This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find each of the following sets.

   a) $A = \{x \in U : x > 6 \text{ or } x \text{ is odd}\}$
   b) $B = \{x \in U : x > 6 \text{ and } x \text{ is odd}\}$
   c) $C = \{x \in U : x \text{ is divisible by 2 or by 3}\}$
   d) $D = \{x \in U : x \text{ is divisible by 2 and by 3}\}$

2. Suppose that $A = \{1, 2, 4, 8, 9\}$, $B = \{1, 4, 5, 7, 9, 10\}$, and $C = \{2, 3, 4, 7, 9\}$. Find each of the following sets.
   a) $A \cap B$ b) $A \cap C$ c) $B \cap C$ d) $A \cap (B \cup C)$ e) $(A \cap B) \cup C$ f) $A \cup (B \cap C)$

3. Given a Venn diagram for A, B, and C, shade the region corresponding to each of the given sets.
   a) $(A \cap B) \cap C$ b) $A \cap (B \cup C)$ c) $(A \cap B) \cup C$ d) $A \cup (B \cap C)$

4. Label each of the following statements as true or false.

   a) If n is an integer such that n^2 is divisible by 12, then n is divisible by 12.
   b) If integer a is divisible by 8 and integer b is divisible by 6, then the product ab is divisible by 48.
   c) If integer n is divisible by 8 and by 6, then it is also divisible by 48.
   d) If the product xy of two integers is divisible by 5, then x is divisible by 5 or y is divisible by 5.
   e) If the product xy of two integers is divisible by 6, then x is divisible by 6 or y is divisible by 6.
   f) The sum of two prime numbers is never prime.
   g) The product of two prime numbers is never prime.
   h) The sum of the first five prime numbers is 28.
   i) Every square is a rectangle.
   j) For every set A and B, $A \subseteq B$ or $B \subseteq A$.

5. Consider the given numbers. 2356 431, 910 190 198, 6760 233, 34 906 355, and 651 168.
   List all numbers from the list that are divisible by: a) 9 b) 11 c) 99

6. Find the prime factorization for each of the following numbers.
   a) 4500 b) 1001 c) 7986 d) 120^{120}

7. Suppose that $n = 2^{30} \cdot 3^{50} \cdot 5 \cdot 11^{100}$. What is the prime-factorization of n^3?

8. Find the digit a in the six-digit number $931a92$ if we know that the number is divisible by 11.

9. Find the last digit of $2^{41} + 2^{42} + 2^{43} + 2^{44}$.

10. Re-write each of the following.

    a) 60% as a reduced fraction
    b) $\frac{12}{25}$ as a percent
    c) $\frac{3}{8}$ with a denominator 32
    d) $\frac{3}{8}$ with a numerator of 21
    e) $1.28 \cdot 10^5$ in regular notation
    f) 480 000 000 000 in scientific notation.

11. Evaluate each of the following.
    a) $|3 - 4| - |6 - 9|$ b) $3 - |4 - 6| - 9$ c) $3\,|-4 - 6| - 9$ d) $3 - 4\,|-6 - 9|$ e) $3 - |4 - |6 - 9||$

12. Evaluate each of the following.
    a) $\left(-4^2 - 3\left(4 - (5 - 8)^2\right)\right)^3$ b) $\dfrac{2 - 3\left(5 - 3^2\right)}{2^3 - (-1)^4}$ c) $\sqrt{7\sqrt{5\sqrt{2 \cdot 5 - 1} + 1} - 3}$

13. Simplify each of the following.
    a) $\sqrt[3]{-8}$ b) $\left(\sqrt[3]{-8}\right)^3$ c) $\sqrt[4]{16}$ d) $-\sqrt[5]{-1}$ e) $\left(\sqrt[4]{2}\right)^{12}$ f) $\left(\sqrt[4]{-2}\right)^{12}$ g) $\left(\sqrt[3]{-2}\right)^{12}$

14. Simplify each of the following expressions.

a) $\dfrac{\left(-2x^4\right)^3 x^5}{\left(-x^3\right)^2}$ b) $\dfrac{x\left(2x\right)^2 \cdot x^3 \cdot 2x^4}{\left(x \cdot 2x^2\right)^3}$ c) $\left(-\left(-x\right)\left(-x^2\right)\left(-x\right)^4\right)^2$ d) $\left(-\left(-x\right)\left(-x^2\right)\left(-x\right)^4\right)^3$

15. Suppose that $x = 8 \cdot 10^{18}$ and $y = 2.5 \cdot 10^5$. Compute each of the following. Present your answer using scientific notation. a) xy b) $\dfrac{x}{y}$ c) $\dfrac{x}{y^2}$ d) $\dfrac{x}{y^3}$

16. Consider the expressions $3 \cdot 2^x$ and 6^x. Evaluate both expressions if a) $x = 1$ b) $x = 2$ c) $x = 3$

17. Consider the expression $\dfrac{2^{n+3} \cdot 5^{n+1}}{10^n}$.

a) Evaluate the expression for each of the values given for n. i) $n = 1$ ii) $n = 2$ iii) $n = 3$

b) Can you explain what you noticed in part a)?

18. Simplify each of the following expressions.

a) $\dfrac{2^x \cdot 5^{x+2}}{10^x}$ b) $\dfrac{3^{x+2} \cdot 4^{x+1}}{6^x \cdot 2^{x+1}}$ c) $\dfrac{8^{x+2}}{2^{3x+1}}$ d) $7 \cdot 3^x - 5 \cdot 3^x + 4 \cdot 3^x$

19. Simplify each of the following expressions.

a) $\left(x^6\right)^2$ b) $\sqrt{x^{12}}$ c) $\sqrt[3]{x^{12}}$ d) $\left(x^5\right)^3$ e) $\sqrt[3]{x^{15}}$ f) $\sqrt{x^{100}}$ g) $\sqrt[5]{x^{100}}$

20. Suppose that we denote 5^{100} by A. Express each of the following in terms of A.

a) 5^{101} b) 25^{100} c) 5^{99} d) $5^{102} - 5^{101}$ e) 5^{50}

21. Solve each of the following equations. Make sure to check your solutions.

a) $5\left(x-2\right) - 3\left(2x+1\right) = -7 - 3\left(x-4\right)$

b) $m + 3\left(4-m\right) = 7 - 2\left(m-3\right)$

c) $3\left(a-1\right) - 3a - 1 = -2\left(3a-1\right)$

d) $5\left(x+1\right) - 3\left(x-2\right) = 5\left(2x-1\right)$

e) $2\left(5x-1\right) - 3\left(-2x+5\right) = 1 + 6\left(x-3\right)$

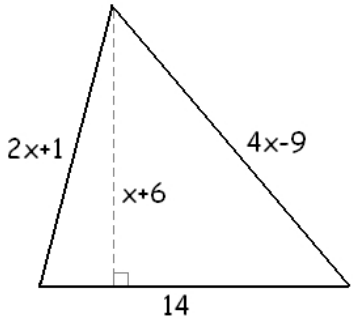
f) $-4 - 5\left(1-2x\right) = 1 + 10\left(x-1\right)$

g) $3\left(\dfrac{\dfrac{5x+1}{4}+3}{-2}+1\right) = 15$

h) $\dfrac{\dfrac{\dfrac{x-1}{3}+1}{2}-1}{5} = -2$

22. Compute the perimeter and area of the right triangle with sides $8\,\text{cm}$, $17\,\text{cm}$, and $15\,\text{cm}$. Include units in your computation and answer.

23. Find the value of x if we know that the triangle shown on the picture has a perimeter A unit and area $2A$ unit^2.



24. The opposite of a number is five less than twice the difference of the number and eight. Find this number.

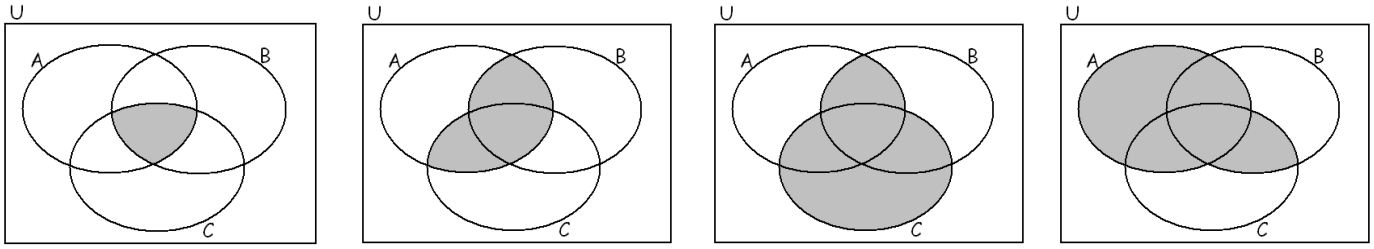
25. There were a lot of coins in that jar, all quarters and dimes. The number of dimes was two less than five times the number of quarters. How many of each coins were there if all the coins in the jar were worth 8 dollars and 80 cents? (Hint: think in terms of cents)

Answers

1. a) $\{1,3,5,7,8,9,10\}$ b) $\{7,9\}$ c) $\{2,3,4,6,8,9\}$ d) $\{6\}$

2. a) $\{1,4,9\}$ b) $\{2,4,9\}$ c) $\{4,7,9\}$ d) $\{1,2,4,9\}$ e) $\{1,2,3,4,7,9\}$ f) $\{1,2,4,7,8,9\}$

3. a) b) c) d)



4. a) false b) true c) false d) true e) false f) false g) true h) true i) true j) false

5. a) 5943 564 297, 6760 233, 651 168 b) 2356 431, 5943 564 297, 34 906 355 c) 5943 564 297

6. a) $2^2 \cdot 3^2 \cdot 5^3$ b) $7 \cdot 11 \cdot 13$ c) $2 \cdot 3 \cdot 11^3$ d) $2^{360} \cdot 3^{120} \cdot 5^{120}$ 7. $2^{90} \cdot 3^{150} \cdot 5^3 \cdot 11^{300}$

8. 3 9. 0 10. a) $\frac{3}{5}$ b) 48% c) $\frac{12}{32}$ d) $\frac{21}{56}$ e) 128 000 f) $4.8 \cdot 10^{11}$

11. a) -2 b) -8 c) 21 d) -57 e) 2 13. a) -1 b) 2 c) 5

14. a) -2 b) -8 c) 2 d) 1 e) 8 f) undefined g) 16 15. a) $-8x^{11}$ b) x c) x^{14} d) $-x^{21}$

15. a) $2 \cdot 10^{24}$ b) $3.2 \cdot 10^{13}$ c) $1.28 \cdot 10^8$ d) $5.12 \cdot 10^2$

16. a) $3 \cdot 2^1 = 6$ and $6^1 = 6$ b) $3 \cdot 2^2 = 12$ and $6^2 = 36$ c) $3 \cdot 2^3 = 24$ and $6^3 = 216$

17. a) i) $\frac{2^4 \cdot 5^2}{10^1} = 40$ ii) $\frac{2^5 \cdot 5^3}{10^2} = 40$ iii) $\frac{2^6 \cdot 5^4}{10^3} = 40$

b) If n is part of an algebraic expression and the expression has the same value for every n, then somehow the value of n doesn't matter. This usually indicates some sort of cancellation.

$$\frac{2^{n+3} \cdot 5^{n+1}}{10^n} = \frac{\left(2^n \cdot 2^3\right)\left(5^n \cdot 5^1\right)}{10^n} = \frac{2^n \cdot 8 \cdot 5^n \cdot 5}{10^n} = \frac{\left(2^n \cdot 5^n\right)(8 \cdot 5)}{10^n} = \frac{10^n \cdot 40}{10^n} = 40$$

18. a) 25 b) 18 c) 32 d) $6 \cdot 3^x$ 19. a) x^{12} b) x^6 c) x^4 d) x^{15} e) x^5 f) x^{50} g) x^{20}

20. a) $5A$ b) A^2 c) $\frac{A}{5}$ d) $20A$ f) $\sqrt{A}$

21. a) 9 b) no solution c) 1 d) 2 e) 0 f) all real numbers g) -9 h) -56

22. $P = 40\,\text{cm}$ $A = 60\,\text{cm}^2$ 23. $x = 6$ 24. 6 25. 12 quarters and 58 dimes

