|n^2 - 14n + 40\right|$. Find all integer values of n for which the value of this expression is a prime number.

Solution: We can factor the expression $n^2 - 14n + 40$ as $(n-4)(n-10)$. That means that for all integer values of n, the expression will factor into a product of two integers. In case of a prime number, one of the factors must be 1 or -1. So, for primes, either $n-4 = \pm 1$ or $n-10 = \pm 1$. We solve each equations for n.

$n-4=1$ or $n-4=-1$ or $n-10=1$ or $n-10=-1$

$n=5$ $n=3$ $n=11$ $n=9$

We evaluate the expression with $n = 5, 3, 9$, and 11, to see which one will work.

If $n=5$ $\left|5^2 - 14\cdot 5 + 40\right| = |-5| = 5$ prime If $n=3$ $\left|3^2 - 14\cdot 3 + 40\right| = |7| = 7$ prime

$n=11$ $\left|11^2 - 14\cdot 11 + 40\right| = |7| = 7$ prime If $n=9$ $\left|9^2 - 14\cdot 9 + 40\right| = |-5| = 5$ prime

So, all of 3, 5, 9, and 11 work.

Last revised: November 15, 2017