Part 1 - The Same Old Equations with Fractions

We will further study solving linear equations. Let us first recall a few definitions.

> **Definition**: An **equation** is a statement in which two expressions (algebraic or numeric) are connected with an equal sign. A **solution** of an equation is a number (or an ordered set of numbers) that, when substituted into the variable(s) in the equation, makes the statement of equality of the equation true. To **solve an equation** is to find *all* solutions of it. The set of all solutions is also called the solution set.

For example, the equation $-x^2+3=4x-2$ is an equation with two solutions, -5 and 1. We leave it to the reader to verify that these numbers are indeed solution. We will have to deploy systematic methods to find all solutions. The methods we will use usually depends on the type of equation. We have been studying the simplest equations, linear equations.

We have seen one- and two-step equations. First we will revisit those again, now that we have a greater number set. The most important thing to realize here is that the appearance of fractions does not change the fundamental methods of solving equations; rather, it makes each of the steps a bit more laborious. However, we should not let ourselves be intimidated by fractions.

Example 1. Solve each of the following equations.

a) $2x-3=15$ b) $\frac{2}{3}x-\frac{1}{2}=-\frac{1}{3}$ c) $\frac{x-4}{3}=-6$ d) $\frac{x+\frac{1}{2}}{\frac{3}{5}}=\frac{1}{3}$

Solution: a) There is nothing new or unusual about this equation. The right-hand side is just a number. The unknown only appears on the left-hand side. There, we see two operations: the unknown was first multiplied by 2 and then 3 was subtracted. To isolate the unknown on the left-hand side, we will perform the inverse operations to both sides, in a reverse order. We will first add 3 and then we will divide by 2.

$$\begin{aligned} 2x-3 &= 15 && \text{add 3} \\ 2x &= 18 && \text{divide by 2} \\ x &= 9 \end{aligned}$$

We check: If $x=9$, then the left-hand side is: $\quad \text{LHS}=2x-3=2\cdot 9-3=18-3=15=\text{RHS}$

Thus our solution, $x=\boxed{9}$ is correct.

b) Consider the equation $\frac{2}{3}x-\frac{1}{2}=-\frac{1}{3}$. There is nothing new or unusual about this equation either. If we could solve $2x-3=15$, then we can solve this equation using the same steps. On the left-hand side, the unknown was multiplied by $\frac{2}{3}$ and then $\frac{1}{2}$ was subtracted. To isolate the unknown, we will perform to both sides the inverse operations, in a reverse order. This means that we will add $\frac{1}{2}$ and then divide by $\frac{2}{3}$. The main computation should be clean; we should just record the result of each step. We will perform computations on the margin.

$$\begin{aligned} \frac{2}{3}x-\frac{1}{2} &= -\frac{1}{3} && \text{add } \frac{1}{2} \\ \frac{2}{3}x &= \frac{1}{6} && \text{divide by } \frac{2}{3} \\ x &= \frac{1}{4} \end{aligned}$$

margin work: $-\frac{1}{3}+\frac{1}{2}=\frac{-2}{6}+\frac{3}{6}=\frac{1}{6}$

$\frac{1}{6}\div\frac{2}{3}=\frac{1}{6}\cdot\frac{3}{2}=\frac{1}{2\cdot\not{3}}\cdot\frac{\not{3}}{2}=\frac{1}{4}$

We check: If $x=\frac{1}{4}$, then the left-hand side is

$$\text{LHS}=\frac{2}{3}\left(\frac{1}{4}\right)-\frac{1}{2}=\frac{\not{2}}{3}\cdot\frac{1}{\not{2}\cdot 2}-\frac{1}{2}=\frac{1}{6}-\frac{1}{2}=\frac{1}{6}-\frac{3}{6}=\frac{-2}{6}=-\frac{1}{3}=\text{RHS}$$

Thus our solution, $x=\boxed{\frac{1}{4}}$ is correct.

c) Consider now the equation $\frac{x-4}{3}=-6$. This is a simple two-step equation, we only have it here to serve as an analogous example for part d. On the left-hand side we have first a subtraction of 4 and then a division by 3. To isolate the unknown, we will multiply by 3 and then add 4. As always, we wil perform all operations to both sides.

$$\begin{aligned} \frac{x-4}{3} &= -6 && \text{multiply by 3} \\ x-4 &= -18 && \text{add } 4 \\ x &= -14 \end{aligned}$$

We check: If $x=-14$, then the left-hand side is: $\text{LHS}=\frac{x-4}{3}=\frac{-14-4}{3}=\frac{-18}{3}=-6=\text{RHS}$

Thus our solution, $x=\boxed{-14}$ is correct.

d) Consider now the equation $\dfrac{x+\frac{1}{2}}{\frac{3}{5}}=\frac{1}{3}$. If we could solve $\frac{x-4}{3}=-6$, then we can solve this equation using the same steps. On the left-hand side, there was an addition of $\frac{1}{2}$, and then a division by $\frac{3}{5}$. To isolate the unknown, we will perform to both sides the inverse operations, in a reverse order. This means that we will multiply by $\frac{3}{5}$ and then subtract $\frac{1}{2}$. The main computation should be clean; we should just record the result of each step. We will perform computations on the margin.

$$\begin{aligned} \frac{x+\frac{1}{2}}{\frac{3}{5}} &= \frac{1}{3} && \text{multiply by } \frac{3}{5} \\ x+\frac{1}{2} &= \frac{1}{5} && \text{subtract } \frac{1}{2} \\ x &= -\frac{3}{10} \end{aligned}$$

margin work: $\frac{1}{3}\cdot\frac{3}{5}=\frac{1}{5}$

$\frac{1}{5}-\frac{1}{2}=\frac{2-5}{10}=-\frac{3}{10}$

We check: If $x=-\frac{3}{10}$, then the left-hand side is

$$\text{LHS} = \frac{x+\frac{1}{2}}{\frac{3}{5}} = \frac{-\frac{3}{10}+\frac{1}{2}}{\frac{3}{5}} = \frac{\frac{-3+5}{10}}{\frac{3}{5}} = \frac{\frac{2}{10}}{\frac{3}{5}} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{5}\cdot\frac{5}{3} = \frac{1}{3} = \text{RHS}$$

Thus our solution, $x = \boxed{-\frac{3}{10}}$ is correct.

Example 2. Solve each of the given equations. Make sure to check your solutions.

a) $\frac{1}{2}m-1=\frac{5}{4}m-\frac{1}{4}$ b) $\frac{2}{3}x-4-\frac{1}{6}(x+6)=\frac{1}{2}(x-10)$ c) $\frac{2}{3}(x-1)-\frac{1}{2}\left(x+\frac{3}{5}\right)=-x+\frac{37}{10}$

Solution:

a)

$$\frac{1}{2}m-1=\frac{5}{4}m-\frac{1}{4} \qquad \text{subtract } \frac{1}{2}m \qquad \text{margin work: } \frac{5}{4}-\frac{1}{2}=\frac{5}{4}-\frac{2}{4}=\frac{3}{4}$$

$$-1=\frac{3}{4}m-\frac{1}{4} \qquad \text{add } \frac{1}{4} \qquad -1+\frac{1}{4}=\frac{-4}{4}+\frac{1}{4}=-\frac{3}{4}$$

$$-\frac{3}{4}=\frac{3}{4}m \qquad \text{divide by } \frac{3}{4} \qquad -\frac{3}{4}\div\frac{3}{4}=-\frac{3}{4}\cdot\frac{4}{3}=-1$$

$$-1=m$$

So the only solution of this equation is -1. We check; if $m=-1$,

$$\text{LHS} = \frac{1}{2}(-1)-1=-\frac{1}{2}-1=\frac{-1}{2}-\frac{2}{2}=-\frac{3}{2} \text{ and}$$

$$\text{RHS} = \frac{5}{4}(-1)-\frac{1}{4}=-\frac{5}{4}-\frac{1}{4}=-\frac{6}{4}=-\frac{3}{2} \qquad \Longrightarrow \qquad \text{LHS} = \text{RHS}$$

So our solution, $\boxed{m=-1}$ is correct.

b) We first eliminate the parentheses by applying the distributive law.

$$\frac{2}{3}x-4-\frac{1}{6}(x+6) = \frac{1}{2}(x-10) \qquad \text{eliminate parentheses}$$

$$\frac{2}{3}x-4-\frac{1}{6}x-1 = \frac{1}{2}x-5 \qquad \text{combine like terms} \qquad \text{margin work: } \frac{2}{3}-\frac{1}{6}=\frac{4-1}{6}=\frac{3}{6}=\frac{1}{2}$$

$$\frac{1}{2}x-5 = \frac{1}{2}x-5 \qquad \text{subtract } \frac{1}{2}x$$

$$-5 = -5$$

When we tried to eliminate the unknown from one side, it disappeared again from both sides. We are left with the statement $-5=-5$. No matter what the value of the unknown is, this statement is always true. Indeed, our last line is an **unconditionally true statement**. This means that every number makes make this statement true, and so the solution set of this equation is the set of all numbers. An equation like this is called an **identity**, and all real numbers are solutions of it.

c) There are several methods available. The method presented here is focusing on how this equation requires the same steps as before, only, each step will take more work. The computations for each step are shown separate, on the margin. We start by distributing $\frac{2}{3}$ and $-\frac{1}{2}$.

$$\frac{2}{3}(x-1)-\frac{1}{2}\left(x+\frac{3}{5}\right)=-x+\frac{37}{10} \qquad \text{distribute} \quad -\frac{1}{2}\cdot\frac{3}{5}=-\frac{3}{10}$$

$$\frac{2}{3}x-\frac{2}{3}-\frac{1}{2}x-\frac{3}{10}=-x+\frac{37}{10} \qquad \text{combine like terms:} \quad \frac{2}{3}x-\frac{1}{2}x=\frac{4-3}{6}x=\frac{1}{6}x$$

$$\text{and } -\frac{2}{3}-\frac{3}{10}=\frac{-20-9}{30}=-\frac{29}{30}$$

$$\frac{1}{6}x-\frac{29}{30}=-x+\frac{37}{10} \qquad \text{add } x \quad \frac{1}{6}+1=\frac{1}{6}+\frac{6}{6}=\frac{7}{6}$$

$$\frac{7}{6}x-\frac{29}{30}=\frac{37}{10} \qquad \text{add } \frac{29}{30}: \quad \frac{37}{10}+\frac{29}{30}=\frac{111+29}{30}=\frac{140}{30}=\frac{14}{3}$$

$$\frac{7}{6}x=\frac{14}{3} \qquad \text{divide by } \frac{7}{6}: \quad \frac{14}{3}\div\frac{7}{6}=\frac{14}{3}\cdot\frac{6}{7}=\frac{2\cdot 7\cdot 3\cdot 2}{3\cdot 7}=\frac{4}{1}$$

$$x=4$$

We check: if $x=4$, then

$$\text{LHS} = \frac{2}{3}(4-1)-\frac{1}{2}\left(4+\frac{3}{5}\right)=\frac{2}{3}\cdot 3-\frac{1}{2}\left(\frac{20}{5}+\frac{3}{5}\right)=2-\frac{1}{2}\cdot\frac{23}{5}=2-\frac{23}{10}=\frac{20}{10}-\frac{23}{10}=\frac{-3}{10}$$

$$\text{RHS} = -4+\frac{37}{10}=-\frac{40}{10}+\frac{37}{10}=-\frac{3}{10}$$

and so our solution, $\boxed{x=4}$ is correct.

Part 2 - Linear Equations after Multiplying Expressions

Now that we can multiply algebraic expressions, we can solve equations that include such products.

Example 3. Solve the given equation. Make sure to check your solution.

$$(2x-3)^2-(x+1)(3x-5)=11-(x-1)(3-x)$$

Solution: We carefully expand the indicated products and combine like terms. Notice that even after we expanded $(x+1)(3x-5)$ and $(x-1)(3-x)$, we still need to keep them in parentheses because we are subtracting them. We will first work out the products.

$$\begin{aligned}(2x-3)^2 &= (2x-3)(2x-3)=4x^2-6x-6x+9=4x^2-12x+9\\ (x+1)(3x-5) &= 3x^2-5x+3x-5=3x^2-2x-5\\ (x-1)(3-x) &= 3x-x^2-3+x=-x^2+4x-3\end{aligned}$$

We are now ready to begin solving the equation.

$$\begin{aligned}(2x-3)^2-(x+1)(3x-5) &= 11-(x-1)(3-x)\\ 4x^2-12x+9-\left(3x^2-2x-5\right) &= 11-\left(-x^2+4x-3\right) \qquad \text{to subtract is to add the opposite}\end{aligned}$$

$$\begin{aligned}
4x^2-12x+9-3x^2+2x+5 &= 11+x^2-4x+3 && \text{combine like terms}\\
x^2-10x+14 &= x^2-4x+14 && \text{subtract } x^2\\
-10x+14 &= -4x+14 && \text{add } 10x\\
14 &= 6x+14 && \text{subtract } 14\\
0 &= 6x && \text{divide by } 6\\
0 &= x
\end{aligned}$$

We check: if $x=0$, then

$$\begin{aligned}
\text{LHS} &= (2\cdot 0-3)^2-(0+1)(3\cdot 0-5)=(-3)^2-1(-5)=9+5=14\\
\text{RHS} &= 11-(0-1)(3-0)=11-(-1)3=11+3=14
\end{aligned}$$

and so our solution, $\boxed{x=0}$ is correct.

Example 4. The area of a square would increase by 93 unit^2 if we increased each of its sides by 3 units. How long are the sides of the square?

Solution: Let us denote the side of the square by x. Then its area is x^2. If we increase its sides by 3 units, the new side would by $x+3$, and the new area $(x+3)^2$. The equation would express the difference between the two areas.

$$\begin{aligned}
(x+3)^2 &= x^2+93\\
x^2+6x+9 &= x^2+93 && \text{subtract } x^2\\
6x+9 &= 93 && \text{subtract } 9\\
6x &= 84 && \text{divide by } 6\\
x &= 14
\end{aligned}$$

Thus the square has sides $\boxed{\text{14 units long}}$. We check: The area of the smaller square is $14^2=196$ unit^2. If we increase the sides by 3 units, they would be 17 units. Then the area if $17^2=289$ unit^2. The difference between the two areas is indeed 93unit^2, as $289-196=93$. Thus our solution is correct.

Sample Problems

Solve each of the following equations. Make sure to check your solutions.

1. $3y-9=-2y+4$
2. $4-x=3(x-7)$
3. $\frac{2}{3}(x-1)=\frac{3}{5}(x-4)+1$
4. $\frac{2}{3}(x-7)=\frac{4}{5}(x+1)$
5. $(x-3)^2-(2x-5)(x+1)=5-(x-1)^2$
6. $(x+1)^2-(2x-1)^2+(3x)^2=6x(x-2)$
7. $12-(2p-1)(p+1)=-2(-p+5)^2$

Practice Problems

Solve each of the following equations. Make sure to check your solutions.

1. $-x+12=2x+1$
2. $x+3=6x-28$
3. $-7x-5=3x-11$
4. $\frac{3}{8}x-\frac{1}{2}=-\frac{1}{4}$
5. $\frac{3}{8}x+1\frac{4}{5}=\frac{1}{4}x+1\frac{3}{10}$
6. $\frac{2}{3}x-1=-\frac{2}{3}\left(x+\frac{1}{2}\right)$
7. $\frac{1}{3}\left(2x-\frac{3}{5}\right)-\frac{2}{5}\left(x-\frac{2}{3}\right)=\frac{4}{15}(x+1)$
8. $-\frac{1}{2}\left(3x+\frac{1}{3}\right)+\frac{2}{3}\left(5x-\frac{1}{4}\right)=2x+\frac{1}{6}$
9. $2x(3x-1)-x(5x-2)=(x-1)^2$
10. $y^2-(y-1)^2+(y-2)^2=(y-3)(y-5)$
11. $(3x)^2-(x+3)(5x-3)=(5-2x)^2-16$
12. $(w+4)(1-2w)=3w-2(w-3)^2$
13. $(2x-3)^2-3(x-2)^2=10-(x-2)(7-x)$
14. $(2-w)^2-(2w-3)^2+7=(w-2)(5-3w)$
15. $3(a+11)-a(8-3a)=3(a-2)^2$
16. $-5(2x-1)-(4-x)^2=3-(x+1)^2$
17. $5(-3-x)-3x(x-2)=x-3(x+2)(x-5)$
18. $2(-m-2)^2-(m-2)^2=8m+(m+2)^2$
19. $(3a-5)(2-a)-(2a-1)(a+3)=-5a^2-7$
20. $\frac{1}{2}(x-3)^2-\frac{1}{2}(x+1)^2=4(x-7)$
21. The area of a square would increase by 14 unit^2 if we increased each of its sides by 2 units. How long are the sides of the square?

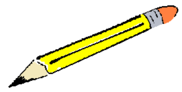
Answers

Sample Problems

1. $\frac{13}{5}$ 2. $\frac{25}{4}$ 3. -11 4. -41 5. 2 6. 0 7. 3

Practice Problems

1. $\frac{11}{3}$ 2. $\frac{31}{5}$ 3. $\frac{3}{5}$ 4. $\frac{2}{3}$ 5. -4 6. $\frac{1}{2}$ 7. no solution 8. -3 9. $\frac{1}{2}$ 10. 2 11. 0 12. 1
13. 3 14. 4 15. -3 16. there is no solution 17. -5 18. all real numbers are solution 19. 0
20. 4 21. $\frac{5}{2}$ units

Sample Problems  Solutions

1. $3y-9=-2y+4$

   Solution:

$$\begin{aligned} 3y-9 &= -2y+4 && \text{add } 2y \text{ to both sides} \\ 5y-9 &= 4 && \text{add } 9 \text{ to both sides} \\ 5y &= 13 && \text{divide both sides by } 5 \\ y &= \frac{13}{5} \end{aligned}$$

We check. If $y=\frac{13}{5}$, then

$$\begin{aligned} \text{LHS} &= 3\left(\frac{13}{5}\right)-9=\frac{3}{1}\cdot\frac{13}{5}-9=\frac{39}{5}-\frac{9}{1}=\frac{39}{5}-\frac{45}{5}=\frac{-6}{5}=-\frac{6}{5} \\ \text{RHS} &= -2\left(\frac{13}{5}\right)+4=\frac{-2}{1}\cdot\frac{13}{5}+\frac{4}{1}=\frac{-26}{5}+\frac{20}{5}=\frac{-6}{5}=-\frac{6}{5} \end{aligned}$$

Thus $\boxed{y=\frac{13}{5}}$ is the correct solution.

2. $4-x=3(x-7)$

   Solution: We first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 4-x &= 3(x-7) && \text{distribute } 3 \\ 4-x &= 3x-21 && \text{add } x \text{ to both sides} \\ 4 &= 4x-21 && \text{add } 21 \text{ to both sides} \\ 25 &= 4x && \text{divide both sides by } 4 \\ \frac{25}{4} &= x \end{aligned}$$

We check. If $x=\frac{25}{4}$, then

$$\begin{aligned} \text{LHS} &= 4-x=4-\frac{25}{4}=\frac{4}{1}-\frac{25}{4}=\frac{16}{4}-\frac{25}{4}=\frac{16-25}{4}=\frac{-9}{4}=-\frac{9}{4} \\ \text{RHS} &= 3(x-7)=3\left(\frac{25}{4}-7\right)=3\left(\frac{25}{4}-\frac{7}{1}\right)=3\left(\frac{25}{4}-\frac{28}{4}\right)=3\left(\frac{25-28}{4}\right) \\ &= 3\left(\frac{-3}{4}\right)=\frac{3}{1}\cdot\frac{-3}{4}=\frac{-9}{4}=-\frac{9}{4} \end{aligned}$$

Thus our solution, $\boxed{x=\frac{25}{4}}$ is correct.

3. $\dfrac{2}{3}(x-1)=\dfrac{3}{5}(x-4)+1$

Solution:

$$\frac{2}{3}(x-1)=\frac{3}{5}(x-4)+1 \qquad \text{distribute} \quad \frac{3}{5}\cdot(-4)=\frac{-12}{5}$$

$$\frac{2}{3}x-\frac{2}{3}=\frac{3}{5}x-\frac{12}{5}+1 \qquad \text{combine like terms:} \quad -\frac{12}{5}+1=\frac{-12}{5}+\frac{5}{5}=-\frac{7}{5}$$

$$\frac{2}{3}x-\frac{2}{3}=\frac{3}{5}x-\frac{7}{5} \qquad \text{subtract } \frac{3}{5}x \quad \frac{2}{3}x-\frac{3}{5}x=\frac{10-9}{15}x=\frac{1}{15}x$$

$$\frac{1}{15}x-\frac{2}{3}=-\frac{7}{5} \qquad \text{add } \frac{2}{3} \quad -\frac{7}{5}+\frac{2}{3}=\frac{-21+10}{15}=-\frac{11}{15}$$

$$\frac{1}{15}x=-\frac{11}{15} \qquad \text{divide by } \frac{1}{15} \quad -\frac{11}{15}\div\frac{1}{15}=\frac{-11}{15}\cdot\frac{15}{1}=-11$$

We check. If $x=-11$, then

$$\begin{aligned}\text{LHS} &= \frac{2}{3}(-11-1)=\frac{2}{3}(-12)=-8\\ \text{RHS} &= \frac{3}{5}(-11-4)+1=\frac{3}{5}(-15)+1=-9+1=-8\end{aligned}$$

Thus our solution, $\boxed{x=-11}$ is correct.

4. $\dfrac{2}{3}(x-7)=\dfrac{4}{5}(x+1)$

Solution:

$$\frac{2}{3}(x-7)=\frac{4}{5}(x+1) \qquad \text{distribute}$$

$$\frac{2}{3}x-\frac{14}{3}=\frac{4}{5}x+\frac{4}{5} \qquad \text{subtract } \frac{2}{3}x \quad \frac{4}{5}x-\frac{2}{3}x=\frac{12-10}{15}x=\frac{2}{15}x$$

$$-\frac{14}{3}=\frac{2}{15}x+\frac{4}{5} \qquad \text{subtract } \frac{4}{5} \quad -\frac{14}{3}-\frac{4}{5}=\frac{-70-12}{15}=-\frac{82}{15}$$

$$-\frac{82}{15}=\frac{2}{15}x \qquad \text{divide by } \frac{2}{15} \quad -\frac{82}{15}\div\frac{2}{15}=\frac{-82}{15}\cdot\frac{15}{2}=-41$$

$$-41=x$$

We check:

$$\text{LHS}=\frac{2}{3}(-41-7)=\frac{2}{3}(-48)=-32 \quad \text{and} \quad \text{RHS}=\frac{4}{5}(-41+1)=\frac{4}{5}(-40)=-32$$

Thus our solution, $\boxed{x=-41}$ is correct.

5. $(x-3)^2-(2x-5)(x+1)=5-(x-1)^2$

Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must keep the parentheses.

$$\begin{aligned}
(x-3)^2-(2x-5)(x+1) &= 5-(x-1)^2 \\
x^2-3x-3x+9-\left(2x^2+2x-5x-5\right) &= 5-\left(x^2-x-x+1\right) && \text{combine like terms} \\
x^2-6x+9-\left(2x^2-3x-5\right) &= 5-\left(x^2-2x+1\right) && \text{distribute} \\
x^2-6x+9-2x^2+3x+5 &= 5-x^2+2x-1 && \text{combine like terms} \\
-x^2-3x+14 &= -x^2+2x+4 && \text{add } x^2 \\
-3x+14 &= 2x+4 && \text{add } 3x \\
14 &= 5x+4 && \text{subtract 4} \\
10 &= 5x && \text{divide by 5} \\
2 &= x
\end{aligned}$$

We check. If $x=2$, then

$$\begin{aligned}
\text{LHS} &= (2-3)^2-(2\cdot 2-5)(2+1)=(-1)^2-(4-5)(2+1)=(-1)^2-(-1)\cdot 3=1-(-3)=4 \\
\text{RHS} &= 5-(2-1)^2=5-1^2=5-1=4
\end{aligned}$$

Thus $\boxed{x=2}$ is indeed the solution.

6. $(x+1)^2-(2x-1)^2+(3x)^2=6x(x-2)$

Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must keep the parentheses.

$$\begin{aligned}
(x+1)^2-(2x-1)^2+(3x)^2 &= 6x(x-2) \\
x^2+x+x+1-\left(4x^2-2x-2x+1\right)+9x^2 &= 6x^2-12x \\
x^2+2x+1-\left(4x^2-4x+1\right)+9x^2 &= 6x^2-12x && \text{distribute} \\
x^2+2x+1-4x^2+4x-1+9x^2 &= 6x^2-12x && \text{combine like terms} \\
6x^2+6x &= 6x^2-12x && \text{subtract } 6x^2 \\
6x &= -12x && \text{add } 12x \\
18x &= 0 && \text{divide by 18} \\
x &= 0
\end{aligned}$$

We check. If $x=0$, then

$$\begin{aligned}
\text{LHS} &= (0+1)^2-(2\cdot 0-1)^2+(3\cdot 0)^2=1^2-(-1)^2+(0)^2=1-1+0=0 \\
\text{RHS} &= 6\cdot 0\cdot(0-2)=6\cdot 0\cdot(-2)=0
\end{aligned}$$

Thus $\boxed{x=0}$ is indeed the solution.

7. $12-(2p-1)(p+1)=-2(-p+5)^2$

Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must keep the parentheses.

$$\begin{aligned}
12-(2p-1)(p+1) &= -2(-p+5)^2 \\
12-\left(2p^2+2p-p-1\right) &= -2\left(p^2-5p-5p+25\right) && \text{combine like terms} \\
12-\left(2p^2+p-1\right) &= -2\left(p^2-10p+25\right) && \text{distribute} \\
12-2p^2-p+1 &= -2p^2+20p-50 && \text{combine like terms} \\
-2p^2-p+13 &= -2p^2+20p-50 && \text{add } 2p^2 \\
-p+13 &= 20p-50 && \text{add } p \\
13 &= 21p-50 && \text{add } 50 \\
63 &= 21p && \text{divide by } 21 \\
3 &= p
\end{aligned}$$

We check. If $p=3$, then

$$\begin{aligned}
\text{LHS} &= 12-(2\cdot 3-1)(3+1)=12-(6-1)(3+1)=12-5\cdot 4=12-20=-8 \\
\text{RHS} &= -2(-3+5)^2=-2\cdot 2^2=-2\cdot 4=-8
\end{aligned}$$

Thus $\boxed{p=3}$ is indeed the solution.

